Control of longitudinal movement of a plane using combined model reference adaptive control

Manuel A. Duarte-Mermoud, Jaime S. Riosco and Rodrigo J. González
Department of Electrical Engineering, University of Chile, Santiago, Chile

Abstract
Purpose — To apply and simulate under different conditions, the combined model reference adaptive control (CMRAC) technique to control the pitch angle in a subsonic plane. Comparisons with the classical PID controller and the adaptive direct MRAC are also performed.

Design/methodology/approach — The methodology used in this work is the CMRAC. This is a relatively new adaptive control technique which combines the information coming from the identification procedure as well as that from the direct control scheme, and use it in the adaptive laws. The identification parameters and the controller parameters are simultaneously adjusted using the identification error, the control error and the so-called close-loop identification error. This combination has shown to improve the transient behavior of the adaptive systems.

Findings — This control scheme has been tested by simulation on a model of a CESSNA 182 plane, to control the pitch angle (longitudinal movement). The results have been compared with other control approaches such as the classical PID and the adaptive direct MRAC. Although the PID control satisfies all the control specifications as much as the CMRAC, it is not able to adapt when changes in the operating conditions occur, as in the case of the CMRAC. The direct MRAC does not perform well in this study.

Research limitations/implications — The implementation at practical level remains to be studied and analyzed, to verify the theoretical and simulation results presented here.

Practical implications — The main advantage of the proposed method is that it behaves well even under different operating conditions, which is one of the most important characteristics for an implementation at practical level.

Originality/value — It is the first time in the control literature that the CMRAC is applied to control the pitch angle of a plane in a longitudinal movement. The results are quite promising remaining the practical implementation to verify the performance of the proposed scheme under real conditions.

Keywords Control, Aircraft, Aerodynamics

Paper type Research paper

1. Introduction

Small planes use mainly classical controllers to maintain the airplane under control (Boeing Corporations, 2000). Because of its simplicity, PID controllers are widely used. The main disadvantage is that they have fixed parameters with no adaptation capabilities to face variations in operating conditions or eventually plane parameter variations.

In controlling some plane characteristics, rather simple controller has been reported in control literature. Only a few attempts have been done in using adaptive or more complex control strategies (Modi et al., 1990; Pedreiro et al., 1998; Simonich et al., 1993; Departamento de Investigación Académica Politécnica Aeronáutica, 1999). Lately, techniques like μ-synthesis (Mangiaracareale, 1996) and nonlinear control Al-Hiddabi and McClamroch (2002) have been used to control the airplane with good results. In Al-Hiddabi and McClamroch (2002) a tracking controller consisting of feedforward and static state feedback is designed to guarantee uniform asymptotic trajectory tracking. Then a maneuver regulation controller is obtained by introducing a suitable state projection that is related to the LQR feedback gains.

The combined model reference adaptive control (CMRAC) is a relatively new control technique that has been thoroughly studied from the theoretical viewpoint for continuous-time systems (Duarte and Narendra, 1989a; Duarte, 1995; Duarte and Narendra, 1989b) and also for discrete time systems (Narendra and Lin, 1980; Duarte and Ponce, 1997). Only few applications have been reported on this method, in particular the application of the CMRAC has not been used in facing these types of problems. In this paper we investigate the application of the CMRAC to control the longitudinal movement of a plane. We show, throughout extensive simulations, that the CMRAC is able to suitably control the longitudinal movement of the plane, exhibiting good tracking and regulation characteristics.

The evaluation of the CMRAC at simulation level is compared with the performance of conventional PID control
and standard MRAC. We study the CMRAC technique to control the pitch ($\delta_p$) of small airplanes using as control variable the lift angle ($\delta_e$).

In Section 2 the airplane model is presented for certain flight conditions. Section 3 is devoted to a brief description of the CMRAC for completeness. In Section 4 simulation results are presented for several cases. Finally some conclusions are drawn in Section 5. Numerical values of the parameters corresponding to a Cessna 182 plane are used in this study (FAR 23, 1991; Federal Aviation Administration, US Department of Transportation, 1991).

2. Model of the longitudinal movement of a plane

2.1 Description of the plane dynamics

The dynamics of a plane can be described in different forms. Several authors have derived different models for airplanes (Roskam, 1995a; Scicke, 1964; Blakelock, 1963; Dole, 1984; Bryson, 1994; Fuerza Aérea de Chile, 1987; Roskam, 1995b; Van Sicidel, 1965). A plane in flight is a system of six degrees of freedom. The equations describing the plane dynamic behavior are three force equations, three momentum equations, and a seventh equation relating the kinetic energy (Roskam, 1995b).

If we consider a coordinate system of perpendicular axes rotating with angular velocity $\omega$ as shown in Figure 1, the equations describing the movement are:

$$\ddot{F} = m \left( \frac{\partial \dot{V}}{\partial t} + \omega \times \dot{V} \right)$$

where $\dot{V}$ is the velocity of the center of gravity with respect to the rotating axes and $m$ represents the mass of the body. Furthermore the momentum balance equation is:

$$\dot{M} = \frac{\partial \dot{H}}{\partial t} + \omega \times \dot{H}$$

where $\dot{H}$ represents the kinetic moment of the body with respect to the rotating axes.

The orthogonal systems used to describe the plane behavior are the terrestrial axes (coordinate system $XYZ$) and rotating axes (coordinate system $XYZ$). We will assume that the plane total mass and mass distribution remain constant. This is a reasonable assumption for mass that changes less than 5 percent within the first 30-60s of flight, with respect to the fuel consumption. We apply Newton’s second law to a wing profile over the non-inertial rotating axes $XYZ$ (Figure 1), characterizing each vector component on the coordinate axes system (as shown in Figure 2). Noting that the inertia moment $I_{xx} = I_{yy} = 0$, because of the plane symmetry in the $XZ$ plane, we obtain the following linear and angular momentum equations (Roskam, 1995b):

- Scalar equations of linear momentum
  
  \[ m(\ddot{U} - VR + WQ) = mg_x + FA_x + FT_x \]
  \[ m(\ddot{Y} - UR + WP) = mg_y + FA_y + FT_y \]
  \[ m(\ddot{W} - UQ + VP) = mg_z + FA_z + FT_z \]

- Scalar equations of angular momentum
  
  \[ \dot{I}_{xx} - I_{xx}PR + (I_{x} - I_{yy})RQ = MA_x + MT_x \]
  \[ \dot{I}_{yy} + (I_{yy} - I_{xx})PQ + I_{xx}(P^2 - R^2) = MA_y + MT_y \]
  \[ \dot{I}_{zz} - I_{zz}PR + I_{xx}QR = NA_z + NT_z \]

where $I$ denotes the moment of inertia. The plane forces, momentum and velocities are defined as follows (Figure 2):

- Forces
  
  Aerodynamic force components:
  
  \[ \mathbf{F}_A = i \times F_{Ax} + j \times F_{Ay} + k \times F_{Az} \]

  Thrust force components:
  
  \[ \mathbf{F}_T = i \times F_{Tx} + j \times F_{Ty} + k \times F_{Tz} \]

- Gravitational acceleration components:
  
  \[ \mathbf{g} = i \times g_x + j \times g_y + k \times g_z \]

- Momentum
  
  Aerodynamic momentum components:
  
  \[ \dot{M}_A = i \times L_A + j \times M_A + k \times N_A \]

  Thrust momentum components:
  
  \[ \dot{M}_T = i \times L_T + j \times M_T + k \times N_T \]
Control of longitudinal movement of a plane

Manuel A. Duarte-Memouda et al.

• Velocities
Angular velocity components: roll rate, pitch rate and yaw rate, respectively,

\[ \dot{\omega} = i \times P + j \times Q + k \times R \]

Linear velocity components: forward, side and downward velocities, respectively,

\[ \dot{V}_P = i \times U + j \times V + k \times W \]

Distance (Position)
Plane’s local mass distance components

\[ \vec{r} = i \times x + j \times y + k \times z \]

In order to define the plane angular orientation relative to the terrestrial axes, we have to define the plane’s orientation with respect to the rotating axes. To this extent, we translate the terrestrial coordinate axes system to the plane’s center of mass (point P) and we rename it X1Y1Z1. In this system we distinguish three successive transformations through Euler’s angles \( \Psi, \Theta, \) and \( \Phi \) (as shown in Figure 3). The first transformation consists in rotating the coordinate axes system \( X_1Y_1Z_1 \) about the axis \( Z_1 \), given an angle \( \Psi \) called the yaw angle. This system, renamed as \( X_2Y_2Z_2 \), is then rotated about the axis \( Y_2 \), given an angle \( \Theta \) called the pitch attitude angle. This system, renamed as \( X_3Y_3Z_3 \), is rotated about the axis \( X_3 \), given an angle \( \Phi \) called the roll angle. From the study of the plane’s flight curve relative to the earth, we obtain the plane’s kinematics equations, which are described according to Euler’s angles \( \Psi, \Theta, \Phi \) by (Roskam, 1995b)

\[ \Phi = P + Q \sin \Psi \tan \Theta + R \cos \Psi \tan \Theta \]
\[ \Theta = Q \cos \Phi - R \sin \Phi \]
\[ \Psi = (Q \sin \Phi + R \cos \Phi) \sec \Theta \]

It is important to observe from the previous definitions that:
1. The roll ratio is not the same as the ratio of change of the roll angle, i.e. \( P \neq \dot{\Phi} \).
2. The pitch ratio is not the same as the ratio of change of the pitch attitude angle, i.e. \( Q \neq \dot{\Theta} \).
3. The yaw ratio is not equal to the ratio of change of the yaw angle, i.e. \( R \neq \dot{\Psi} \).

Thus, we have a system of nine differential equations with nine variables. By applying the flight conditions, we can reduce the order of the system by three degrees, leaving only six variables and six equations. The two most studied types of flight conditions are:
1. steady-state flight condition;
2. disturbed steady-state flight condition.

A stable steady-state flight can be defined as one in which all movement variables remain constant with respect to the XYZ coordinate system. This implies \( \dot{P} = 0 \) and \( \dot{\omega} = 0 \). Three typical stable-steady state flight conditions are (Roskam, 1995b)
1. straight and horizontal motion;
2. coordinated turning; and
3. upward and downward motion.

A disturbed steady-state flight is defined as one in which all movement variables can be described relative to the stable-steady state flight condition, as shown in Figure 4. In this paper our interest is in the disturbed state flight condition.

---

Figure 3 Plane orientation with respect to Euler’s angles
The disturbed movement, force and momentum variables will be defined as the difference between its absolute value and its steady-state value. We will use the disturbed variables instead of the variable absolute value.

The disturbed variables will be defined as:

- **Movement variables**
  
  \[
  u = U - U_1 \quad v = V - V_1 \quad w = W - W_1 \\
  p = P - P_1 \quad q = Q - Q_1 \quad r = R - R_1 \\
  \psi = \Psi - \Psi_1 \quad \theta = \Theta - \Theta_1 \quad \phi = \Phi - \Phi_1
  \]

- **Force variables**
  
  \[
  f_{A_2} = F_{A_2} - F_{A_2} \quad f_{A_1} = F_{A_1} - F_{A_1} \quad f_{A_2} = F_{A_2} - F_{A_2} \\
  f_{T_2} = F_{T_2} - F_{T_2} \quad f_{T_1} = F_{T_1} - F_{T_1} \quad f_{T_2} = F_{T_2} - F_{T_2}
  \]

- **Momentum variables**
  
  \[
  I_A = L_A - L_{A_1} \quad m_A = M_A - M_{A_1} \quad n_A = N_A - N_{A_1} \\
  I_T = L_T - L_{T_1} \quad m_T = M_T - M_{T_1} \quad n_T = N_T - N_{T_1}
  \]

The subindex 1 indicates the stable steady-state condition, the uppercase indicates the absolute value of the variables and the lowercase corresponds to the perturbed condition. In order to get a suitable model of the plane dynamic behavior, the following assumptions will be introduced:

1. **Disturbances in Euler's angles within 15°** will be considered. With this assumption we can approximate the trigonometric functions sine and cosine as follows:

   \[
   \beta : \text{small angle} \Rightarrow \sin(\beta) = \beta \quad \cos(\beta) = 1
   \]

2. **The cross product between the disturbed motion variables** generates non-linear terms in the equations. However if we consider small disturbances, these second order terms become insignificant with respect to the linear terms.

3. **Based on practice**, the following initial conditions will be used:

   - In steady-state the initial plane's lateral velocity is equal to zero, \( V_1(0) = 0 \).
   - The initial roll angle is equal to zero, \( \Phi_1(0) = 0 \).
   - The initial angular velocities are all equal to zero, i.e., \( P_1(0) = Q_1(0) = R_1(0) = \omega_1(0) = \Phi_1(0) = 0 \).

4. Without loss of generality we can assume that the aerodynamic forces and moments in stable steady-state become insignificant.

Finally the general equations describing the plane's dynamic of the perturbed movement become

\[
\begin{align*}
\dot{u}(t) &= -g \cos(\theta_1) \dot{\theta}(t) + X_{u_1} \cdot u(t) + X_{T_u} \cdot T_u(t) + X_{a} \cdot \alpha(t) \\
&\quad + X_{\theta} \cdot \delta(t) \\
U_1 \dot{\alpha}(t) &= -g \sin(\theta(t)) + Z_{a} \cdot u(t) \\
&\quad + Z_{\alpha} \cdot \alpha(t) + Z_{\theta} \cdot \theta(t) \\
&\quad + Z_{\delta} \cdot \delta(t) \\
\dot{\theta}(t) &= M_{u} \cdot u(t) + M_{T_u} \cdot T_u(t) + M_{a} \cdot \alpha(t) \\
&\quad + M_{\theta} \cdot \theta(t) + M_{\delta} \cdot \delta(t)
\end{align*}
\]

Equation (2.4) is written in terms of the velocity \( u(t) \), pitch angle \( \theta(t) \), the elevator angle \( \delta(t) \), the angle of attack \( \alpha(t) \) and the plane derivative coefficients, \( X_{u_1} \), \( X_{T_u} \), \( X_{a} \), \( X_{\theta} \), \( Z_{a} \), \( Z_{\alpha} \), \( Z_{\theta} \), \( M_{a} \), \( M_{T_u} \), \( M_{\theta} \), \( M_{\delta} \), that will be defined later in Table I (Roskam, 1955; Sceke, 1964; Riesecko, 2000). If we define the state variables \( z_1 = \dot{\theta}(t) \)
Control of longitudinal movement of a plane

Table 1 Derivative coefficients for Cessna 182 airplane

<table>
<thead>
<tr>
<th>Derivative coefficient</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_\theta$</td>
<td>ft/sec/ft/sec</td>
</tr>
<tr>
<td>$X_a$</td>
<td>ft/sec/ft/sec</td>
</tr>
<tr>
<td>$X_u$</td>
<td>ft/sec-rad</td>
</tr>
<tr>
<td>$X_h$</td>
<td>ft/sec/seg</td>
</tr>
<tr>
<td>$Z_\theta$</td>
<td>ft/sec/ft/sec</td>
</tr>
<tr>
<td>$Z_u$</td>
<td>ft/sec/seg</td>
</tr>
<tr>
<td>$Z_h$</td>
<td>ft/sec/seg</td>
</tr>
<tr>
<td>$M_\alpha$</td>
<td>rad/seg/ft/seg</td>
</tr>
<tr>
<td>$M_{\alpha\alpha}$</td>
<td>rad/seg/seg/seg</td>
</tr>
<tr>
<td>$M_{\alpha\alpha}$</td>
<td>rad/seg/seg/seg</td>
</tr>
<tr>
<td>$M_a$</td>
<td>19259.39</td>
</tr>
<tr>
<td>$M_{\alpha\alpha}$</td>
<td>2542.51</td>
</tr>
<tr>
<td>$M_{\alpha\alpha}$</td>
<td>4336.60</td>
</tr>
<tr>
<td>$M_{\alpha\alpha}$</td>
<td>35251.27</td>
</tr>
</tbody>
</table>

\[ \ddot{x}_2 = \dot{\theta}(t) \quad x_3 = u(t) \quad x_4 = \alpha(t), \] the state equations are:

\[ \dot{x}_1(t) = x_2(t) \]

\[ \dot{x}_2(t) = -g \sin \Theta \frac{M_{\alpha}}{Z_a + U_1} x_1(t) + \left( M_{\alpha} + M_a \right) \frac{Z_u + U_1}{Z_a + U_1} x_2(t) + \left( M_{\alpha} + M_{\alpha\alpha} \right) \frac{M_{\alpha}}{Z_a + U_1} \dot{\theta}(t) \]

\[ \dot{x}_3(t) = -g \cos \Theta \frac{M_{\alpha}}{Z_a + U_1} x_1(t) + \left( M_{\alpha} + M_a \right) \frac{Z_u + U_1}{Z_a + U_1} x_2(t) + \left( M_{\alpha} + M_{\alpha\alpha} \right) \frac{M_{\alpha}}{Z_a + U_1} \dot{\theta}(t) \]

\[ \dot{x}_4(t) = -g \cos \Theta \frac{M_{\alpha}}{Z_a + U_1} x_1(t) + \left( M_{\alpha} + M_a \right) \frac{Z_u + U_1}{Z_a + U_1} x_2(t) + \left( M_{\alpha} + M_{\alpha\alpha} \right) \frac{M_{\alpha}}{Z_a + U_1} \dot{\theta}(t) \]

and yield the following matrix representation:

\[
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\ddot{x}_3 \\
\ddot{x}_4 \\
\end{bmatrix} =
\begin{bmatrix}
K_1 & \cdots & \cdots & K_4 \\
\vdots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
K_{13} & \cdots & \cdots & K_{16} \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix}
+ \begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\end{bmatrix}
\]

The state vector is:

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix}
\]

is the state vector.

The values of the derivative coefficients are specified in Table 1.

Applying the plane flight conditions, which will be defined later, the transfer function relating the pitch $\theta(t)$ and input $\delta_{\alpha}(t)$ is:

\[
G(s) = \frac{x_1(s)}{\delta_{\alpha}(s)} = \frac{\theta(s)}{\delta_{\alpha}(s)} = \frac{1}{\left( s^2 + 2 \xi_{ph} \omega_{nph} s + \omega_{nph}^2 \right) \left( s^2 + 2 \xi_{ph} \omega_{ph} s + \omega_{ph}^2 \right)}
\]

The denominator of the transfer function is a fourth order polynomial in the complex variable "s" and is represented by:

\[
\left( s^2 + 2 \xi_{ph} \omega_{nph} s + \omega_{nph}^2 \right) \left( s^2 + 2 \xi_{ph} \omega_{ph} s + \omega_{ph}^2 \right)
\]

The model of the plane motion in this work corresponds to a Cessna 182, built by the Cessna Aircraft Company. The technical and operative characteristics of this plane can be consulted in FAR 23 (1991). For this study we are going to consider the following straight-line and leveled flight conditions:

- Attitude (feet) - 5.000
- Velocity (m.p.h.) - 130
- Weight (pounds) - 2.650
- Dynamic pressure (p.s.i) - 49.6
- Center of gravity percentage (percent) - 26.4

The parameters of the transfer function (2.5) are computed from these flight conditions evaluating first the derivative coefficient (Roskam, 1995a). This transfer function represents the relationship in the frequency domain between the plane attitude and the elevator angle of deflection (Rioseco, 2000).

As far as the sensor is concerned, to measure the attitude, the most common device is the gyroscope. The transfer function of the sensor is modeled as a pure gain, since the smallest cut frequency of the gyroscope is large than the largest cut frequency of the plant transfer function. Therefore we have:

\[
G_{s}(s) = 1
\]

The servocable is a power amplifier used to command the servomotors or servoelevators. In most cases the actuator is a
Control of longitudinal movement of a plane

Manuel A. Duarte-Mermeud et al.

servo-elevator and acts directly over the plane's controls, because the only aerodynamic surface used in the pitch control is the elevator $\delta_\theta$. The servo-elevator is commonly modeled as a second order transfer function, but its natural frequency is higher than the plant's frequencies (Roskam, 1995b). That is why the servo-elevator will be modeled as a first order system and its transfer function is:

$$G_A(s) = \frac{a}{s + a}$$

where $a$ is the magnitude of the cut frequency and indicates the servo-elevator response velocity, which in fact determines the velocity of the control surface. In this paper we are going to consider $a = 10$, the most typical value for control strategies development.

The plant transfer function including the sensor and actuator is then (Riosco, 2000):

$$G_{APS}(s) = \frac{347.3544(s + 0.05902)(s + 2.001)}{(s + 10)(s^2 + 0.04417s + 0.02933)(s^2 + 8.902s + 27.79)}$$

(2.6)

2.3 Dynamic and technical specifications

The value of the damping factor $\xi$ of the plane response should satisfy the aviation standards and for a straight-line and level flight uniformly accelerated must fulfill that the natural frequency $\omega_n$ must be lower than 1 rad/seg Roskam (1995b). The plane has to be controllable and maneuverable in all flight phases, not exceeding the structural limits of charge (FAR 23, 1991). In Van Sickele (1965) it is stated that corrective action due to disturbances acting on the plane, should not exceed $+2g$. The plane tail is the part under the largest accelerations during the longitudinal movement, since the plane oscillates around the center of gravity (altitude). In our study we consider the following limits for tangential and normal accelerations:

$$a_t = l\dot{\theta}, \quad a_n = l\dot{\theta}^2$$

(2.7)

Using a value $l = 5$ m for the distance between the nose and the rudder (Riosco, 2000), we obtain the following maximum values for the accelerations in equation (2.7)

$$\dot{\theta} = 3.92 \frac{\text{rad}}{s^2} = 224.59 \frac{\text{degree}}{s^2}$$

$$\ddot{\theta} = 1.9798 \frac{\text{rad}}{s} = 113.44 \frac{\text{degree}}{s}$$

Another constraint is that the maximum rudder angular velocity is:

$$\delta_r = 1.57 \frac{\text{rad}}{s} = 89.11 \frac{\text{degree}}{s}$$

The operative range attitude and deflection angle reached by the elevator is given by:

$$\theta_{\text{max}} = 7.562[\text{degrees}], \quad \delta_{\text{max}} = -9.712[\text{degrees}]$$

$$\theta_{\text{min}} = -7.565[\text{degrees}], \quad \delta_{\text{min}} = 9.712[\text{degrees}]$$

2.4 Control process description

In this section we present the basic control structure for the process under study. In Figure 7 we can distinguish the different blocks of the control loop. The mathematical description of the different components has already been discussed. Different controllers can be used to solve this problem, classical control strategies including PID and frequency domain methods are presented in Riosco and Duarte (2002). In this paper we will use an adaptive controller which is described in the next section.

On this process a reference $\theta_{\text{ref}} [\text{degrees}]$ is compared with the plant's output to generate an error signal $\delta_e [\text{degrees}]$ sensed by the vertical twist. The controller generates an electrical signal $\delta_e [\text{volts}]$, this signal is the input of the actuator which produce an angle $\delta_e [\text{degrees}]$ to correct the elevator position in the tail rudder by a generally electromechanical valve.

3. Control strategies

A brief description of the CMRAC algorithm used to control the plane is given in what follows, for sake of completeness. For a more detailed explanation of the algorithm, the reader is referred to the original sources (Duarte and Narendra, 1989a; Duarte, 1995; Duarte and Narendra, 1989b; Duarte and Ponce, 1997). Since they are used for comparison purposes, the PID and DMRAC (Narendra and Anaswamy, 1989) are also presented.

3.1 CMRAC algorithm

Generally speaking, the main objective of the MRAC is to minimize the error between the plant output and the model reference output. The controller parameter adjustment is based on the control error in the DMRAC (Narendra and Anaswamy, 1989), on the identification error in the IMRAC (Narendra and Anaswamy, 1989) and on the control, identification and closed loop estimation errors in the case of a CMRAC (Duarte and Narendra, 1989a; Duarte and Narendra, 1989b).

For the particular case of the longitudinal movement of the plane under investigation, we will use a fifth order model for the plant around an operating point defined by equation (2.6). Under well known hypothesis, the control law for the CMRAC has the form (Duarte and Narendra, 1989a, b)

$$u(t) = \theta^T(t) \cdot w(t)$$

where $\theta(t) = [\theta(t), \theta^T(t), \theta^T(t), \theta^T(t)] \in \mathbb{R}^{10}$ and $w(t) = [r(t), w^T(t), \gamma(t), w^T(t)] \in \mathbb{R}^{10}$ are the controller parameters and the auxiliary signal vector, respectively. The signal vector is defined by:
Figure 5 Unit feedback schematic loop

\[
\begin{align*}
&\text{Reference} \quad \text{Sum} \quad \text{Amp} \quad \text{Servo-elevator} \\
&-10 \quad \text{Scope}
\end{align*}
\]

\[
\begin{align*}
\text{Transfer Function:} & \quad \frac{-34.74s^2-71.56s+102}{s^4+8.56s^3+28.21s^2+1.48s+0.815} \\
\text{Plane's Transfer Function} & \quad \frac{-34.74s^2-71.56s+102}{s^4+8.56s^3+28.21s^2+1.48s+0.815}
\end{align*}
\]

Figure 6 Five degrees step unit feedback system response

\[
\begin{align*}
&\text{degrees} \\
&\text{seconds}
\end{align*}
\]

Figure 7 Control process for control of a subsonic plane

\[
\begin{align*}
\dot{\theta}_1(t) &= \Lambda \dot{w}_1(t) + k_4 \dot{y}_p(t) \\
\dot{w}_2(t) &= \Lambda \dot{w}_2(t) + \delta_y(t)
\end{align*}
\]

with \( k(t), \theta_0(t), r(t), y_p(t) \in \mathbb{R} \), and \( \theta_1(t), \theta_2(t), \alpha_1(t), \alpha_2(t) \in \mathbb{R}^4 \). \((\Lambda, I)\) is an arbitrary controllable pair, with \( \Lambda \in \mathbb{R}^{4 \times 4} \) an asymptotically stable matrix. For simplicity, in our study we choose \((\Lambda, I)\) in the controllable canonical form. We additionally define:

\[
\begin{align*}
\dot{\alpha}(t) &= \Lambda \dot{w}_1(t) + k_4 \dot{y}_p(t) \\
\delta_y(t) &= \alpha^T \dot{w}_2(t)
\end{align*}
\]

where \( \dot{\alpha}(t) = y_p(t) - y_m(t) \) is the control error, \( \delta_y(t) \) is the augmented error, \( \dot{\alpha}(t) \) is the auxiliary error and \( k_4(t) \) is an additional adjustable parameter. \( I_{10} \) denotes the identity matrix of dimension 10.

The CMRAC needs a plant parameter estimator which is built based on the following plant representation:

\[
\begin{align*}
\dot{y}_p(t) &= \frac{k_p}{k_m} \dot{u}(t) + \dot{\beta} \dot{w}_1(t) + \alpha \dot{y}_p(t) + \alpha^T \dot{w}_2(t)
\end{align*}
\]

where

\[
\begin{align*}
\dot{\theta}_1(t) &= \theta_1(t) + \delta_1(t) \\
\dot{\theta}_2(t) &= \theta_2(t) + \delta_2(t)
\end{align*}
\]
The adaptive laws for controller and identification parameters are given by:

\[ k(t) = -\frac{\text{sgn}(k_p)\alpha_s e_s(t)\eta_m(t)}{N_i(t)k_m} - \frac{\text{sgn}(k_p)\beta_s e_s(t)}{N_i(t)k_m} \]

\[ \dot{b}_1(t) = -\frac{\text{sgn}(k_p)\alpha_e e_e(t)\eta_0(t)}{N_i(t)k_m} - \beta_s e_{\theta_0}(t) \]

\[ \dot{b}_2(t) = -\frac{\text{sgn}(k_p)\alpha_e e_e(t)\eta_2(t)}{N_i(t)k_m} - \frac{\text{sgn}(k_p)\beta_s e_{\theta_0}(t)}{N_i(t)k_m} \]  \hspace{1cm} (3.1)

\[ \dot{b}_3(t) = -\frac{\text{sgn}(k_p)\alpha_e e_e(t)\eta_3(t)}{N_i(t)k_m} - \frac{\text{sgn}(k_p)\beta_s e_{\theta_0}(t)}{N_i(t)k_m} \]

\[ \dot{b}_4(t) = -\frac{\text{sgn}(k_p)\alpha_e e_e(t)\eta_4(t)}{N_i(t)k_m} - \frac{\text{sgn}(k_p)\beta_s e_{\theta_0}(t)}{N_i(t)k_m} \]

\[ \dot{k}_1(t) = -\alpha_s e_s(t) e_1(t) \]

\[ \dot{k}_2(t) = -\frac{\alpha_s e_s(t) e_2(t)}{N_i(t)e_m} - \beta_s e_{\theta_0}(t) \]

\[ \dot{k}_3(t) = -\frac{\alpha_s e_s(t) e_3(t)}{N_i(t)e_m} - \beta_s e_{\theta_0}(t) \]

\[ \dot{k}_4(t) = -\frac{\alpha_s e_s(t) e_4(t)}{N_i(t)e_m} - \beta_s e_{\theta_0}(t) \]  \hspace{1cm} (3.2)

\[ \dot{a}_0(t) = -\frac{\alpha_s e_s(t) e_1(t)}{N_i(t)} - \beta_s e_{\theta_0}(t) \]

\[ \dot{a}_1(t) = -\frac{\alpha_s e_s(t) e_2(t)}{N_i(t)} - \beta_s e_{\theta_0}(t) \]

where \( N_e(t) \) and \( N_i(t) \) are normalization factors defined as:

\[ N_e(t) = 1 + \bar{e}_i(t) \bar{e}_i(t) + \bar{e}_{i1}(t) \in \mathbb{R} \]

\[ N_i(t) = 1 + (\bar{e}_i^2(t)\bar{e}_i(t) + \bar{u}(t))^2 + \bar{e}_{i1}(t)\bar{e}_{i1}(t) \]

\[ + \bar{e}_{i2}(t)\bar{e}_{i2}(t) \in \mathbb{R} \]

Factors \( \alpha_s, \beta_s, \alpha_e, \) and \( \beta_e \) are the adaptive gains. The general scheme of the CMRAC is shown in Figure 8. The DMRAC is obtained from the previous development by making zero the adaptive gains \( \alpha_e \) and \( \beta_e \). The dynamic IMRAC is obtained from the previous development by making zero the adaptive gains \( \alpha_e \) and \( \beta_e \).

3.2 PID control

The PID controller used in computer simulations is defined in terms of the error produced by the difference between the reference signal \( \theta_0 \) and the plane's output \( \theta \). The PID transfer function is given by:

\[ G_c(s) = k_p + \frac{k_i}{\tau_i s} + k_d \tau_d s \]  \hspace{1cm} (3.3)

Several techniques can be used to tune the coefficients of a PID controller. In this work we used the second method of Ziegler-Nichols (Franklin et al., 1991; Ogata, 1993) and we obtain the parameters value presented in Table II.

In Figure 9 the structure used in SIMULINK to simulate the closed-loop PID control is presented.

4. Simulations results and comparisons

Computer simulations were performed in Matlab-Simulink (The Math Works Inc, 1998; Ranw Marc, 1998). To solve the differential equations, a Runge-Kutta of fifth order method was used. The relative tolerance was set to \( 10^{-12} \). For all simulations zero initial conditions were generally considered for the plane, unless other nonzero IC are explicitly mentioned.

4.1 PID control

By using PID control we asymptotically stabilize the system. The controller was implemented using the parameters shown in Section 3.2. The time response of the closed-loop-controlled system is shown in Figure 10 when a unit step is considered as reference.

From the time response we obtain the following system characteristics satisfying the technical specifications:

- overshoot of 3.9 percent;
- settling time 0.75 s;
- rise time 0.51 s; and
- Steady-state error 1 \times 10^{-1} percent.

4.2 Adaptive control

According to Narendra and Anaswamy (1989) the reference model and the plant should have at least the same relative degree. It means that the reference model has to have a relative degree three or greater. We will choose two different reference models; the first one has a fast time response, with rise time of 0.63 s, delay of 0.27 s and without overshoot. With this kind of reference model we obtain controlled systems with fast response, taking into account the structural and physical charge limits on the plane and passengers. The second reference model has a slower response than the first one, with rise time of 1.28 s, delay 0.98 s and with an overshoot of 3.7 percent. With this model, plane's wing charge does not exceed its maximum value. These reference models are shown in Figure 11.

The unit step responses of both reference models are shown in Figure 12.

To implement the adaptive algorithms, first of all we need to choose the controllable pair \( (\lambda, \beta) \), which will be chosen in controllable canonical form. Since the dimension or order of the plant together with the sensor and actuator is 5 \((n = 5)\), then the dimension of matrix \( \lambda \) is \((4 \times 4)\) and vector \( \lambda \) is \((4 \times 1)\). The values considered in this study are:

\[ \lambda = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -10 & -60 & -40 \end{bmatrix}, \quad \lambda = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \]

First we simulate the DMRAC, considering the slow reference model (Model Reference 1) as seen in Figures 13 and 14, and later the fast reference model latter (Model Reference 2) as seen in Figures 15 and 16. For the reference input a constant attitude of one degree with respect to the artificial horizon is considered. All the adaptive gains are set to 1, unless other value is explicitly stated.

In Figure 13 we can see that the plant has damped oscillations. The behavior of the system is not as good as we want, but, if we consider a set of initial conditions different
from zero for the controller parameters, then a better response can be obtained. In particular one can set the initial conditions as the final values obtained from this simulation (Figure 13) getting an improvement in the system behavior as seen in Figure 14, where the plant react 34 percent faster than the model reference 1 with an overshoot of 28 percent and a rise time of 0.85 s. Also one can achieve better responses by changing the values of the adaptive gains, this in fact improve the transient behavior (Rioseco, 2000) (simulations not showed here for sake of space).

From Figure 15 we can see that the overshoot is quite large, with a maximum value of 2° approximately. The steady-state error tends to zero after 13.5 s. The rise time was 0.62 s bigger than the 0.53 s of the reference. The settling time was 4.23 s, which means that the plant reacts 15 percent slower with respect to the reference model. We can also improve the response by changing the initial conditions or the adaptive gains (Rioseco, 2000).

The adaptive gain \( \alpha \) can be divided into two parts one named it \( \gamma \), controlling the parameters present on the control law, and a second part that is the gain on the parameter \( k_1 \) used in the augmented error. In Figure 16, the adaptive gain \( \gamma \) was changed, resulting in a worst system response. But reducing the value of \( \gamma \) to a value 0.05, then a better response can be achieved (Rioseco, 2000) (simulation not showed here for sake of space).

In Figures 17–19 we present the simulation results corresponding to the CMRAC control scheme. In Figure 17 the overshoot is quite large, representing an error of 800 percent. Also, we can see how the identification model try to follow the reference, the settling time in this case is large too, so we can say that this is not the best response for the Model Reference 1. By considering initial conditions different from zero we can reduce the overshoot in 500 percent (Rioseco, 2000). In Figure 18 it is clear that the plant cannot follow the Model Reference 2 when zero initial conditions are chosen. This is the worst case in all simulations presented, however one of the major advantages of the CMRAC is the capability of considering the identification part and the control part with different weights. This property allows us to considerably improve the response as shown in Figure 19.
In the simulation shown in Figure 9 we change the adaptive gains to the following values $\alpha = 280, \beta = 10^{-3}, \gamma = \beta = 10^{-2}$. These adaptive gains weight more heavily the identification part than the control part in the CMRAC scheme. With these values of the adaptive gains the overshoot is only 2 percent, the rise time is 3.8 s and the settling time is 4.9 s. This is without doubt the best response obtained in all simulations and it satisfies all the technical specifications of Section 2.3.
5. Conclusions

Control strategies applied nowadays in small planes like Cessna 182 used to control the pitch angle are mainly of PID type (Boeing Corporations, 2000). In Section 4.1 we saw that this kind of control produce a good plant response satisfying all technical and dynamics specifications with a steady-state error smaller than 0.001 percent. One of the principal disadvantages of this type of controller is that they cannot consider changes in plant parameters which for the particular case of a plane in flight, these type of changes are normal. That is why an adaptive controller would be better and why we study the response of the plane when an adaptive controller is considered.
Control of longitudinal movement of a plane
Manuel A. Duarte-Mermoud et al.

Figure 15 DMRAC for model reference 2 with zero initial conditions

Figure 16 DMRAC with model reference 2, $\gamma_1 = 100$ and zero initial conditions

In this paper we study two different kinds of adaptive controllers. Using the direct adaptive controller the plant response did not behave well and we could not satisfy the specifications of Section 2.3. However, using a combined direct and indirect adaptive controller the plant response was quite good and all the specifications were satisfied. Comparing PID control results with those of CMRAC, we can observe that PID controller produce a good plant response but it cannot consider changes in parameters value.
By changing initial conditions in the controller parameters we can improve the response, but also we can worsen. The same happens with the adaptive gains so we must conclude that the adaptive control scheme works well for some specific range of the parameters.

The results presented in this work are clearly better than those reported in Ricoeco and Duarte (2002), where classical frequency control strategies were used. A good transient response is observed and a more rapid control action is obtained. (Results of Ricoeco and Duarte (2002) are not shown here).
Figure 19 CMRAC for model reference 2, zero initial conditions and adjusted adaptive gains

Note: This diagram is reproduced from the best original supplied.

References


Franklin, G., Powell, J. and Emami-Naeini, A. (1991), Control de sistemas dinámicos con retroalimentación (in Spanish), Addison-Wesley, Glen View, IL.

Fuerza Aérea de Chile (1987), Comando de Personal Escuela de Especialidades, Diccionario técnico aeronáutico inglés-español.


Riosceo, J.S. and Duarte, M.A. (2002), “Time and frequency classical control techniques applied to the model of the
Control of longitudinal movement of a plane

Manuel A. Duarte-Mermoud et al.

longitudinal movement of a subsonic plane”; Anales del instituto de Ingenieros de Chile, Vol. 114 No. 1, pp. 3-17 (in Spanish).


Seckel, E. (1964), Stability and Control of Airplane and Helicopters, Department of Aeronautical Engineering, The James Forrestal Research Center School of Engineering and Applied Science, Princeton University, Princeton, NJ.


Van Sickle, N.D. (1965), Aeronautica moderna (in Spanish), UTEHA.

About the authors

Manuel A. Duarte-Mermoud He received the degree of Civil Electrical Engineer from University of Chile in 1977 and the MSc, MPhil and the PhD degrees, all in Electrical Engineering, from Yale University in 1985, 1986 and 1988, respectively. From 1977 to 1979, he worked as Field Engineer at Santiago Subway. In 1979 he joined the Electrical Engineering Department of University of Chile, where he is currently Professor. His main research interests are in robust adaptive control (linear and nonlinear systems) and system identification. Dr Duarte is member of the IEEE and past President of ACCA, Chilean National Member Organization of IFAC.

Jaime S. Rioseco He received the degree of Civil Electrical Engineer from University of Chile in 2001 and the Aeronautical Propulsion Master degree from University of Paris VI in 2003. Since 2004 he is working as power plant Engineer at LAN airlines. His main research interests are in robust adaptive control of planes and rockets.

Rodrigo I. González He received the degree of Civil Electrical Engineer and the MSc degree in Electrical Engineering, from the University of Chile in 2004. In 2002 he joined the Control Systems Department of Tuning Engineering Ltd, where he is currently technical assessor for Nestlé Chile S.A. His main research interests are in robust adaptive control (linear and nonlinear systems) and motion planning. Mr González is member of the Control Systems and Computer Societies of IEEE.