Localized states in bistable pattern forming systems
Super Peak

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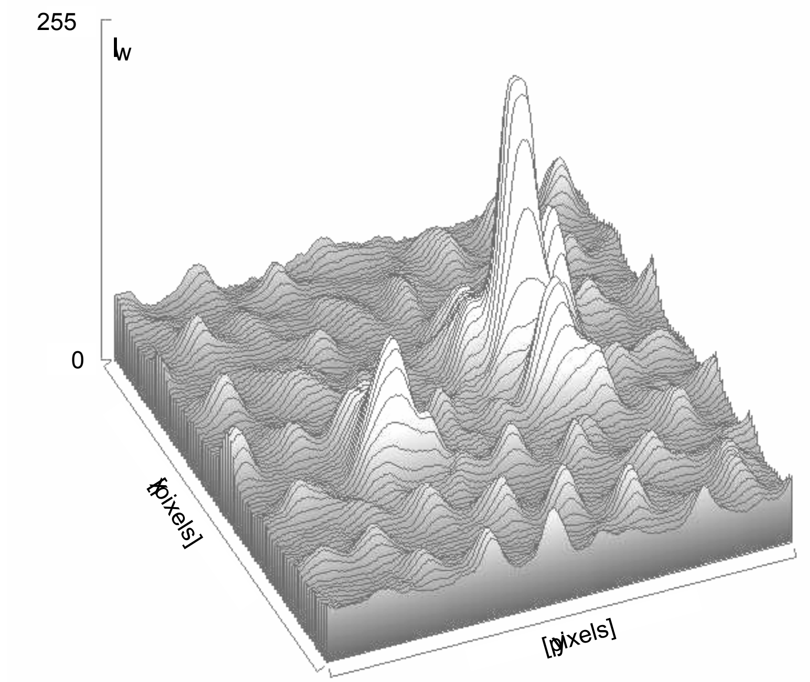


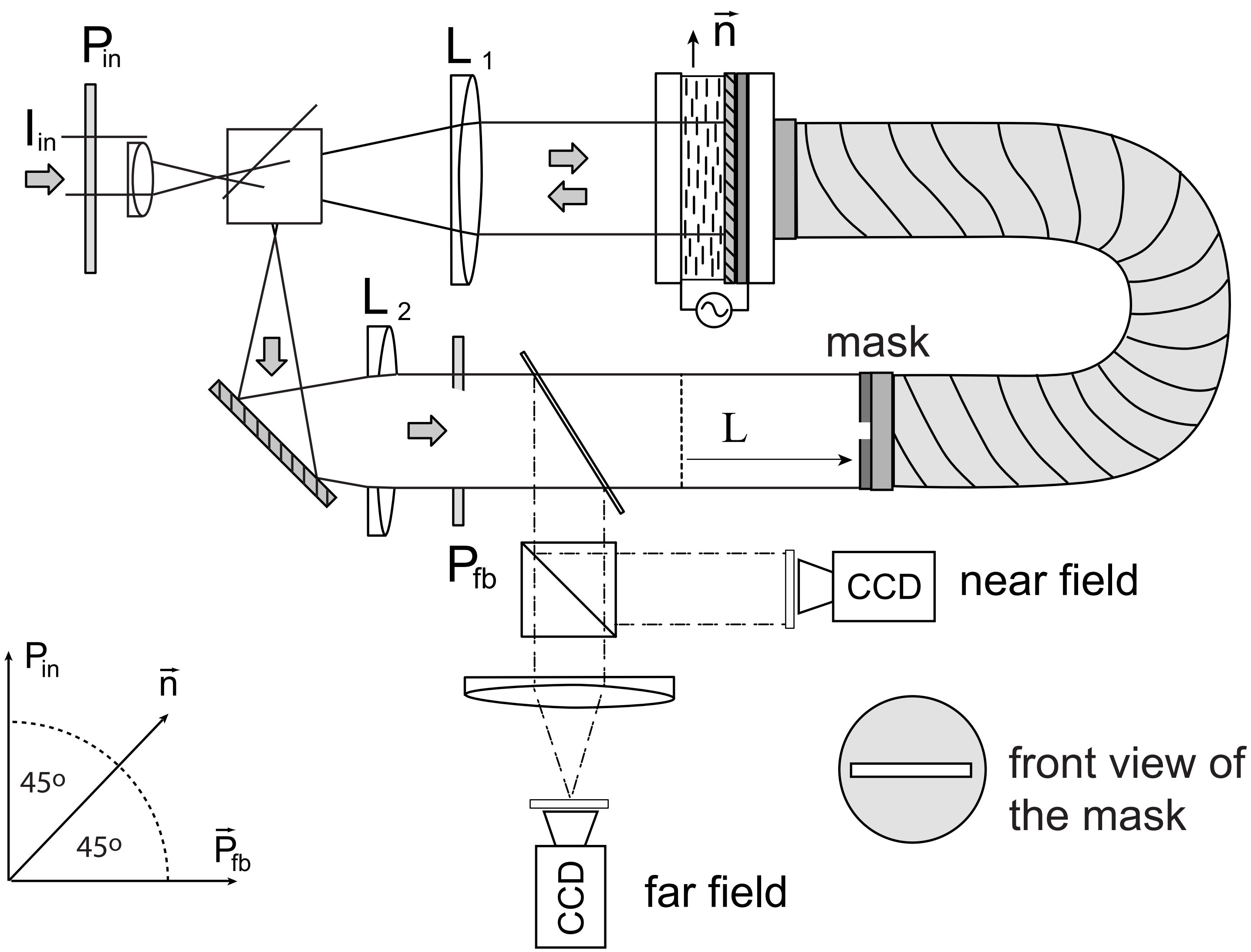
DFI, INLN, Fondecyt, CIMAT, and Ecos-conicyt

Outline

- Introduction to Liquid Cristal Light Valve with optical feedback(LCLV).
- Theoretical description (LCLV).
- Bifurcation diagram of uniform states.
- Bifurcation diagram of pattern states (Bi-pattern).
- Main ingredients of super peak.
- Parametrically driven Newtonian fluid.
- Universal theoretical description.
- Amplitude equation with non-resonant term.
- Super-peak.
- Conclusion.

Introduction to Liquid Cristal Light Valve with optical feedback(LCLV)

- Experimental set up



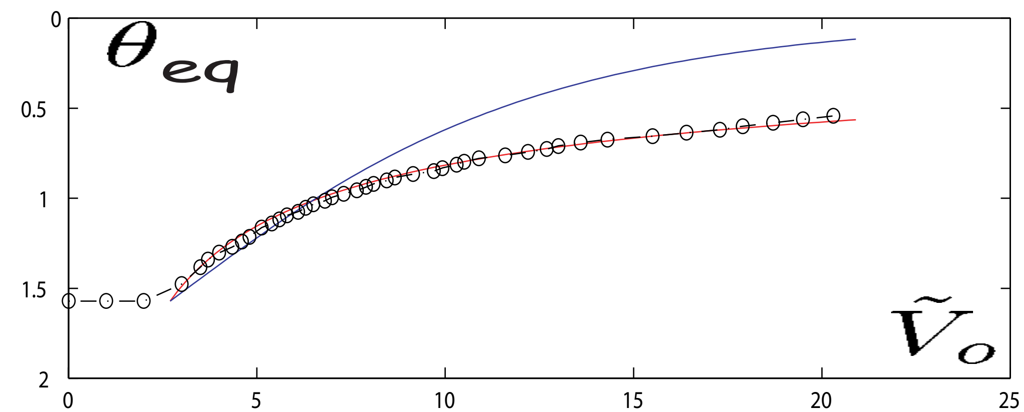
$$\Delta n = 0.2$$
$$\psi_1 = \psi_2 = 45^o$$
$$d = 30\mu m$$
$\lambda = 633$ nm
L =1-4 cm

Theoretical description (LCLV)

- The dynamic of average tilt θ is described by local relaxation model of the form (applied voltage larger than voltage of Freederickz transition)

$$\tau\partial_t\theta = l^2\partial_{xx}^2\theta - \theta + \frac{\pi}{2}\left(1 - \sqrt{\frac{V_{FT}}{\tilde{V}_o + \alpha I_w(\theta, \partial_x)}}\right)$$

- Experimental measurement of equilibruim average tilt as function of applied voltage without feedback.



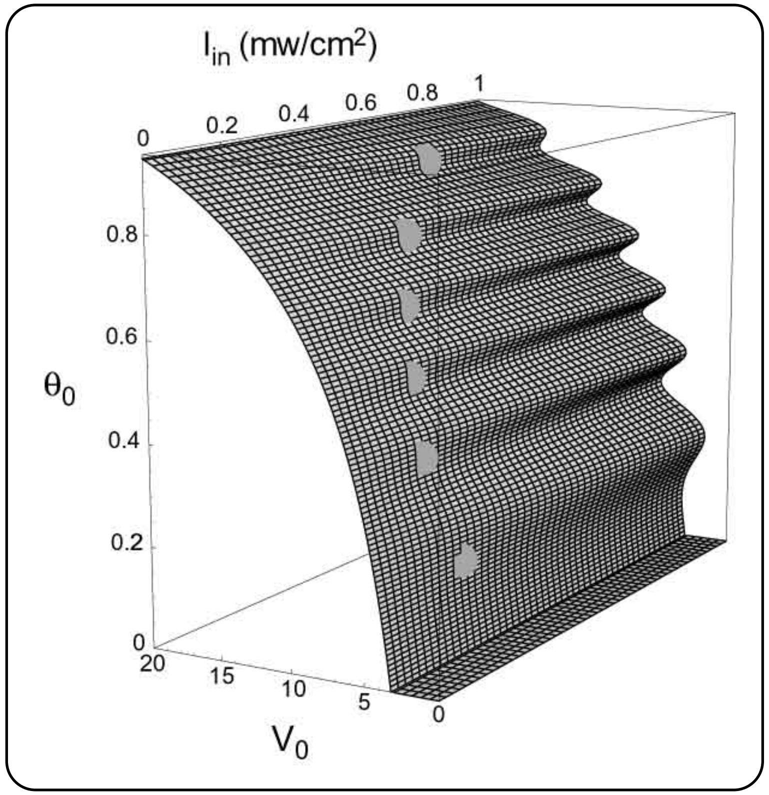
- The model is non-local in the space and non-variational

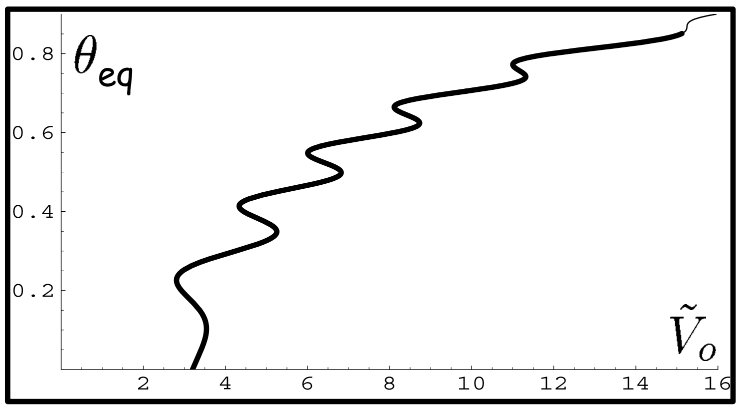
$$I_w(\theta, \partial_x) = I_{in}\left|e^{-\frac{L}{2k}\partial_{xx}}\left\{\sin\psi_1\sin\psi_2 + \cos\psi_1\cos\psi_2 e^{-i2kd\Delta n\cos^2\theta}\right\}\right|^2$$

Phys. Rev. E 71, 015205-1 (2005); Physica D 199, 149-165 (2004);
J. Opt. B: Quantum Semiclass. Opt. 6, S169 (2004).

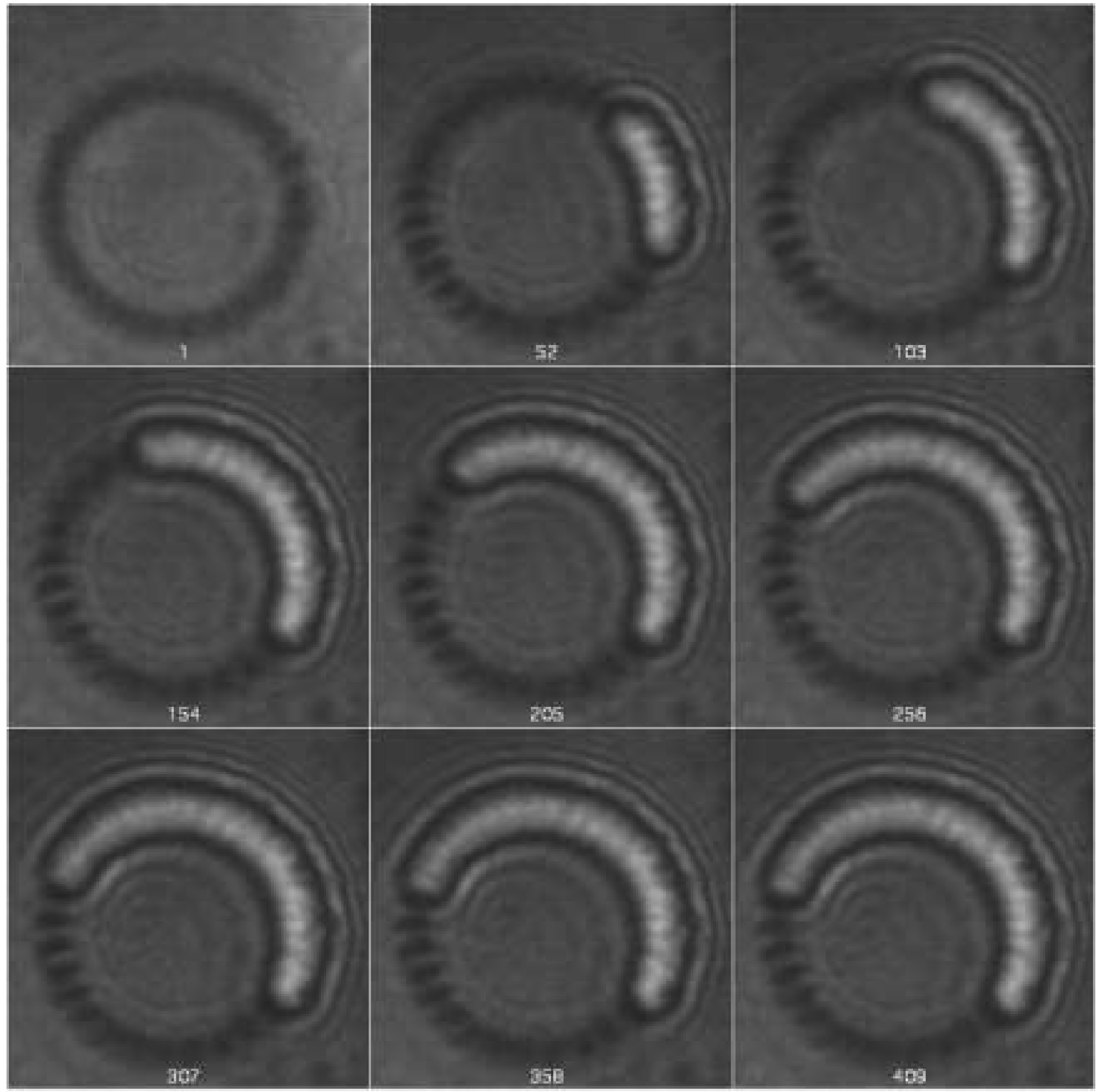
Bifurcation diagram of uniform states

- Bifurcation diagram





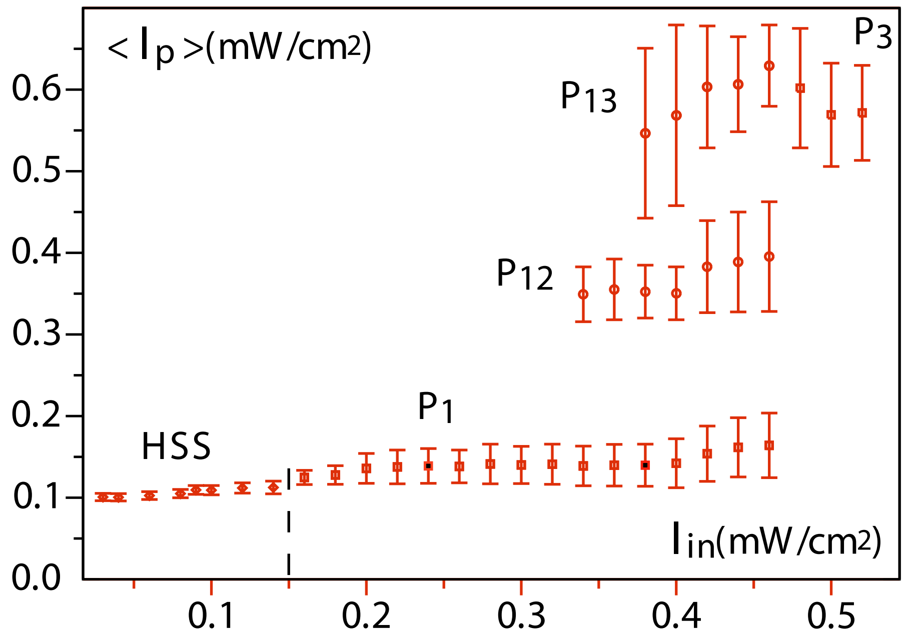
- Front Solution

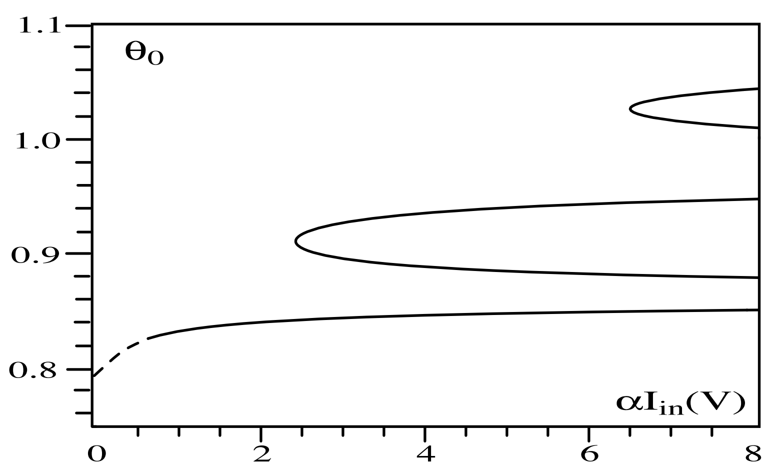


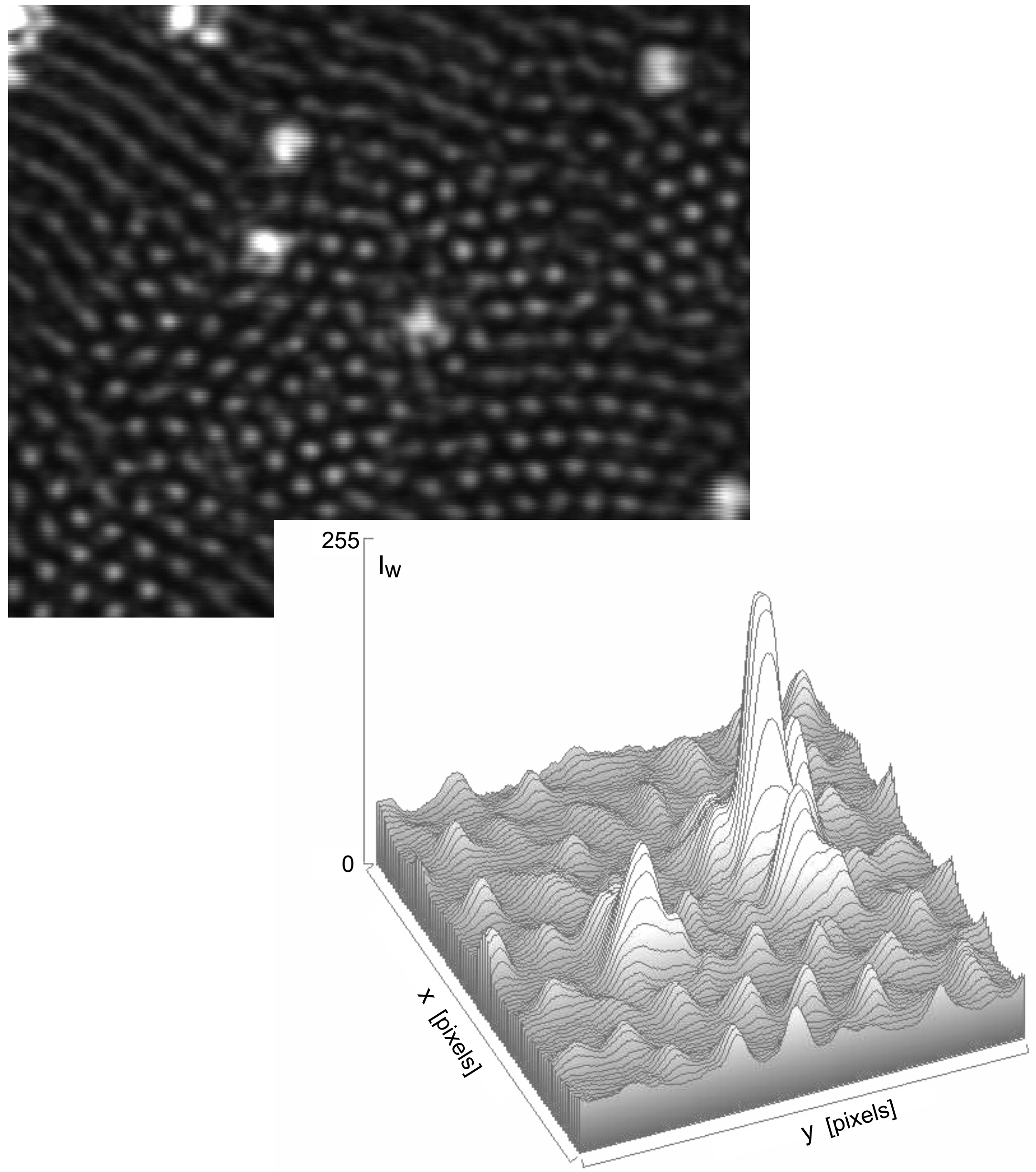
Eur. Phys. J. D 28, 435 (2004).

Bifurcation diagram of pattern states (Bi-pattern)

When L is increasing

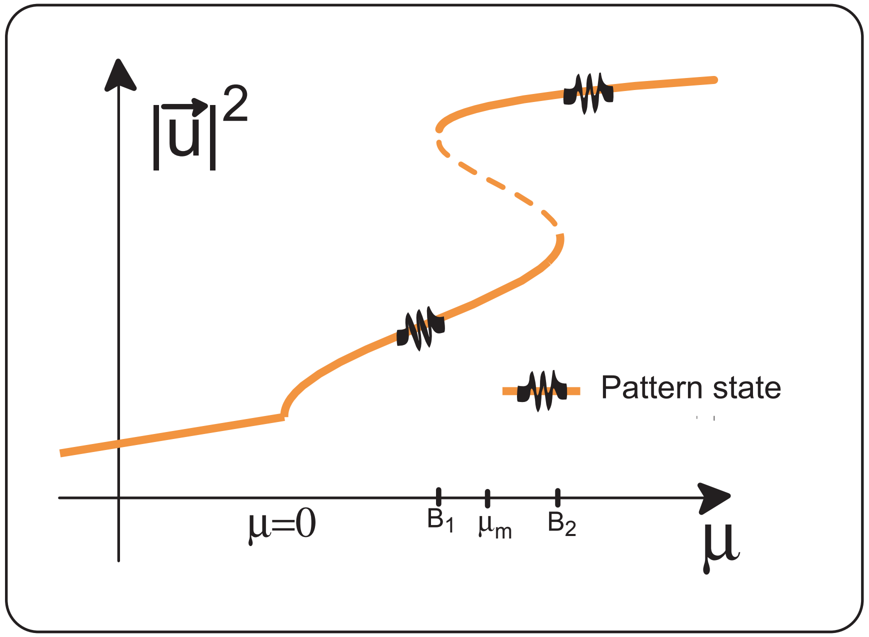


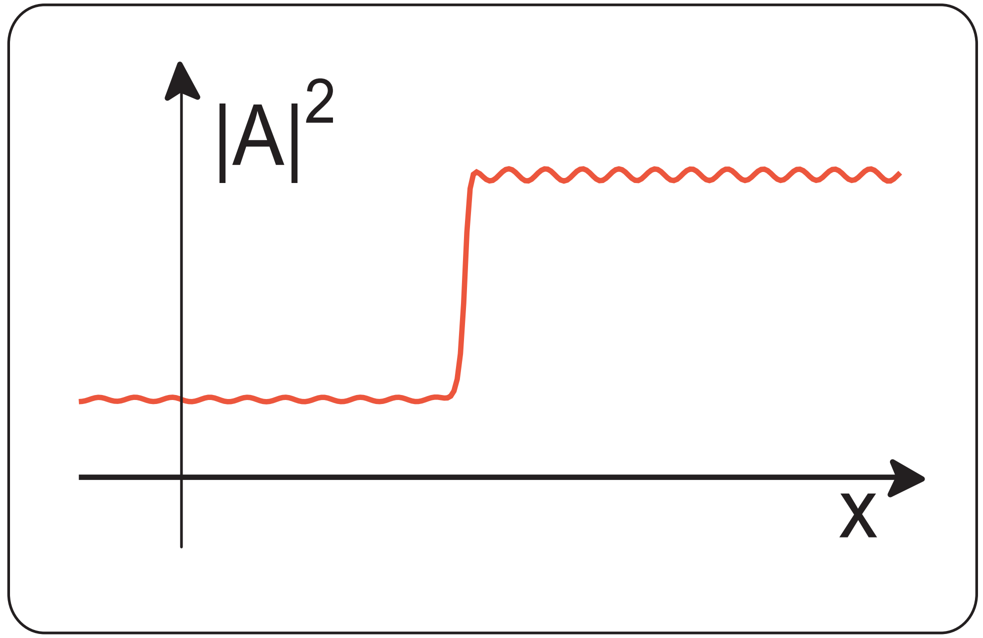




Main ingredients of super peak

- Coexistence between two patterns (bistable pattern forming systems)

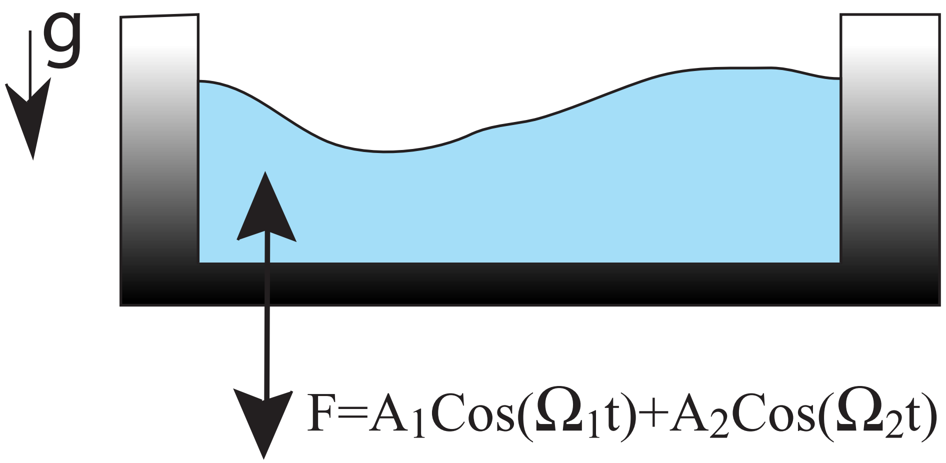


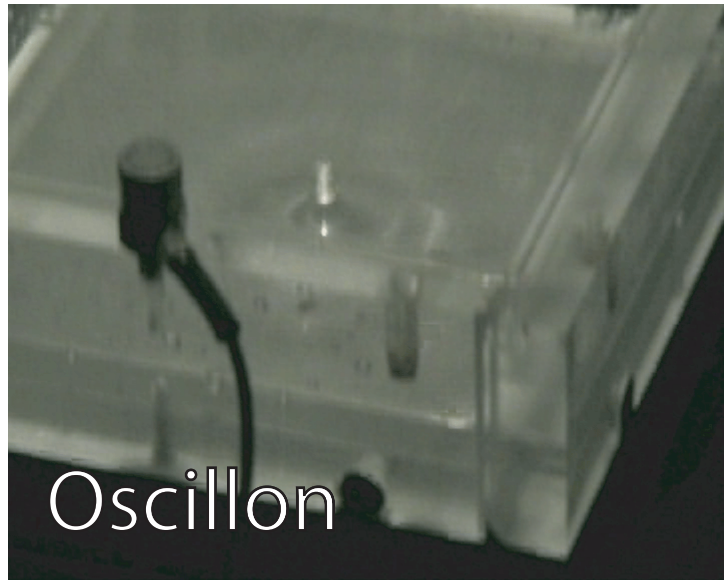


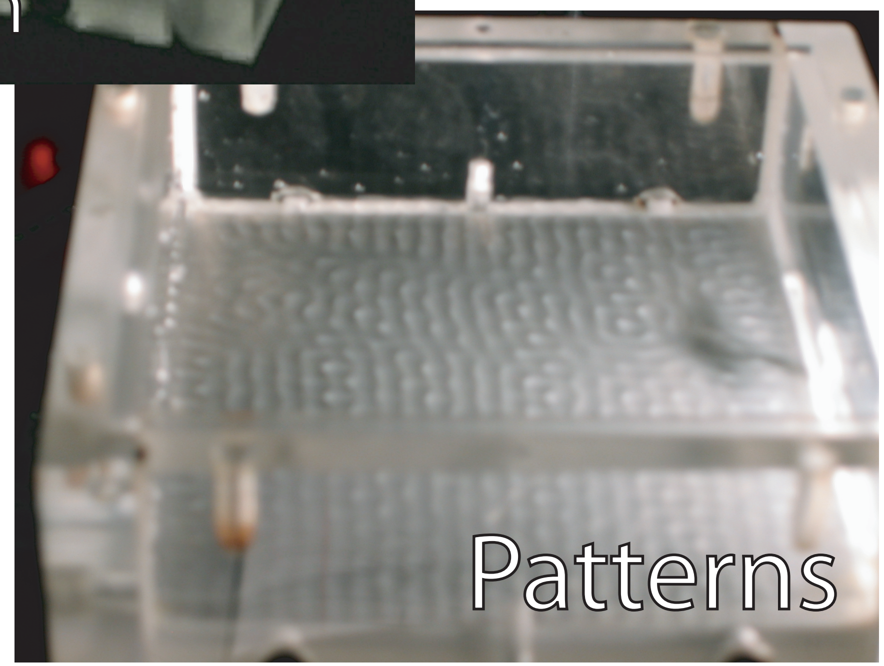
Hence, this phenomenon is Universal

Parametrically driven Newtonian fluid

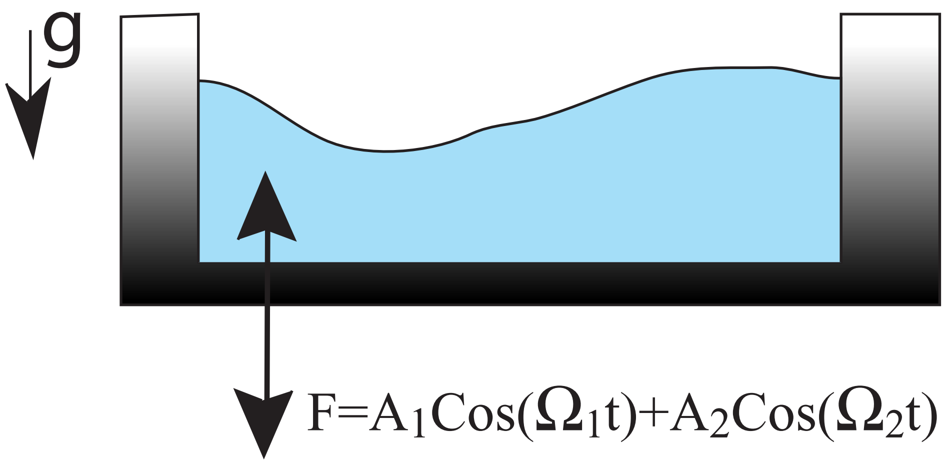
We consider a Newtonian Fluid (water-glycerine) parametrically excited with two frequencies ($\Omega_1/\Omega_2=3/2$)

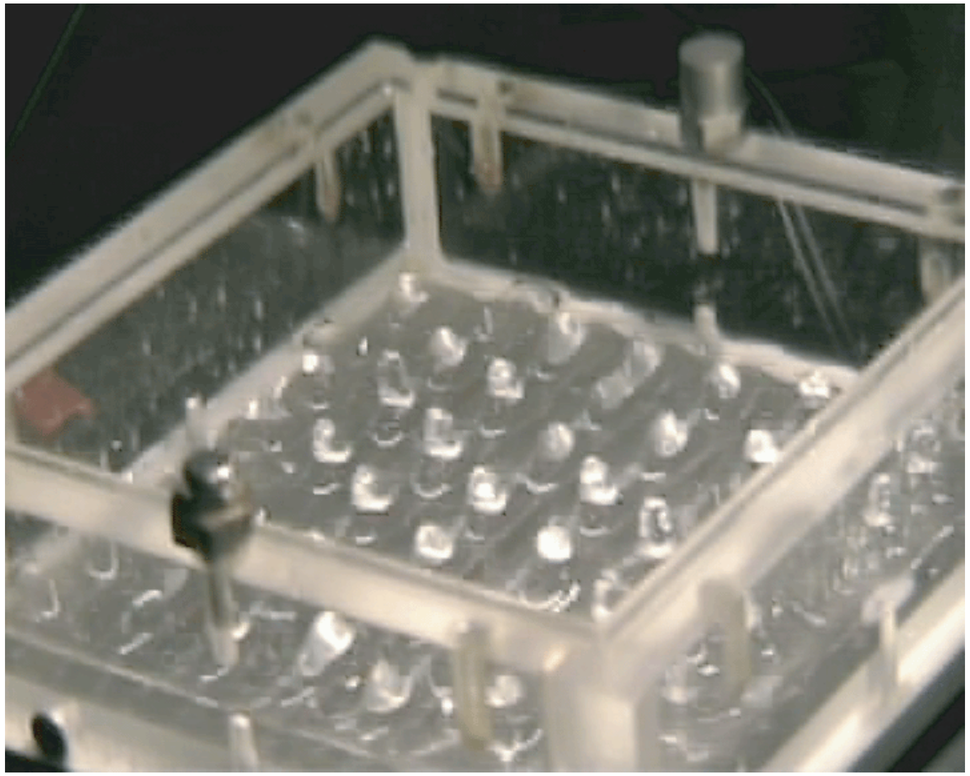






Super-peak, parametrically driven Newtonian fluid

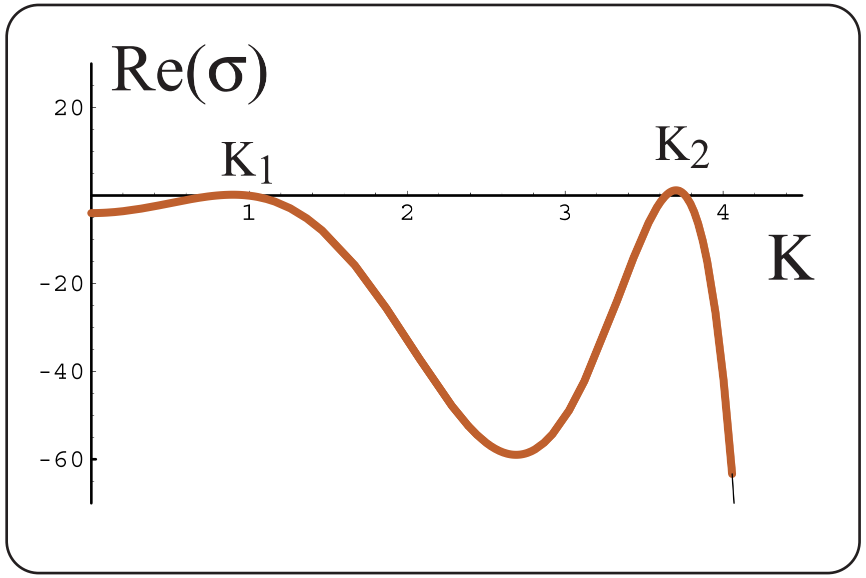




Experimental observation of Super-peak

Universal theoretical description

- One possibility is to consider the bifurcation of an uniform state characterized by to critical wave-lenght



Using the ansatz

$$u = u_o + A(X,T)e^{ik_1 x} + B(X,T)e^{ik_2 x} + c.c + h.o.t.$$

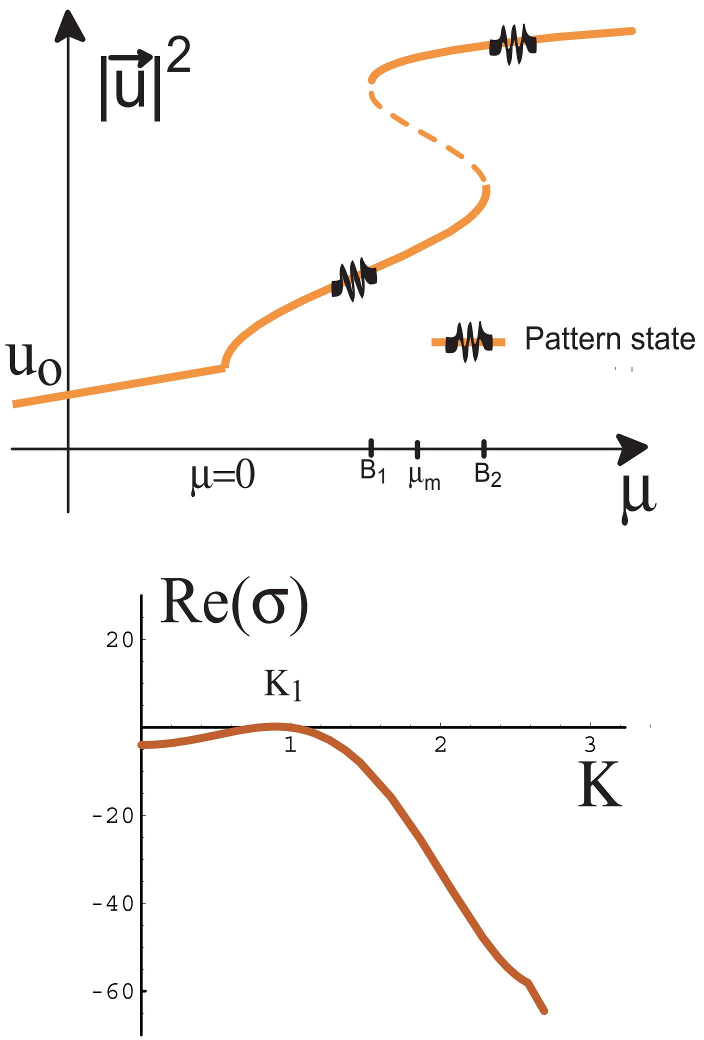
one obtain the coupled amplitude equation close to bifurcation

$$\partial_T A = \mu A - (|A|^2 + \gamma |B|^2)A + \partial_{xx} A$$
$$\partial_T B = \varepsilon B - (|B|^2 + \delta |A|^2)B + D\partial_{xx} B$$

Experimentally, in the LCLV close to the bifurcation there is only one frequency, and the second frequency do not correspond to the experimental observed.

Super-sub-critical bifurcarion

- Another possibility is consider a super-sub-critical bifurcation



Given a dynamical system of the form

$$\partial_t \vec{u} = \vec{f}\,(\vec{u}, \partial_x, \{\lambda_i\}),$$

this system has a uniform state (u_o), which exhibits a spatial super-sub-critical bifurcation. Using the ansatz

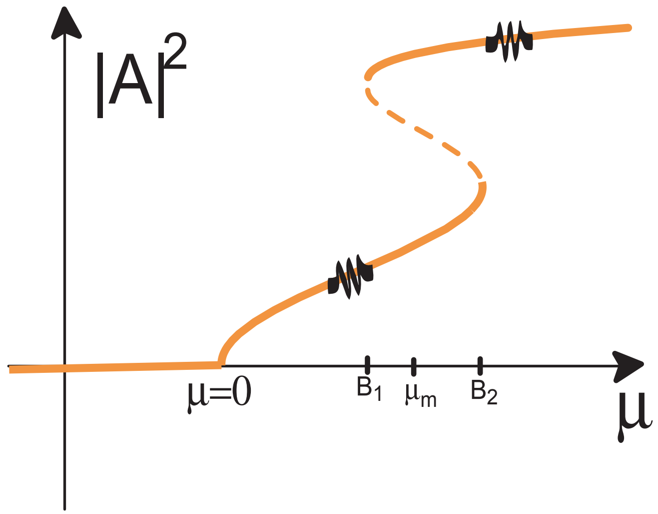
$$\vec{u} = A(X,T)e^{iqx}\hat{u} + \bar{A}(X,T)e^{-iqx}\hat{u} + \cdots$$

one obtain the amplitude equation close to bifurcation

$$\partial_T A = \mu A - \nu|A|^2 A + \alpha|A|^4 A - |A|^6 A + \partial_{XX} A,$$

where μ, ν, and α are positive.

- Bifurcation diagram of super-sub-critical model

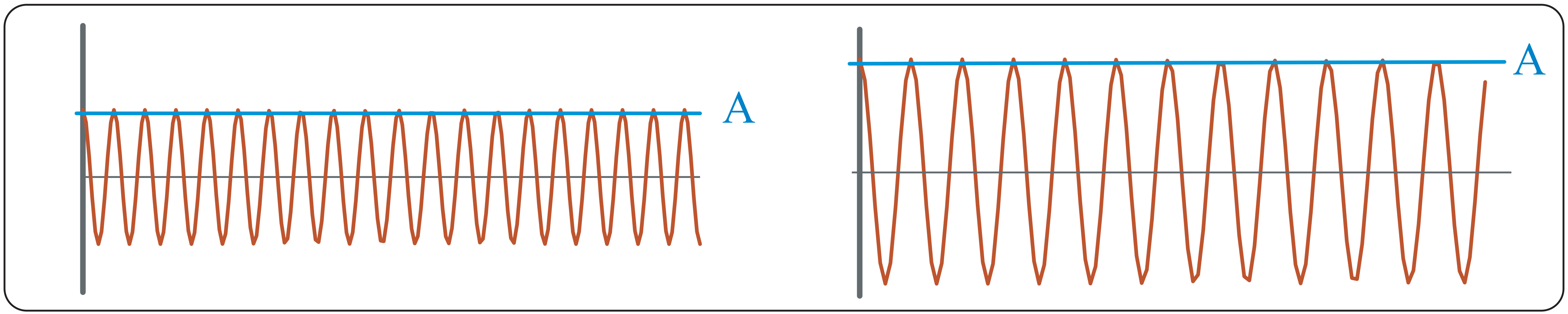


$$\partial_T A = \mu A - \nu |A|^2 A + \alpha |A|^4 A - |A|^6 A + \partial_{XX} A,$$

The homogenous or uniform states satisfy

$$A = R_o e^{i \frac{a}{R_o^2} x},$$

where $0 = \mu - a^2/R_o^4 - \nu R_o^2 + R_o^4 - R_o^6$ and "a" is a constant of montion. Therefore, if the amplitude of the pattern is increasing the wave length also increases.

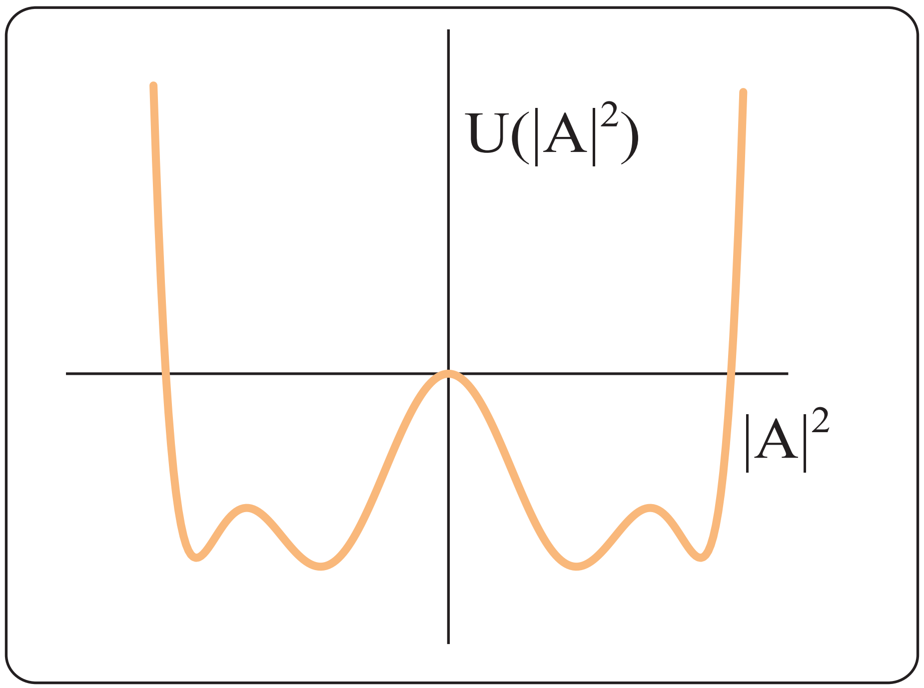


- The model is variational

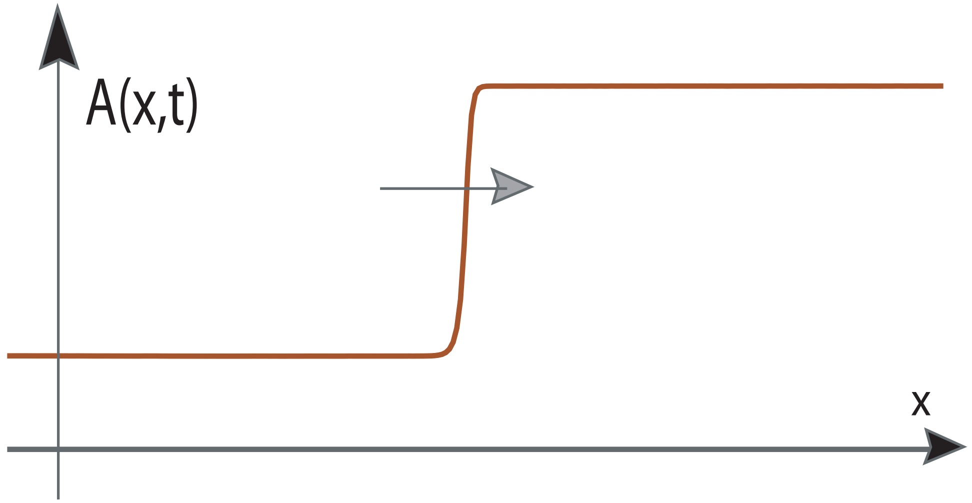
$$\partial_t A = -\frac{\delta \mathcal{F}\left[A, \overline{A}\right]}{\delta \overline{A}},$$

where $$\mathcal{F} = -\int \left(\mu \frac{|A|^2}{2} - \nu \frac{|A|^4}{4} + \alpha \frac{|A|^6}{6} - \frac{|A|^8}{8} + \frac{|\,\partial_X A\,|^2}{2} \right) dx.$$

Hence, the dynamic of this model is relaxational type.

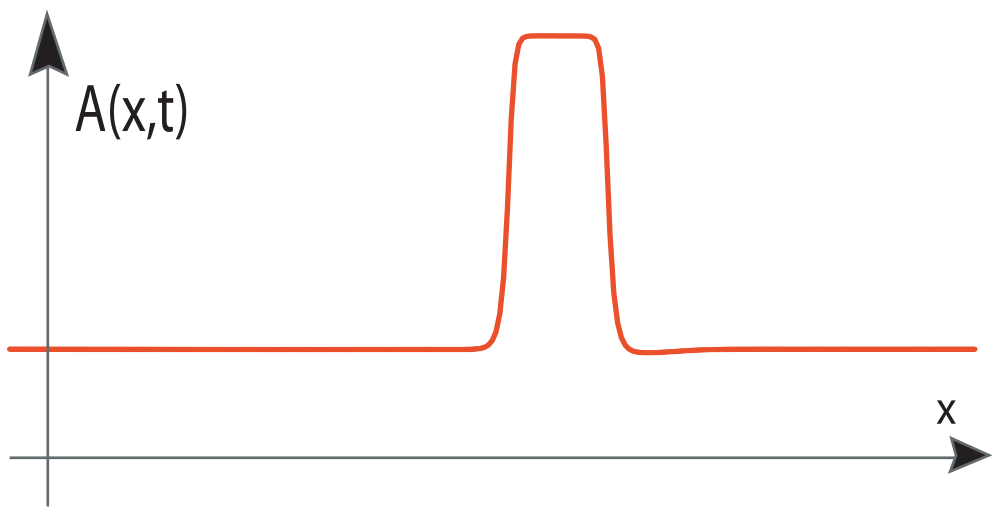


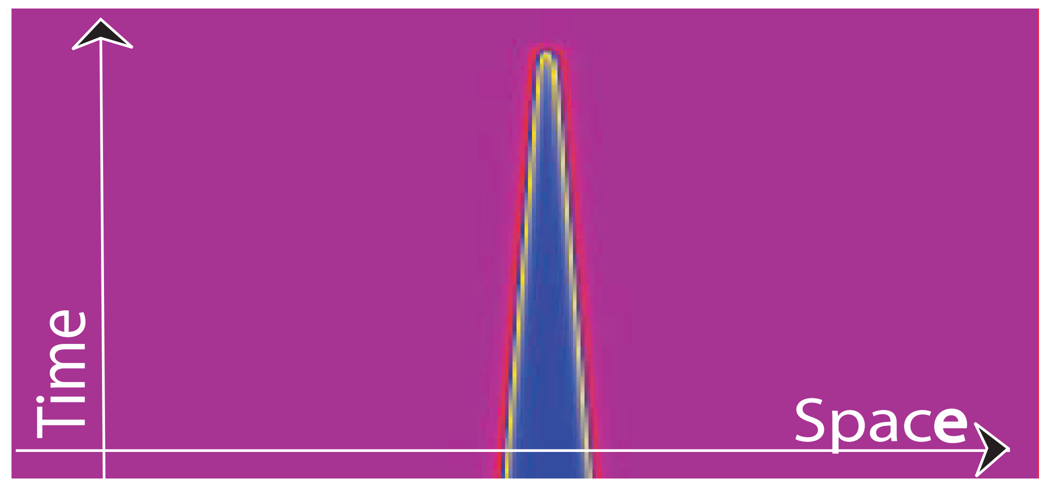
- Particle-type solution of the model (Front connection)



The system does not exhibit localized particle as consequence of kink interaction

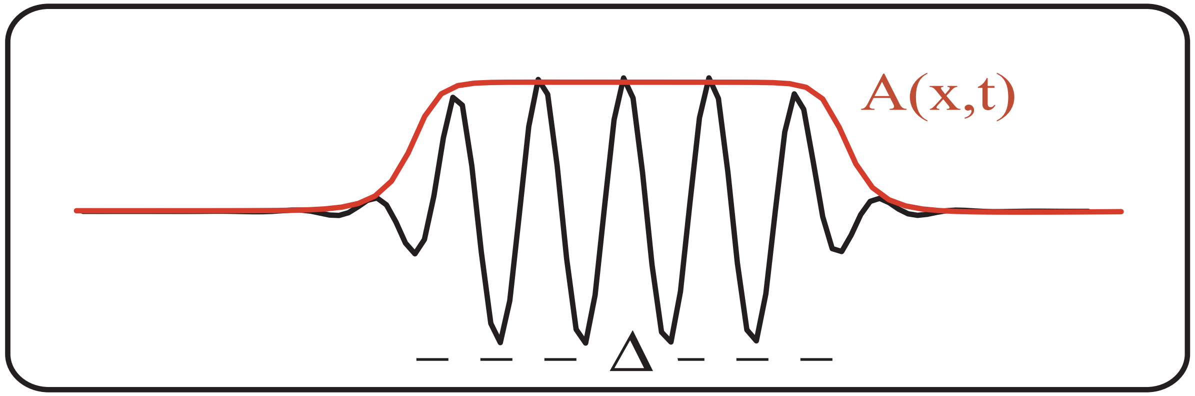
$$\dot{\Delta} = -a e^{-\lambda \Delta} + \delta,$$



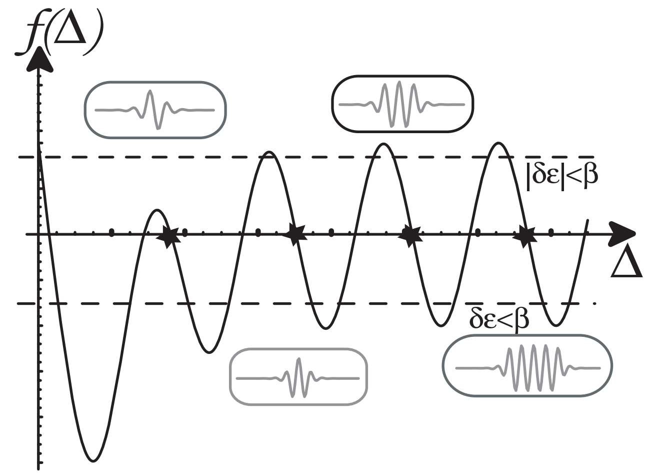


- It is well-know that the resonant amplitude equations do not describe the particle-type solutions (adiabatic effects).
- Recently using non resonant terms, we have explained the localized patterns and noise induces front propagation

Localized patterns

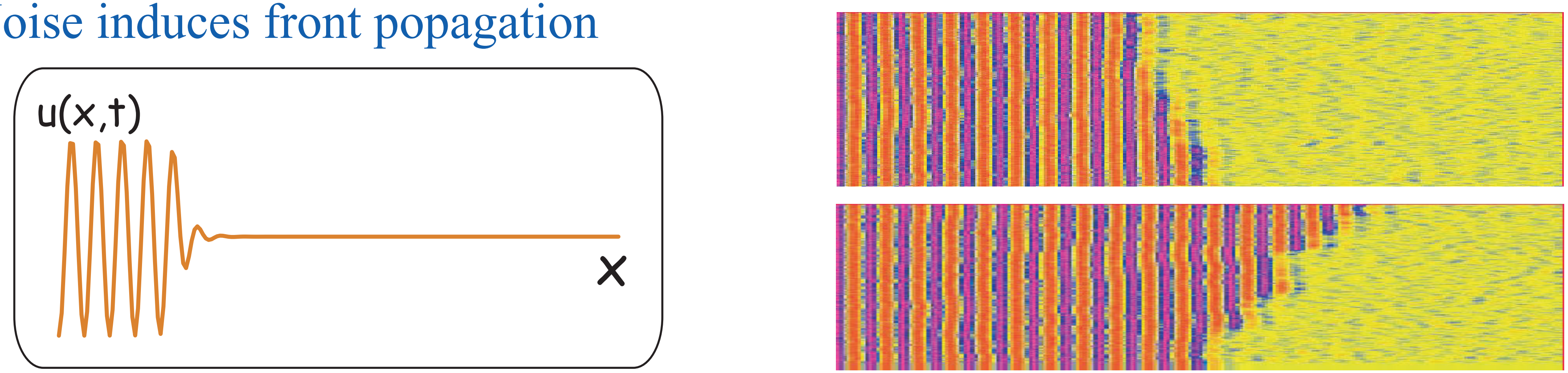


Kink interaction



To appear in Physica A

Noise induces front popagation



Phys. Rev. Lett. 94, 148302 (2005).

Amplitude equation with non-resonant term

The non resonant terms are consequence of the interaction of the large scale envelope A(x,t) with the small scale underlying in the spatial periodic solution. The amplitude equation takes the form

$$\begin{aligned}\partial_T A &= \mu A - \nu |A|^2 A + \alpha |A|^4 A - |A|^6 A + \partial_{XX} A \\ &+ \sum_{m,n \geq 0} g_{mn} A^m \bar{A}^n e^{iq(1+n-m)x}\end{aligned}$$

For the sake of simplicity we consider

$$\boxed{\begin{aligned}\partial_T A &= \mu A - \nu |A|^2 A + \alpha |A|^4 A - |A|^6 A + \partial_{XX} A \\ &+ \eta A^2 e^{-i\frac{q}{\sqrt{\mu}}X}\end{aligned}}$$

where η is the amplitude of the parametric forcing and $\mu \ll 1$.

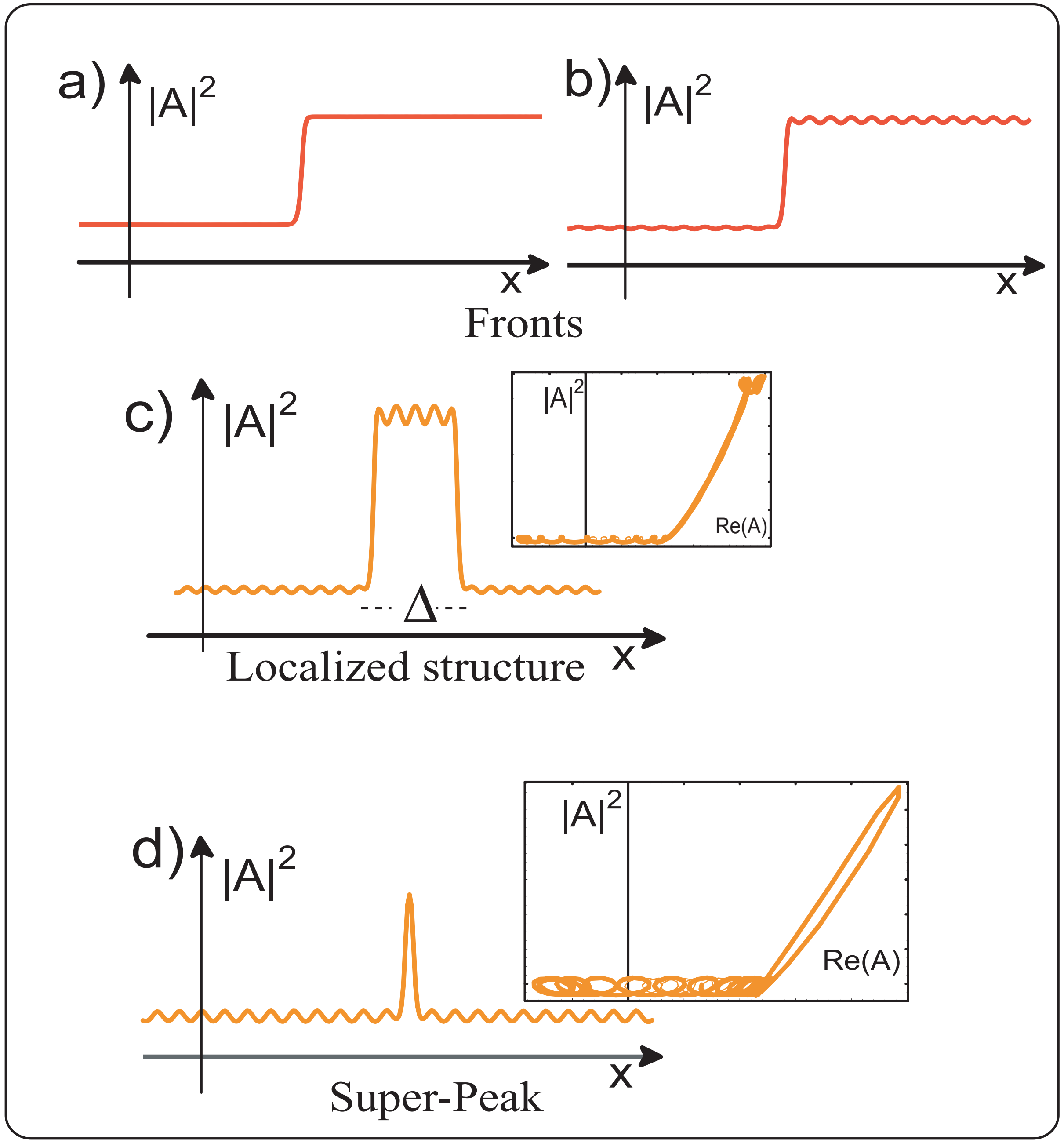
Super- Peak

- As result of parametrical spatial forcing the system exhibits locking phenomena. Hence, there is a pining range for the front conections.

- As consequence of the kink interaction the system presents a familly of localized structure. This interaction has the form

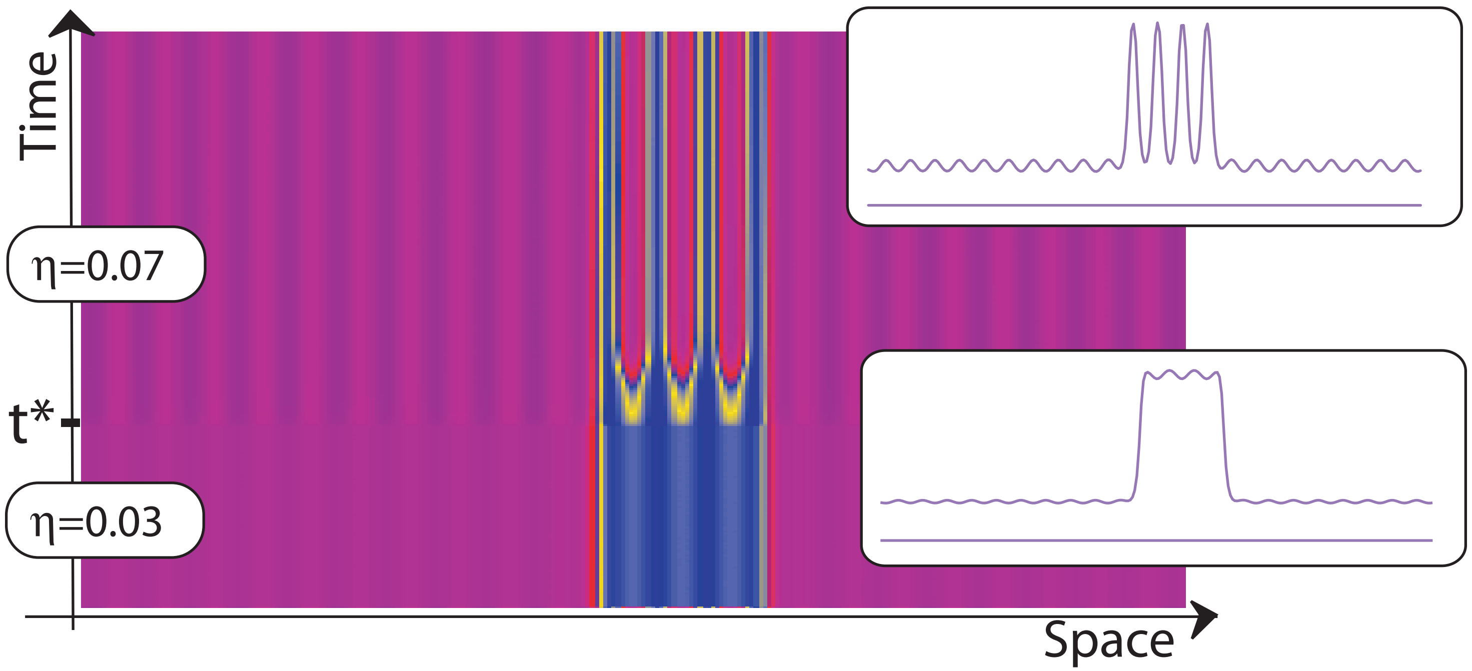
$$\dot{\Delta} = -ae^{-\lambda\Delta} + \delta + \gamma \cos\left(\frac{q}{\sqrt{\mu}}\Delta\right),$$

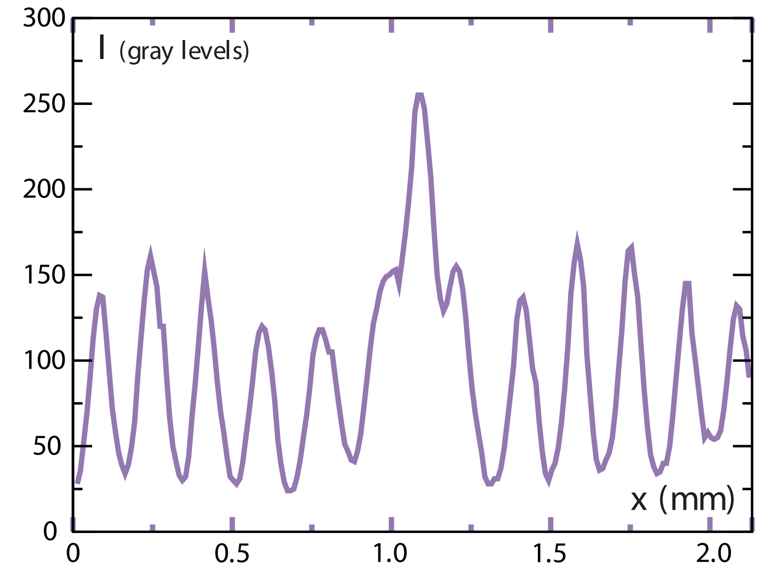
- The smallest of these solution we term SUPER-PEAK



Super- Peaks are robust solution

- The mechanism for super-peak appearance is related to the fact that the spatial forcing is nonlinear. Thus, for the same forcing the amplitude of oscillations for the upper branch is larger than the amplitude of oscillations for the lower branch. Hence, for a critical, and small, value of the forcing the upper oscillations collide with the unstable branch, giving rise the appearance of a super-peak.





Super-peak observed in one-dimensional set up of LCLV with optical feedback

Conclusion

- We have presented an unified description of super-peak, which is a large peak nucleating over a pattern.

- This particle-like state is a generic class of behavior appearing whenever a pattern forming system exhibits coexistence of twos patially periodic states.

- The front solution that connect two different pattern states exhibit a locking phenomena, that is, the front is motionless for a range of parameter.

- We have obtained the front interaction. From this interaction, we deduce the family of localized solution exhibit by the system. As consequence of nonlinear intensity of the forced, localized patterns with a size larger than the shortest length are not robust phenomena and the typical behavior observed in the experiments is the appearance of localized-peaks.

- We also put in evidence this robust phenomenon in different one-dimensional experiments, like a perimetrically driven Newtonian fluid and a nematic liquid crystal valve with optical feedback. The experimental observations are in a good qualitative agreement with the numerical results.