P3

1/2

a) Discos de diámetro D, separación e (líquido de viscosidad μ)
veloc. angular ω_0.

$dT = r\,dF$; $dF = \tau\,dA$; $dA = r\,dr\,d\theta$ (elemento de área en disco, coords polares)

$\Rightarrow dT = r^2 \tau\, dr\, d\theta$

Ley de Newton-Navier: $\tau = \mu \dfrac{du}{dz} = \mu \dfrac{r\omega_0}{e}$ (perfil lineal válido por tratarse de e pequeño)

$\Rightarrow dT = r^2 \cdot \dfrac{\mu \cdot r \cdot \omega_0}{e} \cdot dr\, d\theta$

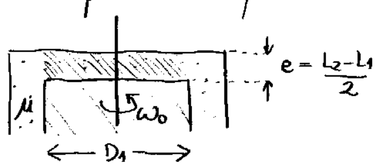
$dT = \dfrac{\mu r^3 \omega_0}{e}\, dr\, d\theta$ ← Integrando en la superficie del disco..

$$\int_\Omega dT = \int_0^{2\pi}\int_0^{D/2} r^3 \cdot dr \cdot d\theta \cdot \frac{\mu \cdot \omega_0}{e}$$

$$T = 2\pi \left.\frac{r^4}{4}\right|_0^{D/2} \cdot \frac{\mu \cdot \omega_0}{e} = \frac{2\pi}{4}\frac{D^4}{16} \cdot \frac{\mu\,\omega_0}{e}$$

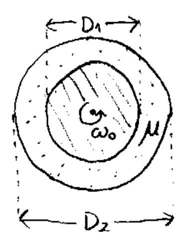
$$\boxed{T = \frac{\pi}{32}\frac{D^4 \mu \cdot \omega_0}{e}}$$

b) El problema se separa en 2 partes: "Tapas" (discos paralelos como en la parte 'a') y "manto"



$$\boxed{T_{Tapa} = \frac{\pi}{32}\frac{D_1^4 \cdot \mu \cdot \omega_0}{e}}$$

Manto:



$dT = r\,dF$, $dF = \tau\,dA$

$dA = r\,d\theta\,dz$ (elemento de área en cilindro)

$\Rightarrow dT = r^2 \tau\, d\theta\, dz$

Ley de Newton en coords. polares
$\tau = \mu r \dfrac{d\omega}{dr}$

$\Rightarrow dT = r^2 \cdot \left(\mu \cdot r \dfrac{d\omega}{dr}\right) d\theta\, dz$ ← Integrando en el manto de un cilindro de radio r...

$$\int_\Omega dT = \mu r^3 \frac{d\omega}{dr} \int_0^{L_1}\int_0^{2\pi} d\theta\, dz$$

$$T = \mu r^3 \frac{d\omega}{dr} L_1 \cdot 2\pi$$

Separando variables:

$$T\,\frac{dr}{r^3} = \mu \cdot L_1 \cdot 2\pi\, d\omega$$

Debe integrarse entre los cilindros; a diferencia de los discos paralelos, en este caso ω varía con r.

Condiciones de borde: $r = \frac{D_1}{2} \Rightarrow \omega = \omega_0$

$r = \frac{D_2}{2} \Rightarrow \omega = 0$

$$\int_{D_1/2}^{D_2/2} T\,\frac{dr}{r^3} = \mu \cdot L_1 \cdot 2\pi \int_{\omega_0}^{0} d\omega$$

$$T \cdot \left(\frac{-1}{2r^2}\right)\Bigg|_{D_1/2}^{D_2/2} = -\mu \cdot L_1 \cdot 2\pi\,\omega_0$$

$$T\left(\frac{2}{D_1^2} - \frac{2}{D_2^2}\right) = -\mu \cdot L_1 \cdot 2\pi\,\omega_0$$

$$\Rightarrow \boxed{T_{manto} = \frac{\mu\, L_1 \cdot \pi\,\omega_0}{\frac{1}{D_2^2} - \frac{1}{D_1^2}}}$$

Torque total = Tmanto + 2 Ttapa

$$T_{tot} = \left|\frac{\mu\, L_1 \cdot \pi\,\omega_0}{\frac{1}{D_2^2} - \frac{1}{D_1^2}}\right| + 2\left|\frac{\pi}{32}\,\frac{D_1^4\,\mu\,\omega_0}{e}\right|$$

(Tomando módulo para asegurar la suma de torques resistivos)

$$T_{tot} = a \cdot \omega_0 \qquad a = cte$$

Inicialmente $T_{tot} = T_0 = T_r \Rightarrow \omega_0 = cte.$

Situación de régimen impermanente:

$$\Sigma T = I\,\frac{d\omega}{dt} \qquad \Sigma T = -T_r$$

(sólo actúa el torque resistivo, no hay torque motriz externo)

$$-T_r = -a \cdot \omega = I \cdot \frac{d\omega}{dt}$$

Separando variables:

$$-a\,dt = I\,\frac{d\omega}{\omega}$$

cond. de borde: $t=0 \quad \omega = \omega_0$

$t = t^* \quad \omega = \omega_0/2$

$$-a\int_0^{t^*} dt = I\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega}$$

$$-a\,t^* = I\,\ln\omega\Big|_{\omega_0}^{\omega_0/2} = I\left[\ln\frac{\omega_0}{2} - \ln\omega_0\right] = I\,\ln\left(\frac{1}{2}\right)$$

$$-a\,t^* = -I\,\ln 2 \quad \Rightarrow \quad t^* = \frac{I}{a}\,\ln 2$$

$$I = \frac{1}{2} M D_1^2 = 0{,}1125\ [Kg \cdot m^2] \ ; \quad a = 0{,}01133\left[\frac{Kg \cdot m^2}{s}\right]$$

$$\Rightarrow \boxed{t^* = 6{,}88\ [seg]}$$