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ORIGINAL PAPER

Characterization of the energy bursts in vibrated shallow granular systems

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Abstract We study the recently reported energy bursts that take place in a granular system confined to a vertically vibrated shallow box containing two types of grains of equal size but different mass (Rivas in Phys Rev Lett 106:088001–1–088001–4, 2011). In a quasi one dimensional configuration, it is possible to characterize the propagating fronts. The rapid expansion and the subsequent compression of the energy bursts take place at roughly constant velocities. The expansion velocity is 40 times larger than the compression velocity. Starting from an initially segregated configuration it is possible to determine the instants at which the energy bursts begin and the mechanisms that trigger them. Two mechanisms are identified: an oblique collision of a heavy grain with a light one in contact with one of the horizontal walls and a slow destabilization produced by light grains that are surrounded by heavy ones.

Keywords Granular mixture · Collective phenomena · Vibrated systems · Shallow geometry

1 Introduction

Granular systems consisting of different types of grains have been the focus of a variety of studies. In ref. [2,3], for example, a gaseous granular mixture presenting segregation is studied. Segregation in granular mixtures is quite common, appearing under a variety of different conditions [4,5].

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In particular, segregation is present when the granular system consisting of two types of particles of different mass are constrained to a vertically vibrated shallow box, namely a box of height comparable to the size of the grains, as we have shown in a recent study [6].

Shallow granular systems have attracted attention because they allow a detailed analysis of both the collective behavior of the system as well as the motion of individual grains [7–12]. Placing monodisperse inelastic spheres in a vertically vibrated shallow box, a particular phase separation takes place. Grains form solid-like regions surrounded by fluid-like ones, presenting high contrasts in density and energy [7–9]. The observed melting transition is similar to the melting transition in two-dimensional equilibrium systems, which are driven by dislocations [13]. It has been shown that waves develop in such systems when the system is phase separated. Such waves are driven by negative compressibility of the effective two dimensional fluid [10]. Superheating and hysteresis have also been observed in shallow monodisperse systems [14]. Binary mixtures, in which grains gain energy by friction with the horizontally vibrated box show segregated fluid and solid phases [15].

It is known that a single particle inside a vibrating box may present vertical motion synchronized with the periodic motion of the box. Both the horizontal velocity and the angular velocity vanish, while the vertical movement becomes synchronized with the periodic motion of the box. We say that the particle has reached the *fixed point* motion.

In shallow systems it may happen that a full cluster of neighboring particles can reach the fixed point and therefore they move as if they were part of a solid layer. These particles typically have a total kinetic energy significantly larger than those which are hitting each other. But the kinetic energy of the first is concentrated in their vertical motion.

In the case of binary systems consisting of two species of inelastic hard spheres, H and L (H for *heavy* and L for *light*), which differ only by their mass—when the ratio m_H/m_L is larger than about 8—segregation is observed, see Fig. 7 in Ref. [6]. The Hs form clusters of particles at the fixed point, perturbed by slight horizontal momentum exchange. The Ls remain in a fluid state, keeping a significant horizontal energy. One interesting effect is that the pressure exerted by the Ls on the clusters of Hs makes the latter ever denser.

At a crucial moment an H belonging to a cluster collides with an L in such a way that the H is taken out of the fixed point motion acquiring a significant horizontal velocity. Once an H hits another H in the same cluster a chain reaction of quite energetic collisions between the Hs takes place causing the cluster to expand and the horizontal energy of the system suddenly increases: this is what we call *energy bursts* or *explosions* [1,16].

In previous articles our attention was focused on the relevance of macroscopic quantities associated to the energy bursts, such as the kinetic energy and the synchronization of the motion of the Hs with the vibration of the box [1,16]. In the present article we analyze, by means of numerical simulations, the dynamics of the explosion itself, that is, the propagation of kinetic energy, momentum and mass through the system when an explosion takes place. We also identify mechanisms that trigger the energy bursts.

2 Numerical method and system setup

The dynamics of the shallow granular media is studied by means of numerical simulations. We model grains as inelastic hard spheres with translational and rotational degrees of freedom. Simulations are performed using our own event driven molecular dynamics algorithm [17]. Collisions are instantaneous and they are characterized by normal and tangential restitution coefficients, r_n and r_t, as well as static and dynamic friction coefficients, μ_s and μ_d [18,19]. For simplicity, collisions among grains and with the top and bottom walls are characterized by the same mechanical parameters. The $L_x \times L_y \times L_z$ box oscillates periodically with amplitude A and angular frequency ω. Finally, there is a vertical gravitational acceleration and we use periodic boundary conditions in the horizontal directions.

Units are chosen such that the mass of the light grains m_L, the diameter of the grains σ, and the gravitational acceleration g are fixed to one. For convenience, time will be measured in periods of the oscillating box T. The other parameters are fixed to $m_H = 10$, $L_z = 1.82$, $r_n = r_t = 0.8$, $\mu_s = 0.3$, $\mu_d = 0.15$, $\omega = 7$ and $A = 0.15$. The total number of particles N is chosen so as to have an area density $n = N\sigma^2/(L_x L_y) = 0.9375$. These values are somehow arbitrary but we keep them to compare with previous studies [1,16]. The energy bursts are observed for a wide range of parameters.

3 The energy bursts

Two energy burst sequences of five snapshots each of a quasi two dimensional system with $L_x = L_y = 40$, and $N = 1,500$ are presented in Fig. 1. In both cases the number of heavy and light grains are $N_H = 500$ and $N_L = 1,000$ and in both sequences it can be seen that the initial state has a global cluster of Hs containing some Ls inside it. This is the typical final cluster configuration and geometry. An explosion begins when an H in the cluster gains—by some perturbation—horizontal energy and starts colliding, initiating a chain reaction of collisions that propagate through the cluster. As the collisions propagate, the cluster expands either locally or globally, loosing its crystallographic order. We refer to this phase as the *expansion phase*. The third snapshot of the top sequence shows that the cluster has ostensibly expanded so that the crystallographic order of the Hs has been almost completely lost and the Ls have been pushed to higher densities. Backing this, at bottom in Fig. 2 it is possible to distinctly see that the mean quadratic radius of the clusters suddenly increases.

At this point, the Hs have already lost their horizontal energy because of dissipation, and the cluster begins to recover its original density by the pressure exerted by the Ls. We call this phase the *compression phase*. Processes of this type were observed to repeat in an almost periodic way for as long as $10^6\ T$.

Global explosions are those which involve almost all the Hs in the cluster—as in the top sequence in Fig. 1—while *local explosions*, involving only a fraction of the total number of Hs in a cluster, take place in clusters of all sizes, and even in global clusters. The bottom sequence in Fig. 1 shows a local explosion in a global cluster. The chain reaction is seen to propagate through a region of the cluster, leaving unperturbed the rest of it. There is no sharp distinction between global and local explosions and the histogram of amplitudes shows a continuous distribution with a clear maximum amplitude.

Explosions are easily identified as abrupt localized peaks of the horizontal kinetic energy K_{Hh} of the Hs. After the process of coalescence, energy peaks dominate the time evolution. The form of K_{Hh} in the explosion regime is shown in Fig. 2. Peaks are easily recognized as a one order of magnitude increase of K_{Hh} that lasts for about 100 oscillation periods T. Associated to each energy peak the radius of the clusters d_{cm} rapidly increases followed by a much slower decrease of it until the initial size is recovered. At this moment typically a new energy burst takes place. Although the energy peaks are roughly periodic, they show a distribution of

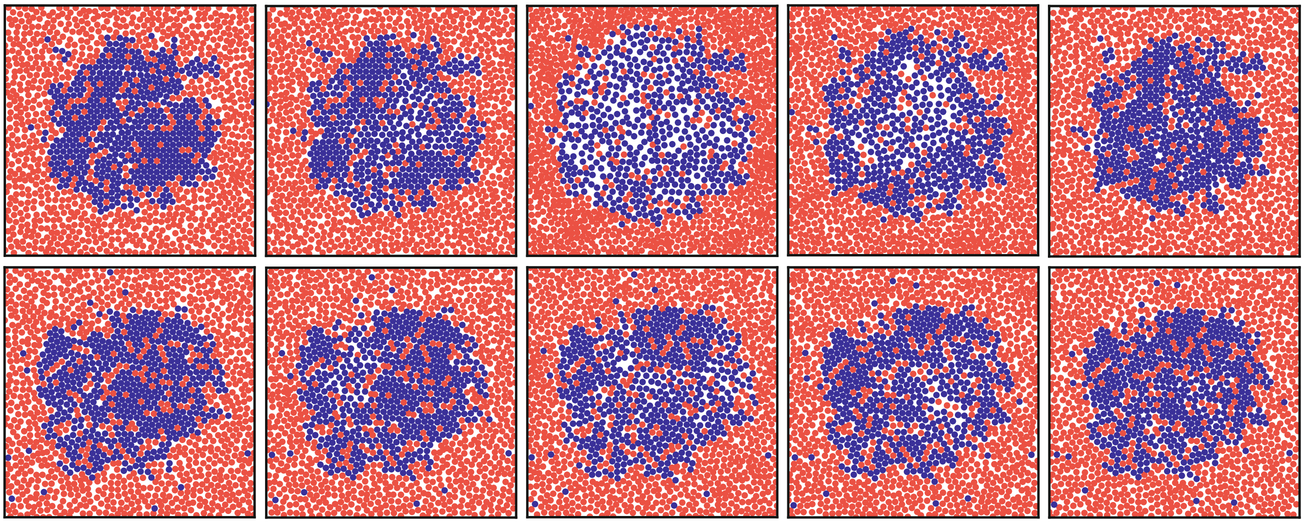

Fig. 1 Sequence of *top-down* views of a simulation during an energy burst in a system with $L_x = L_y = 40$, $N_H = 500$, and $N_L = 1,000$. L-particles are red and H-particles are blue. (*Top*) Global burst. From *left* to *right*: **a** segregated system, where Hs have coalesced forming one large cluster, with light particles in between. Crystallographic order can be appreciated. **b** One particle suddenly gains horizontal energy, transmits this energy through collisions to its neighbors, producing a localized low density zone that propagates. **c** The energy burst has propagated through all the cluster, almost no crystallographic order is present. The cluster is significantly less dense, thus increasing Ls density. The expansion phase has ended. **d** The Hs have lost their horizontal kinetic energy, and the compression phase has begun, where Ls push Hs back to their crystalline order. (*Bottom*) Local burst propagating from the *upper-left* corner of the cluster to the *middle-right* area. Afterwards the cluster returns to its original density. Notice that the final cluster configuration is quite similar to the starting one

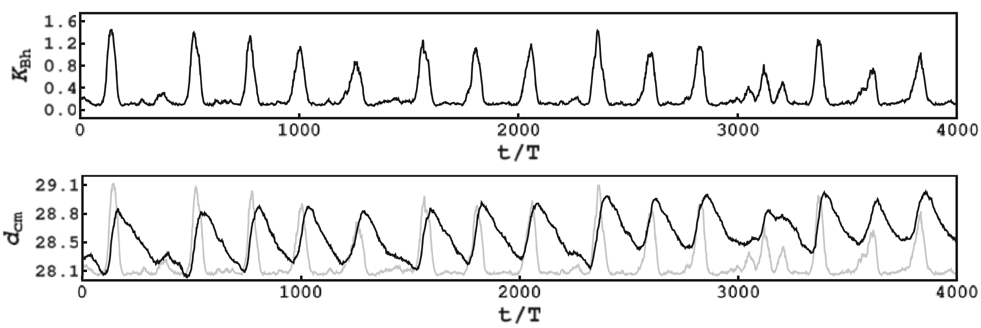

Fig. 2 (*Top*) Evolution of the horizontal kinetic energy of the heavy grains, K_{Hh}, for a lapse of 4, 000 periods in a system with $L_x = L_y = 40$, $N_H = 500$, and $N_L = 1,000$. Explosions are clearly seen as high abrupt peaks of K_{Hh}, the different intensities depend on the number of particles involved in the explosion. In global explosions (when almost all the Hs take part of it) K_{Hh} increases more than an order of magnitude. (*Bottom*) Mean quadratic radius of the heavy particle's cluster, d_{cm}, for the same period of time. K_{Hh} is shown in *gray* for easier comparison

intensities and waiting times. The implied dispersions are connected to the possibility that not all particles are involved in the energy burst.

4 Front propagation

Since the explosion propagates through the cluster in intricate ways, measuring the speed of propagation in the quasi-two-dimensional systems is too difficult. Even identifying the boundary of the explosion can be quite hard and arbitrary. To overcome these difficulties we consider a *channel*, a quasi-one-dimensional (Q1D) system, where $L_x \gg L_y \gg L_z$. The system still presents explosions, but now they propagate mainly in the $\hat{x}$ direction making it easy to measure the front's speed, and identify the expansion and compression phases.

The Q1D system has $L_x = 120$, $L_y = 8$, while the height is maintained at $L_z = 1.82$. To keep the area density $n = 0.9375$ the total number of particles is $N = 900$. In the following simulations the initial conditions are a segregated system with the cluster of Hs placed at $x = L_x/2$ since we are interested in the dynamics of explosions and not in the process of segregation. The number of Hs is chosen to be $N_H = 270$. The length L_x was chosen so that the expanded cluster were smaller than the length of the box. This makes the system practically periodic for the Hs only in the $\hat{y}$ direction.

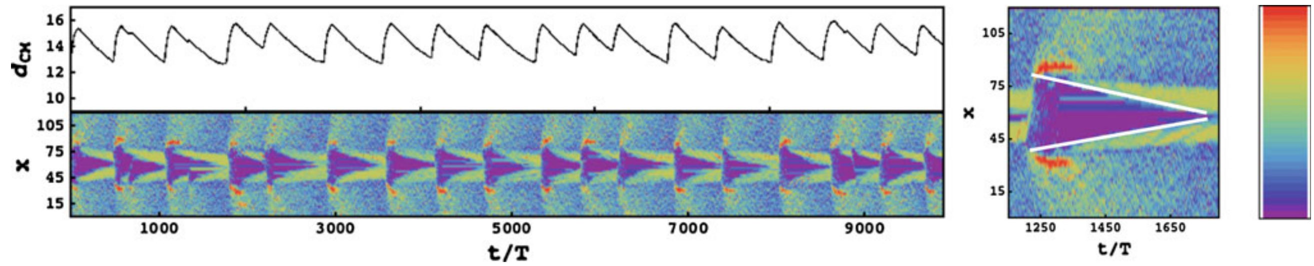

Fig. 3 *Left*: At the top the evolution of the size of the heavy particles cluster, d_{cm}, measured as the square root of the quadratic deviation of the mass distribution, while at bottom the spatio-temporal diagram of the particle density, obtained by coarse-graining a quasi-one-dimensional system. *Violet colors* represent lower values, while *red colors* are high values. *Middle*: Detail of one energy burst event in the spatio-temporal diagram, showing in *white* the *lines* used to compute the compression velocity. *Right*: *Colorbar* used in the spatio-temporal plots. The map is linear from $\rho_{\text{violet}} = 0.25$ to $\rho_{\text{red}} = 1.625$. (Color figure online)

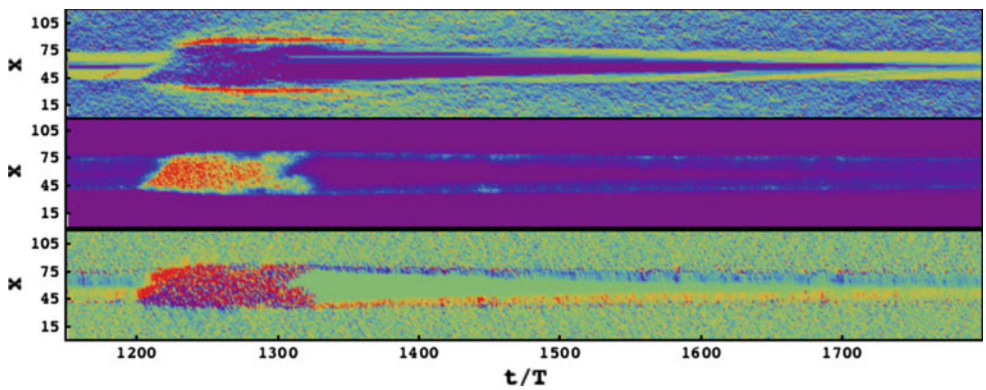

Fig. 4 Spatio-temporal plots of the particle density (*top*), the horizontal kinetic energy density of H-particles (*middle*), and the momentum density in the x-direction (*bottom*), obtained by coarse-graining a quasi-one-dimensional system. *Violet colors* represent lower values, while *red colors* are high values. The *colorbar* is the same as in Fig. 3 with different minimum and maximum values: $\rho_{\text{violet}} = 0.25$ and $\rho_{\text{red}} = 1.625$ (*top*); $K_{\text{violet}} = 0$, and $K_{\text{red}} = 6.25$ (*middle*); $P_{\text{violet}} = -3.12$, and $P_{\text{red}} = 3.12$ (*bottom*). (Color figure online)

Density profiles, $\rho(x)$ can be obtained in the Q1D system by coarse graining the system in bins of $\delta x = 2\sigma$, and counting the number of particles in each bin. The evolution of the density can be seen by arranging the one dimensional profiles in a two dimensional spatio-temporal density plot. This is presented in Fig. 3, together with the instantaneous size of the cluster d_{cm}, $d_{cm} = [\int \hat{\rho}(x)x^2\,dx - (\int \hat{\rho}(x)x\,dx)^2]^{1/2}$, where $\hat{\rho}(x)$ is the normalized coarse grained density. Explosions are easily identified as sudden changes of the density from high values (red) to low ones (blue) in the middle of the system, where the cluster of Hs is located. The expansion phase is not easily seen by looking at the density, but from the figure the compression phase is evident. The highest densities ($\rho \approx 1.625$) are reached by Ls in contact with the cluster's boundary, forming a crystalline order. In Fig. 3 can also be seen, as red patches, that high density fronts propagate through the Ls after each explosion.

The compression phase, which lasts at least an order of magnitude more than the expansion phase, has a roughly constant speed. This velocity depends weakly on time, as Ls slowly enter the H-cluster. An average speed is obtained considering several explosions in just the first 2000 periods for 10 systems with different initial conditions. Adjusting a constant velocity to the spatio-temporal evolution of the cluster's boundary in each explosion, we get an average compression speed $V_c \sim (0.03 \pm 0.01)\sigma/T$ (see Fig. 3).

The particle density $\rho(x)$, together with the horizontal kinetic energy density of the heavy particles $K(x)$ and the x-component of the momentum density $P(x)$ are shown in Fig. 4 for a single explosion. $K(x)$ is computed as the total horizontal kinetic energy of the heavy particles in a bin, divided by the bin width δx and $P(x)$ is the total x-component of the momentum in a bin, divided by δx.

Figure 4 shows that the first field to change because of the explosion is momentum (the lowest plot). This is so because collisions (and not translation) is the main mechanism of momentum propagation. The explosion begins at the lowest end of the cluster and in the three graphs it is possible to see the propagation of the front from $x \approx 45$ to about $x \approx 75$. During the explosion, that lasts for about 100 periods, the horizontal kinetic energy and momentum densities display high values ($|P| = 3.12$, $K = 6.25$). The momentum and energy plots clearly show that for about $100T$ the cluster remains agitated. During half this time the cluster expands reaching a maximum size and then begins to compress. Once this agitation ends the particles in the cluster recover a movement close to the fixed point motion except for the boundary particles, which keep a slight agitation because they are being pushed by the Ls. When the cluster is close to a state like the initial condition this slight agitation invades the whole cluster.

The speed of the cluster boundary in the expansion phase is obtained following the energy density front. To measure it several simulations were performed with the same parameters and different initial conditions. We obtain an average expansion speed $V_e = (1.2 \pm 0.3)\sigma/T$ and the observed maximum is $V_e^{\max} = 1.8\sigma/T$. This average speed is consistent with an estimation made considering the size of the cluster and the duration of the expansion phase (increase of $K_{Hh}(t)$ in Fig. 2), since explosions last for about $35T$ and the cluster size is about 40σ (Fig. 3) the estimated expansion speed is about $V_e^{\text{estimate}} = 1.14\sigma/T$.

5 Triggering mechanisms of the energy bursts

To identify the triggering mechanisms of the energy bursts, a series of simulations in the Q1D geometry were analyzed following the particles motion close to the cluster interface. Snapshot sequences showing top and side views, at a fixed phase of the oscillation, were produced until an explosion took place (identified by the rapid increase of K_{Hh}). The side-view allows to recognize the instant at which heavy particles abandon the fixed point and therefore desynchronize with the rest of the cluster. Having a heavy grain out of phase increases the probability of producing an explosion since out-of-phase H-H collisions are efficient in transferring kinetic energy from vertical to horizontal one. Figures 5 and 6 illustrate two processes of desynchronization of heavy grains that we describe in the next paragraphs.

Figure 5 shows an H-L collision that abruptly starts an explosion. In the first frame, from the side view, an L (red) can be seen to be below the layer of Hs (blue), and from the top view the L is seen to be overlapping two Hs (marked

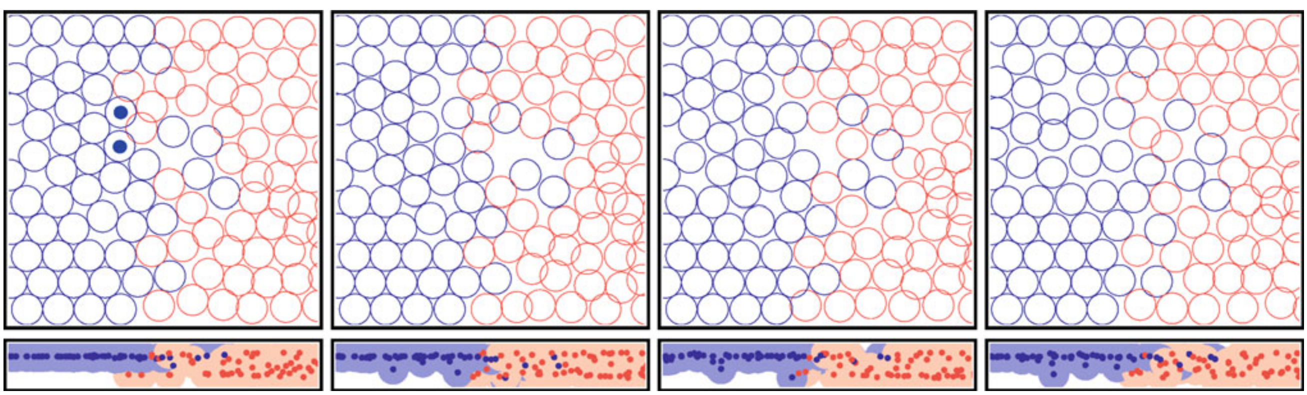

Fig. 5 System configurations showing the beginning of an explosion (only a fraction of the system is shown). *Top-down* view (*top*) and a *side-view* (*bottom*). The first frame shows the heavy particles that start the explosion, marked with a *dot*. Subsequent frames are separated by $2T$

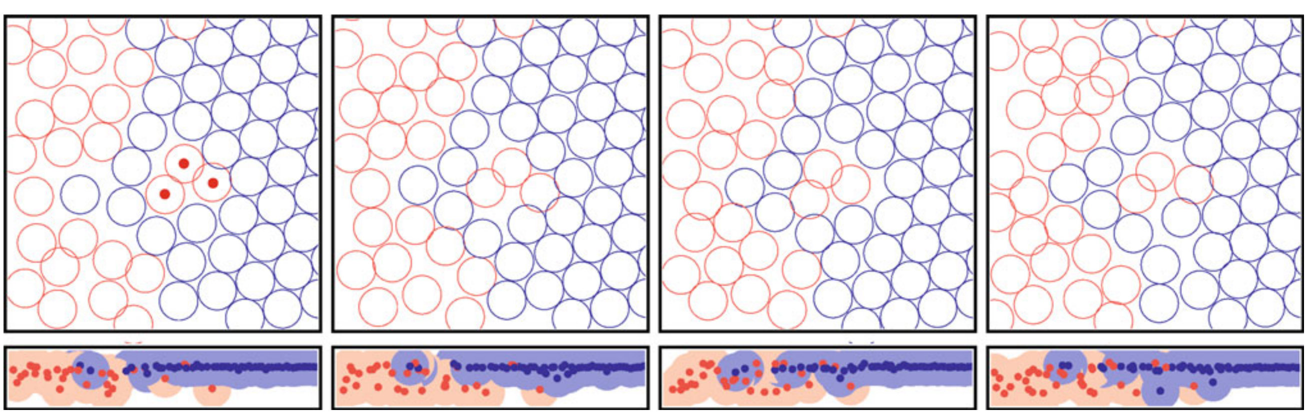

Fig. 6 System configurations showing the beginning of an explosion (only a fraction of the system is shown) triggered by frequent collisions with a small cluster of light grains (*marked with a dot*) inside the heavy cluster grain. *Top-down view* (*top*) and a *side-view* (*bottom*). Frames are separated by $3T$

with a dot), both of which were seen to collide with the L. These collisions, which are not frontal neither tangential, are efficient in transferring vertical to horizontal energy. Besides, the proximity of the L with the plate allows for successive collisions in a very short time, increasing the energy transfer effect. As a result these Hs become out of phase with a large horizontal kinetic energy. In the second frame, just two oscillation periods later, the side view shows that several Hs are already out of phase, and that the crystalline order has been partly lost in the region surrounding the initial collision, namely the chain reaction has begun. The following frames show the lost of order in the neighborhood of the triggering event. Not all collisions of this type produce an explosion, as it also depends on the subsequent collisions of the Hs with their immediate neighbors.

The other more subtle mechanism for triggering explosions is shown in Fig. 6 where three Ls inside the cluster—close to the boundary of it—are responsible for the explosion. These particles continuously perturb their H neighbors: they have a larger horizontal kinetic energy because of collision with Hs. A sequence of these perturbations is shown, ending with the beginning of an explosion. In this case it is much harder to identify an initial event, as it involves several collisions that gradually destabilize the cluster.

6 Discussion

We have studied the horizontal dynamics of a vertically vibrated shallow granular system consisting of heavy and light particles of the same size. In particular we have analyzed in detail the horizontal dynamics associated to the energy bursts.

Energy bursts may involve a considerable fraction of the heavy particles drastically, changing the global horizontal kinetic energy as well as the particle configuration. In this article we show that the bursts mainly depend on the existence of a fixed point in the one-particle dynamics: we were able to track the sequence of collisions back to the triggering events. We have also characterized the expansion and the subsequent compression speeds associated to these bursts.

These bursts are an excellent example of how a localized event, usually irrelevant to macroscopic quantities, can lead to drastic changes in the system. They are possible because the large kinetic energy stored in the coherent vertical motion of the cluster is rapidly transferred to the horizontal motion. Goldhirsch pointed out that granular matter, except in the limit of quasielastic particles, lacked scale separation and thus hydrodynamic models would fail [20,21]. In the quasi two-dimensional geometry studied here, the lack of scale separation is still present but it is well controlled: the vertical dynamics is fast and takes place in a small length scale, while the horizontal dynamics is slow compared to the former and inhomogeneities occur at larger scales. As a result collective hydrodynamic-like models could be built for the horizontal motion, coupled to the non-hydrodynamic motion in the vertical direction.

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References

1. Rivas, N. et al.: Sudden chain energy transfer events in vibrated granular media. Phys. Rev. Lett. **106**, 088001–1–088001–4 (2011)
2. Serero, D., Goldhirsch, I., Noskowick, S.H., Tan, M.-L.: Hydrodynamics of granular gases and granular gas mixtures. J. Fluid Mech. **554**, 237–258 (2006)
3. Serero, D., Noskowicz, S.H., Tan, M.-L., Goldhirsch, I.: Binary granular gas mixtures: theory, layering effects and some open questions. Eur. Phys. J. Special Top. **179**, 221–248 (2009)
4. Kudrolli, A.: Size separation in vibrated granular matter. Rep. Prog. Phys. **67**, 209–248 (2004)
5. Rosato, A., Strandburg, K.J., Prinz, F., Swendsen, R.H.: Why the Brazil nuts are on top: size segregation of particulate matter by shaking. Phys. Rev. Lett. **58**, 1038–1040 (1987)
6. Rivas, N., Cordero, P., Risso, D., Soto, R.: Segregation in quasi-two-dimensional granular systems. New J. Phys. **13**, 055018-1–055018-22 (2011)
7. Olafsen, J.S., Urbach, J.S.: Clustering, order, and collapse in a driven granular monolayer. Phys. Rev. Lett. **81**, 4369–4372 (1998)
8. Prevost, A., Melby, P., Egolf, D.A., Urbach, J.S.: Nonequilibrium two-phase coexistence in a confined granular layer. Phys. Rev. E **70**, 050301(R)-1–050301(R)-4 (2004)
9. Melby, P. et al.: The dynamics of thin vibrated granular layers. J. Phys. Cond. Mat. **17**, S2689–S2704 (2005)
10. Clerc, M.G., Cordero, P., Dunstan, J., Huff, K., Mujica, N., Risso, D., Varas, G.: Liquid-solid-like transition in quasi-one-dimensional driven granular media. Nat. Phys. **4**, 249–254 (2008)
11. Schnautz, T., Brito, R., Kruelle, C.A., Rehberg, I.: A horizontal Brazil-nut effect and its reverse. Phys. Rev. Lett. **95**, 028004 (2005)
12. Chung, F.F., Liaw, S.S., Ju, C.Y.: Brazil nut effect in a rectangular plate under horizontal vibration. Gran. Matter **11**, 79–86 (2009)
13. Olafsen, J.S., Urbach, J.S.: Two-dimensional melting far from equilibrium in a granular monolayer. Phys. Rev. Lett. **95**, 098002-1–098002-4 (2005)
14. Pacheco-Vazquez, F., Caballero-Robledo A, G., Ruiz-Suarez, J.C.: Superheating in granular matter. Phys. Rev. Lett. **102**, 170601–170604 (2009)
15. Reis, P.M., Sykes, T., Mullin, T.: Phases of granular segregation in a binary mixture. Phys. Rev. E **74**, 051306-1–051306-13 (2006)
16. Rivas, N., Risso, D., Soto, R., Cordero, P.: Energy bursts in vibrated shallow granular systems. AIP Conf. Proc. **1332**, 184–189 (2011)
17. Marín, M., Risso, D., Cordero, P.: Efficient algorithms for many-body hard particle molecular dynamics. J. Comput. Phys. **109**, 306–317 (1993)
18. Risso, D., Soto, R., Godoy, S., Cordero, P.: Friction and convection in a vertically vibrated granular system. Phys. Rev. E **72**, 011305-1–011305-6 (2005)
19. Jenkins, J.T., Zhang, Chao: Kinetic theory for identical, frictional, nearly elastic spheres. Phys. Fluids **14**, 1228–1235 (2002)
20. Goldhirsch, I.: Scales and kinetics of granular flows. Chaos **9**, 659–672 (1999)
21. Goldhirsch, I.: Rapid granular flows. Annu. Rev. Fluid Mech. **35**, 267–293 (2003)