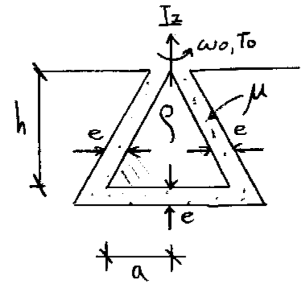
P1)



Se dividirá el problema en dos partes: manto y fondo

Fondo: (2 discos paralelos separados por un espesor e)

$dT = r\,dF$, $dF = \tau\, dA$; $dA = r\,dr\,d\theta$

$\Rightarrow dT = r^2 \tau\, dr\, d\theta$

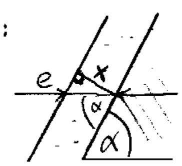
Ley de Newton-Navier: $\tau = \mu \dfrac{du}{dz} = \mu r \dfrac{\omega_0}{e}$

$$\Rightarrow dT = r^2 \frac{\mu \cdot r \cdot \omega_0}{e}\, dr\, d\theta$$

$$\int_\Omega dT = \int_0^{2\pi}\int_0^{a} r^3\, dr\, d\theta \cdot \frac{\mu \cdot \omega_0}{e}$$

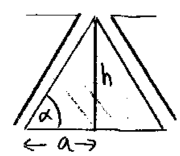
$$T_{fondo} = 2\pi \frac{a^4}{4} \cdot \frac{\mu \omega_0}{e} \Rightarrow \boxed{T_{fondo} = \frac{\pi a^4}{2} \frac{\mu \omega_0}{e}}$$

Manto:



$\operatorname{sen}\alpha = \dfrac{x}{e}$

$\operatorname{sen}\alpha = \dfrac{h}{\sqrt{a^2+h^2}}$



$dT = r\,dF$; $dF = \tau \cdot dA$; $\tau = \mu r \dfrac{d\omega}{dr}$

$dT = r\,\tau\, dA$ con $dA = r\, d\theta\, dz$

$dT = r^2 \tau\, d\theta\, dz = \mu r^3 \dfrac{d\omega}{dr}\, d\theta\, dz$

Notar que $\dfrac{r}{z} = \dfrac{a}{h} \Rightarrow dz = \dfrac{h}{a}\, dr$

$$\frac{d\omega}{dr} = \frac{\omega_0}{e \cdot \operatorname{sen}\alpha}$$

$$\int_\Omega dT = \int_0^{a}\int_0^{2\pi} \mu \frac{h}{a} \frac{\omega_0}{e \cdot \operatorname{sen}\alpha} r^3\, d\theta\, dr$$

$$T_{manto} = \frac{\mu \cdot \cancel{h}}{a} \frac{\omega_0}{e\, \cancel{h}} \sqrt{a^2+h^2} \cdot 2\pi \cdot \frac{a^4}{4}$$

$$= \frac{\mu \sqrt{a^2+h^2} \cdot \pi a^3}{2e}\, \omega_0$$

$$\Rightarrow \text{Torque total} = T_{fondo} + T_{manto} = \omega_0 \cdot \frac{\pi \mu a^3}{2e}\left[a + \sqrt{a^2+h^2}\right]$$

Evaluando con datos entregados:

$$T_{total} = \frac{2\pi\,[rad/s] \cdot \pi \cdot 0{,}007 \left[\frac{Kg}{m\, s}\right] \cdot 0{,}2^3\,[m^3]}{2 \cdot 0{,}007\,[m]} \left[0{,}2 + \sqrt{0{,}2^2 + 0{,}3^2}\right] [m]$$

$$T_{total} = 0{,}0443 \;[N \cdot m]$$

b) $\sum T_{externos} = I_z \cdot \frac{d\omega}{dt}$

Inicialmente $\sum T_{externos} = 0$, $\dot{\omega} = 0$, $\omega = cte.$

Cuando deja de aplicarse el torque antes calculado, $\frac{d\omega}{dt} < 0$, por el efecto de la viscosidad del líquido.

$$T_r = -\omega \frac{\pi \mu a^3}{2e}\left[a + \sqrt{a^2+h^2}\right] = -\omega \cdot b \qquad b = cte.$$

$$I_z = M \cdot \frac{3}{10} a^2 \qquad M = Vol_{cono} \cdot \rho = \frac{\pi a^2 h}{3} \cdot \rho$$

$$I_z = \frac{\pi}{10} a^4 h \cdot \rho$$

Evaluando: $b = \frac{\pi \mu a^3}{2e}\left[a + \sqrt{a^2+h^2}\right] = 0{,}00704 \;\left[Kg \cdot \frac{m^2}{s}\right]$

$$I_z = 0{,}1206 \;[Kg \cdot m^2]$$

$$-\omega \cdot b = I_z \cdot \frac{d\omega}{dt}$$

separando variables: $-b\,dt = I_z \frac{d\omega}{\omega}$ $\qquad t=0 \quad \omega = \omega_0$; $\quad t = t^* \quad \omega = \omega_0/2$

$$-b \int_0^{t^*} dt = I_z \int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega}$$

$$-b t^* = I_z \left[\ln \frac{\omega_0}{2} - \ln \omega_0\right]$$

$$t^* = -\frac{I_z}{b} \ln\left(\frac{1}{2}\right) = \frac{I_z}{b} \ln 2$$

$$\Rightarrow t^* = 11{,}87 \;[seg]$$