NOISE INDUCES FRONT PROPAGATION

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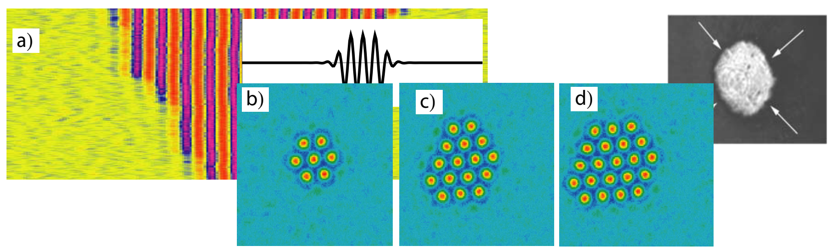
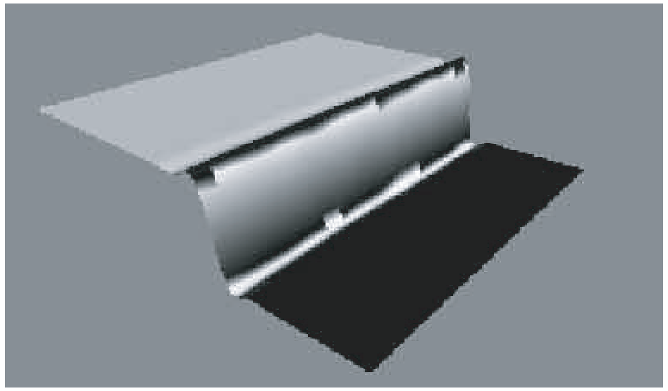


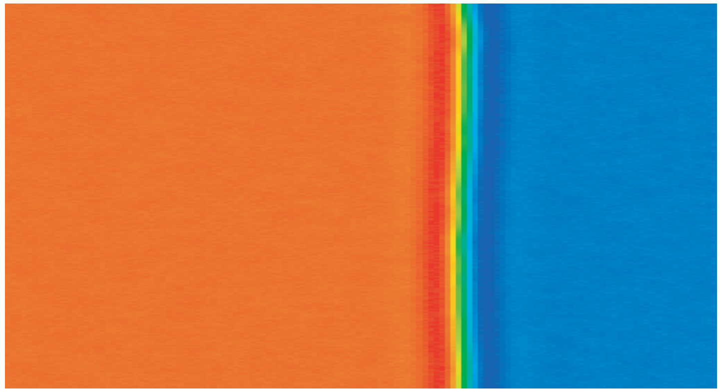
Outline

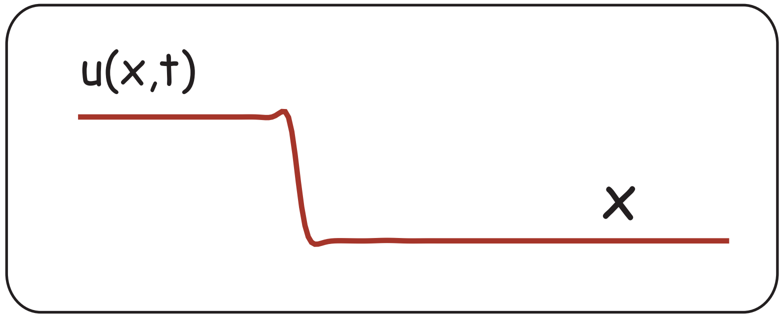
- Front solutions, dynamics behaviors close to Freederickz transition.
- Front connection between an spatial periodic states and homogrenous one.
- Noise
- Noise +Front
- Mechanism of Noise induces Front propagation
- Front stochastic speed
- Generalization
- Conclusions
- Outlook

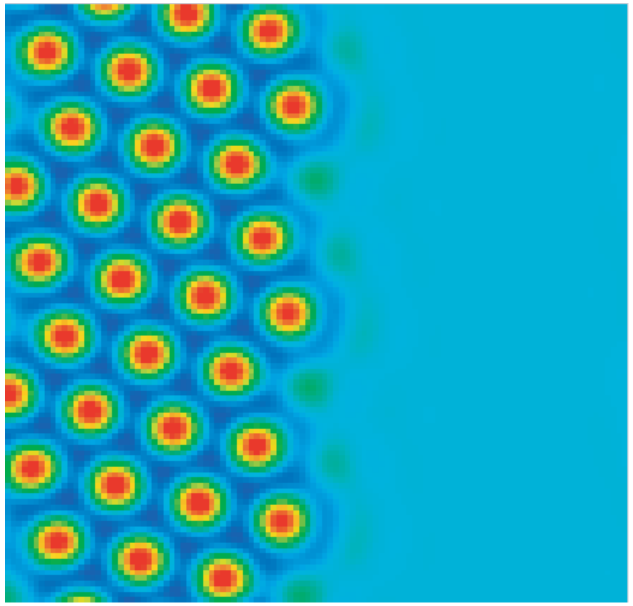
Front solution

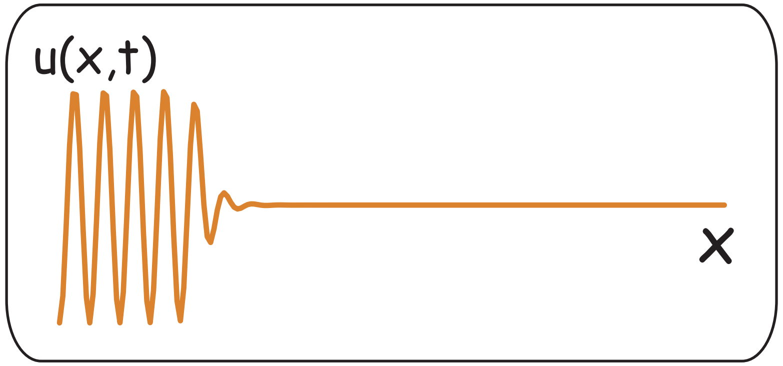
- "Stationary solution that links two steady states".
- "Heteroclinic connection of stationaries states in the stationary extended system or moving reference frame" (multi-stability).





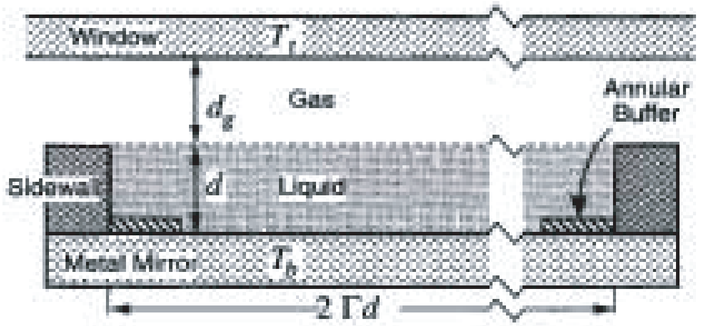




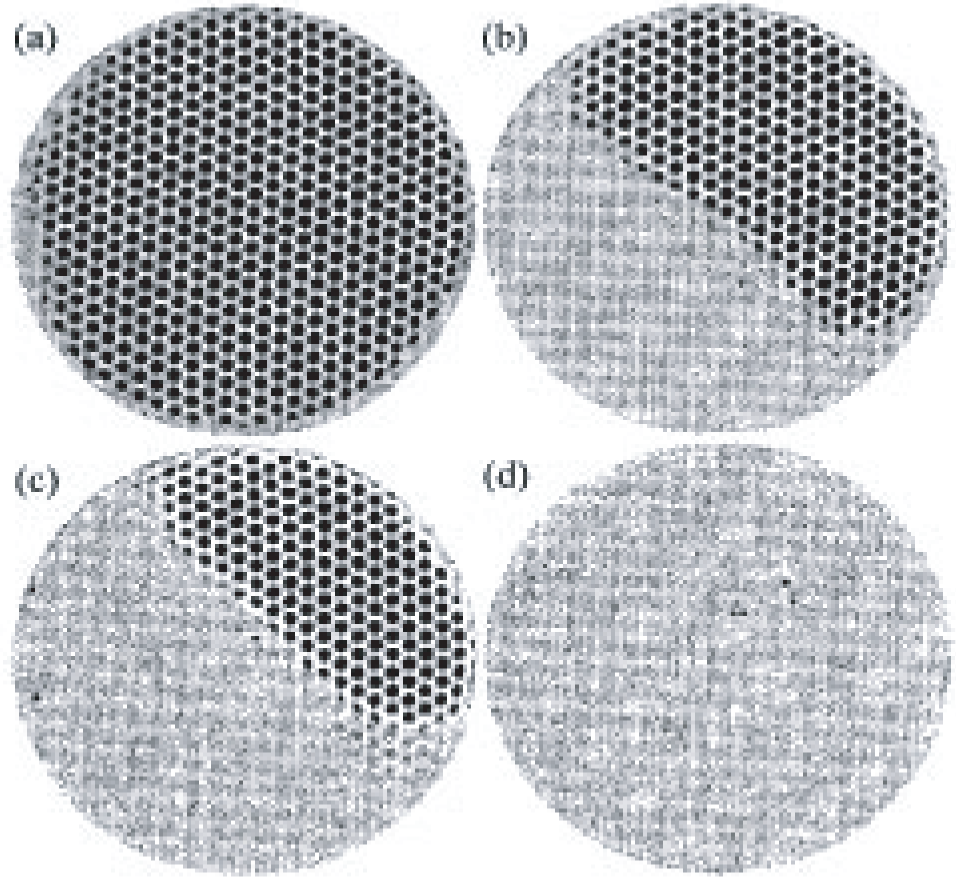


Fronts and experiments

- Benard-Marangoni



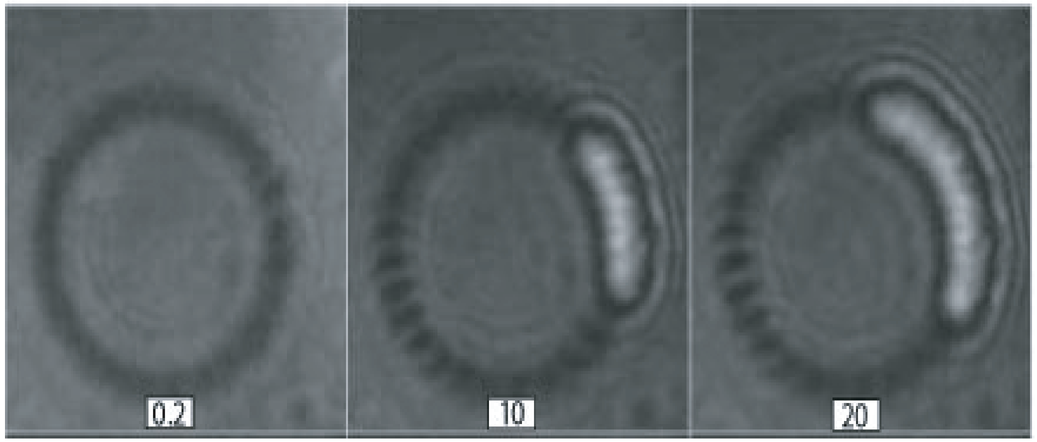
Front propagation

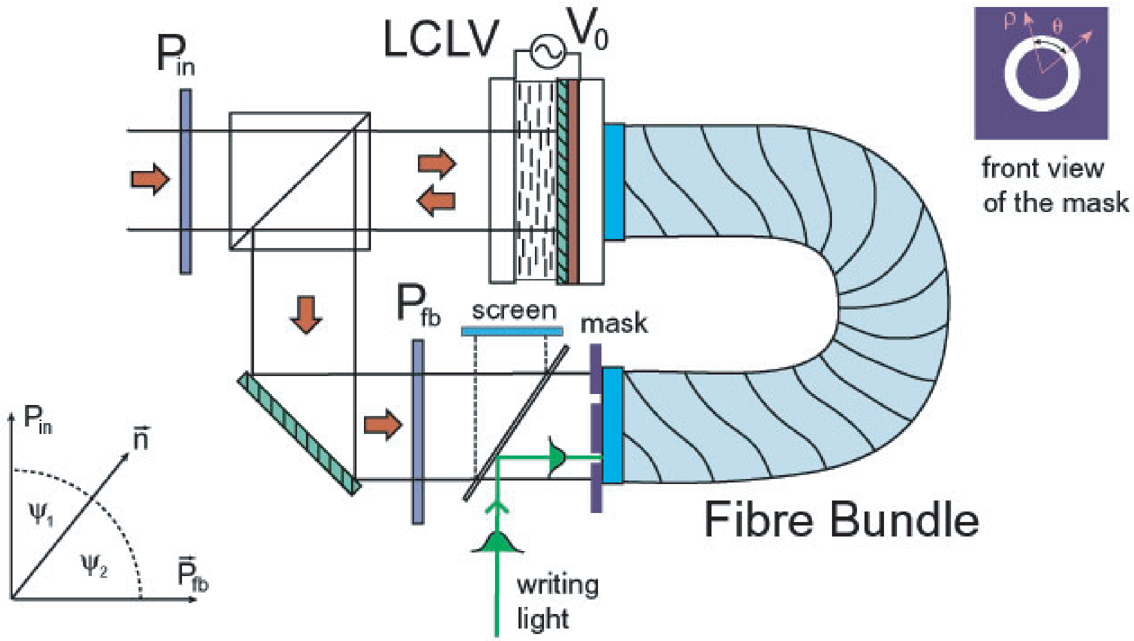


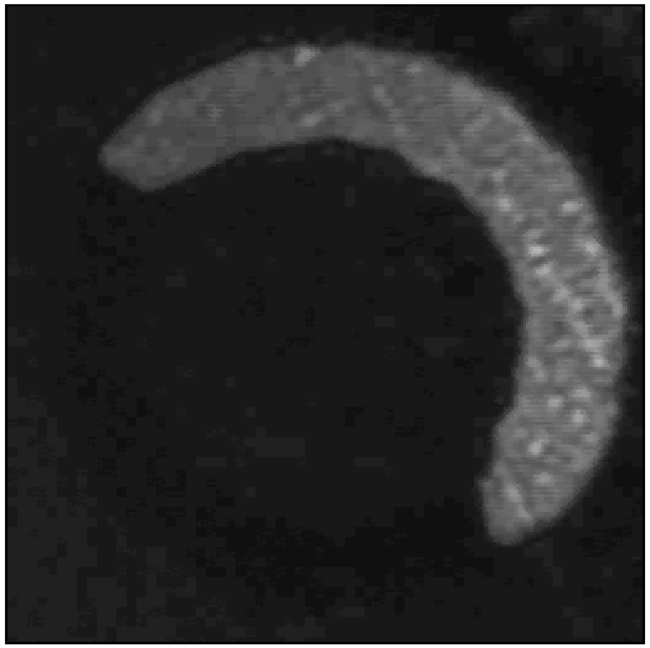
MF Schatz et at, Phys. Rev. Lett. 75, 1938 (1995)

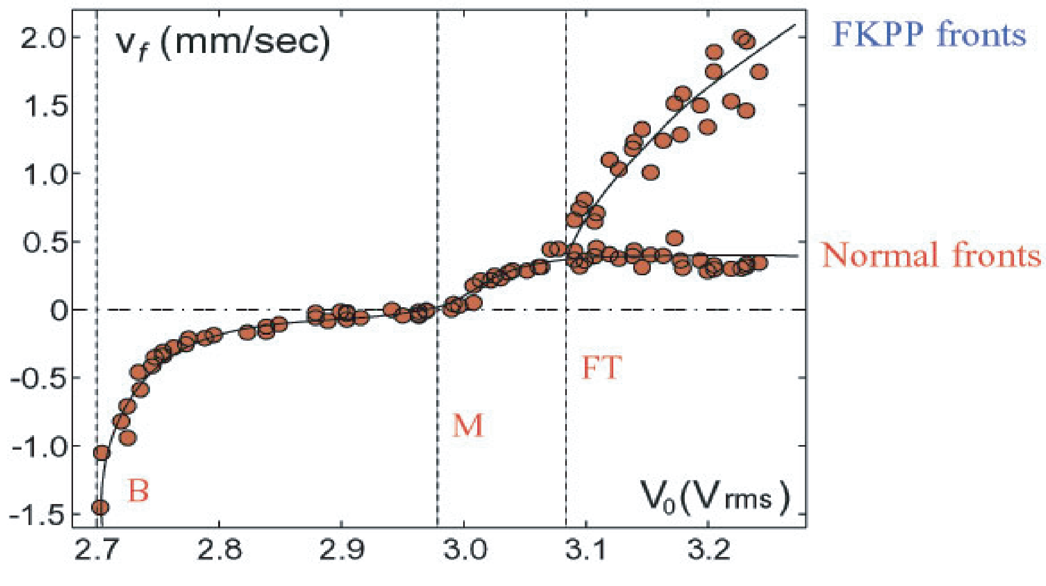
Experimental measurement of front velocity

- Liquid crystal light valve with optical feedback (1-D experiment)





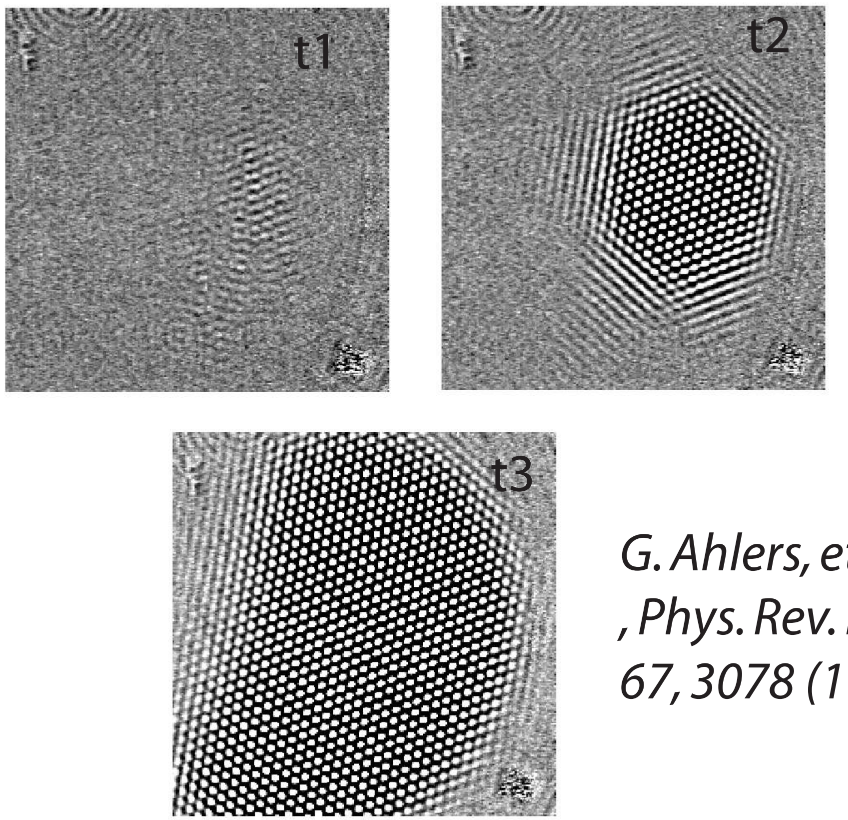




M.G. Clerc et al, Eur. Phys. J. D 28, 435 (2004).

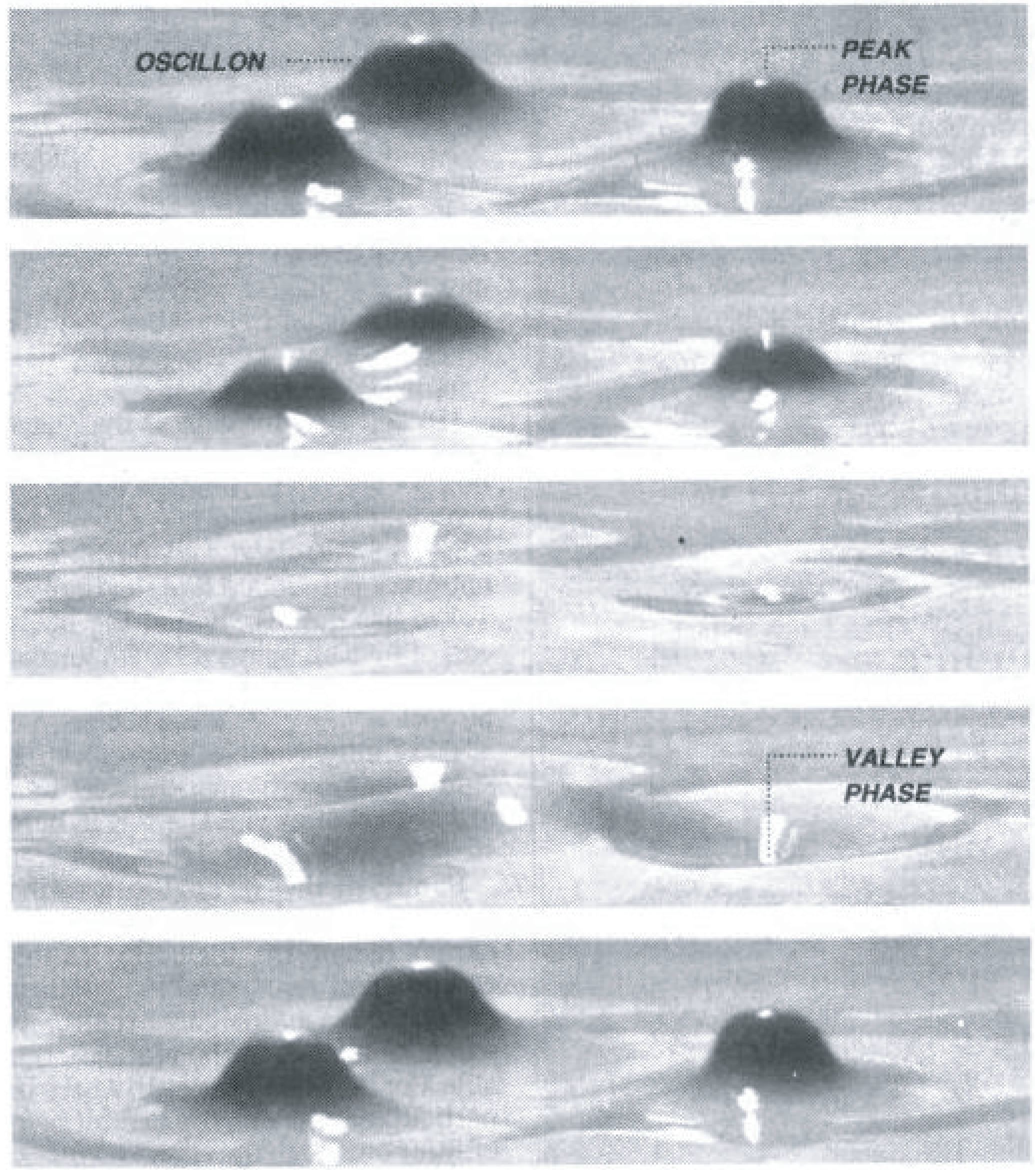
Front connection between an spatial periodic states and homogrenous one

- Pattern in Benard -Marangoni convection



G. Ahlers, et al , Phys. Rev. Lett. 67, 3078 (1991)

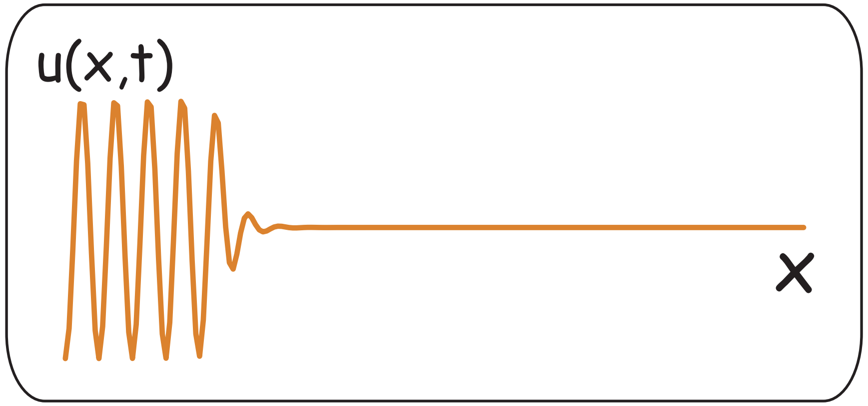
- Oscillon in a colloidal fliud



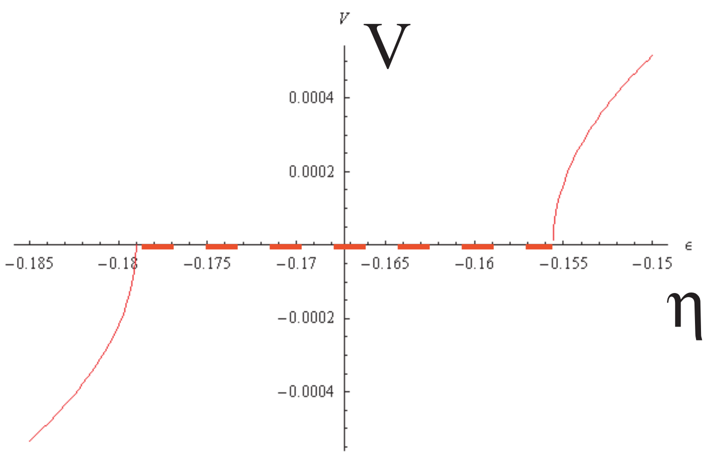
Agnon et al, PRL, 83,16 (1999)

Properties in one extended systems

- Locking phenomenon and pinning range

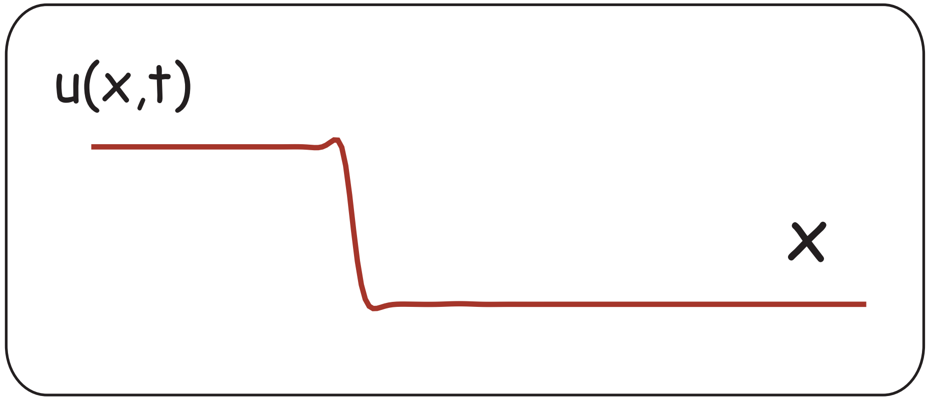


The front is stationary in a width range of parameters, pinning range

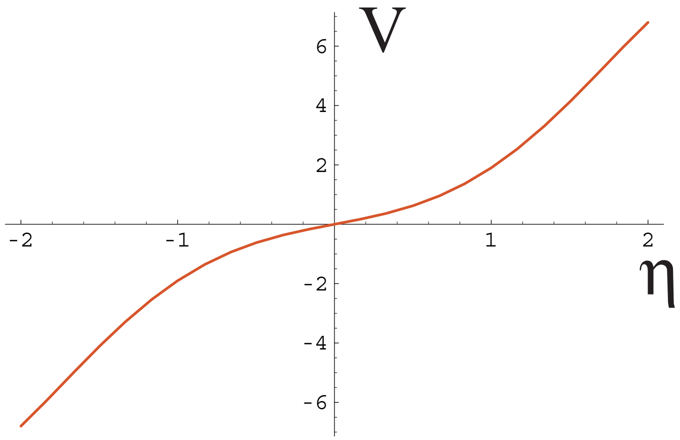


Y Pomeau, Physica D 23, 3 (1986)

- Normal form



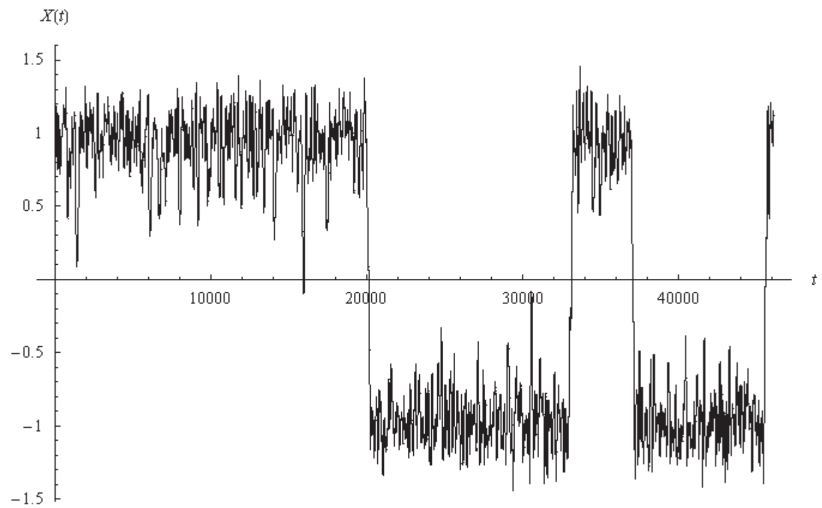
The front is stationary in one point, Maxwell point.



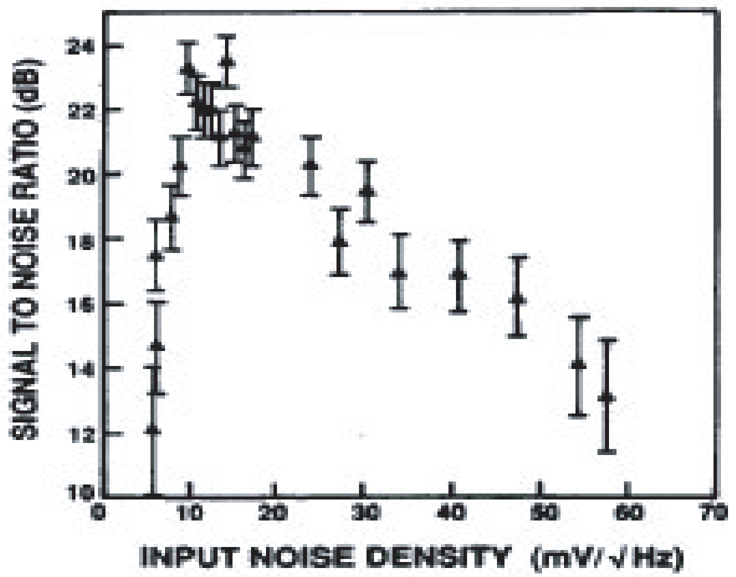
Noise

- The influence of noise in nonlinear systems has been the subject of intense experimental and theoretical investigations.
- Far from being merely a perturbation to the idealized deterministic evolution or an undesirable source of randomness and disorganization, noise can induce specific and even counterintuitive dynamical behavior.

Noise induced transition

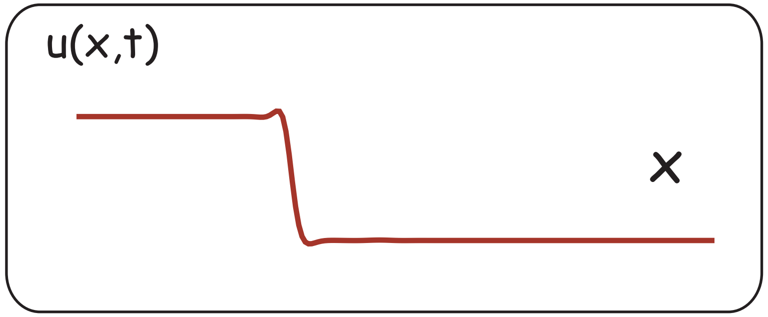


Stochastic resonances

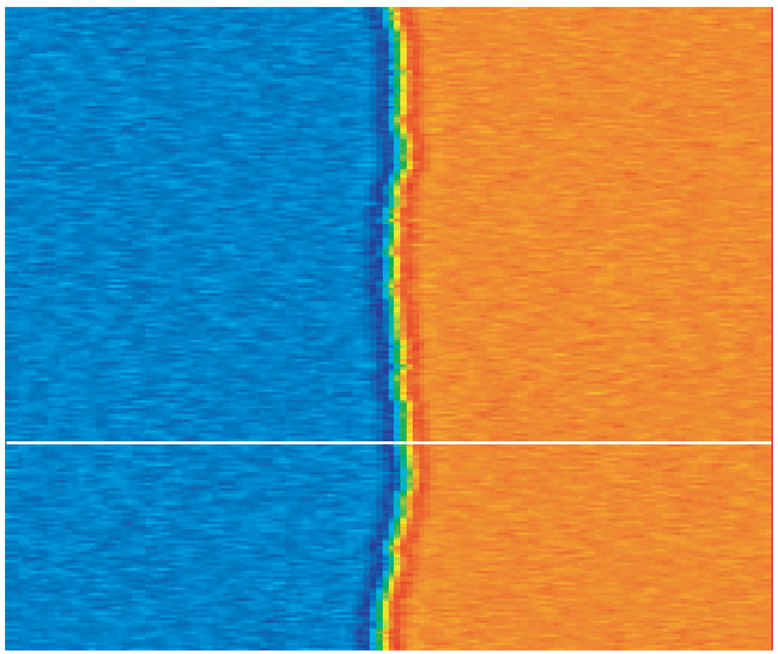


Noise+Front

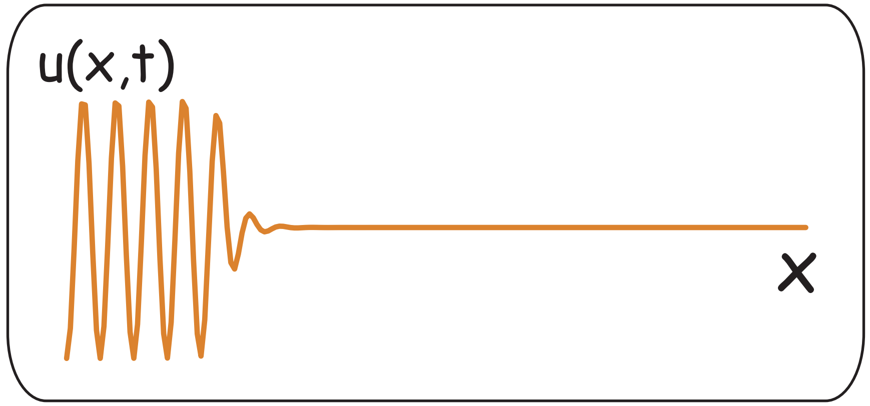
- Normal front

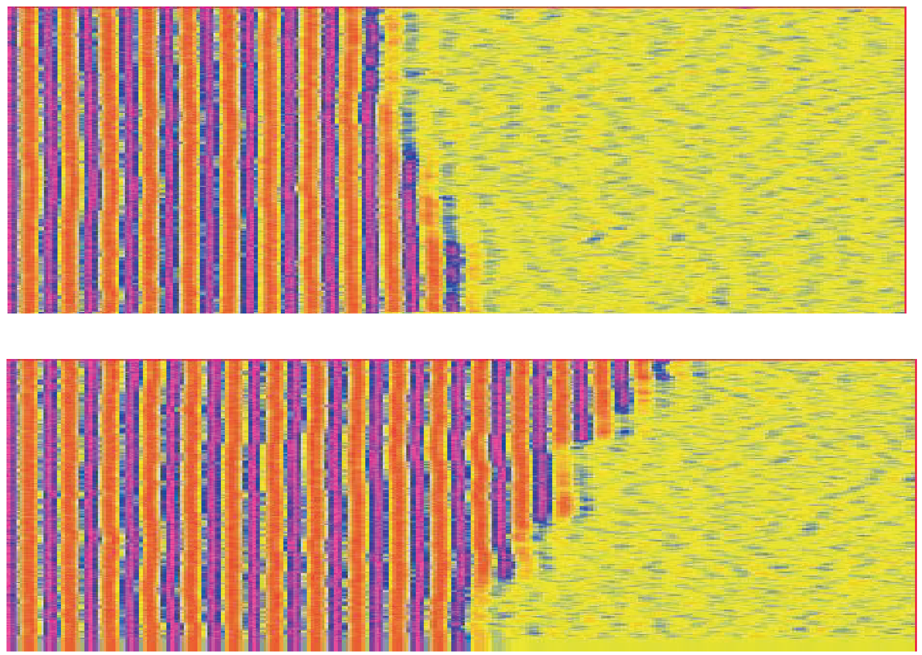


Spatio-temporal diagram



- Patterns-homogeneous





NOISE INDUCES FRONT PROPAGATION

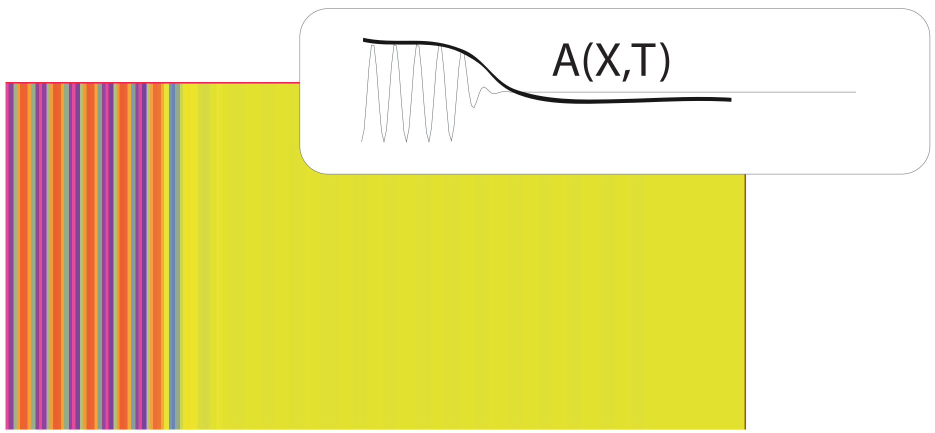
Mechanism of Noise induces Front propagation

- Swift-Hohenberg Model

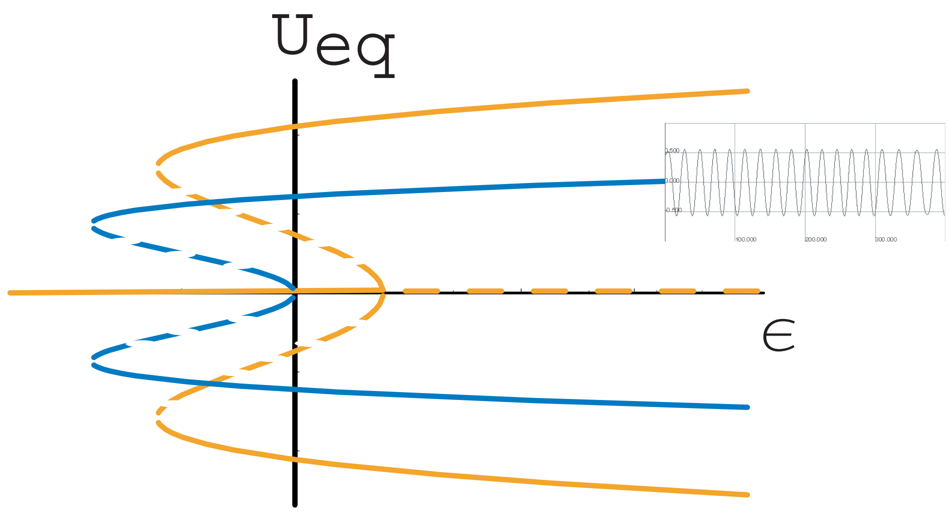
$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - (\partial_{xx} + q^2)^2 u + \sqrt{\eta}\zeta(x,t)$$

where $\langle \zeta(x,t)\,\zeta(x',t')\rangle = \delta(x-x')\,\delta(t-t')$

- Front soltions



- Bifurcation Diagram



Mechanism of Noise induces Front propagation

- Using the ansatz

$$u(x,t)=\nu^{1/2}A(X,T)e^{iqx}+cc.+\nu^{5/2}W(X,x,T)e^{3iqx}+h.o.t.$$

one obtains the envelope equation

$$\partial_\tau A = \epsilon A + |A|^2 A - |A|^4 A + \partial_{yy} A$$
$$+\left(\frac{A^3}{9\nu} - \frac{A^3|A|^2}{2}\right)e^{\frac{2iqy}{a\sqrt{|\varepsilon|}}} - \frac{A^5}{10}e^{\frac{4iqy}{a\sqrt{|\varepsilon|}}} + \frac{\sqrt{\eta}b}{|\varepsilon|^2}e^{\frac{iqy}{a\sqrt{|\varepsilon|}}}\zeta(y,\tau)$$

when the non-resonant termrs are negligible, the system has an analytical front solutions

$$A_\pm = \sqrt{\frac{3/4}{1+e^{\pm\sqrt{3/4}(y-y_o)}}}e^{i\theta}$$

Mechanism of Noise induces Front propagation

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Non-resonant terms

when the non-resonant termrs are negligible, the system has an analytical front solutions

$$A_\pm = \sqrt{\frac{3/4}{1+e^{\pm\sqrt{3/4}(y-y_o)}}}e^{i\theta}$$

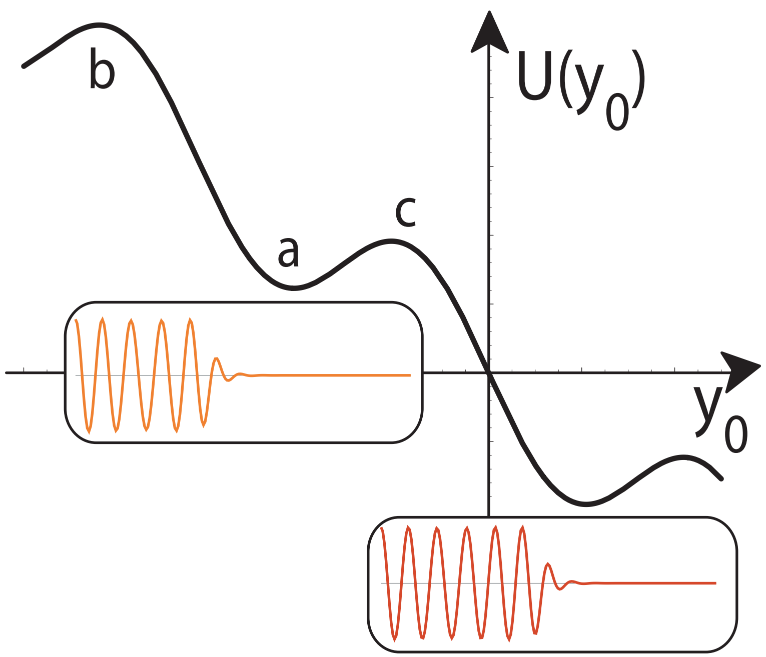
Mechanism of Noise induces Front propagation

- In order to study the dynamics of the core front

$$A(y,\tau) = (A_+(y - y_o(\tau)) + \delta\rho)e^{i\delta\Theta}$$

where

$$\begin{aligned}\dot{y}_o &= -\frac{\partial U(y_0)}{\partial y_o} + \frac{ab}{|\epsilon|^2}\sqrt{\frac{\eta}{2d}}\zeta(\tau) \\ &= \Delta + \Gamma\cos\left(\frac{2q}{d\sqrt{|\epsilon|}}y_o - \varphi\right) + \frac{ab}{|\epsilon|^2}\sqrt{\frac{\eta}{2d}}\zeta(\tau)\end{aligned}$$



- Brownian motor: The conversion of random fluctuations into direct motion of front core is responsible of the propagation.

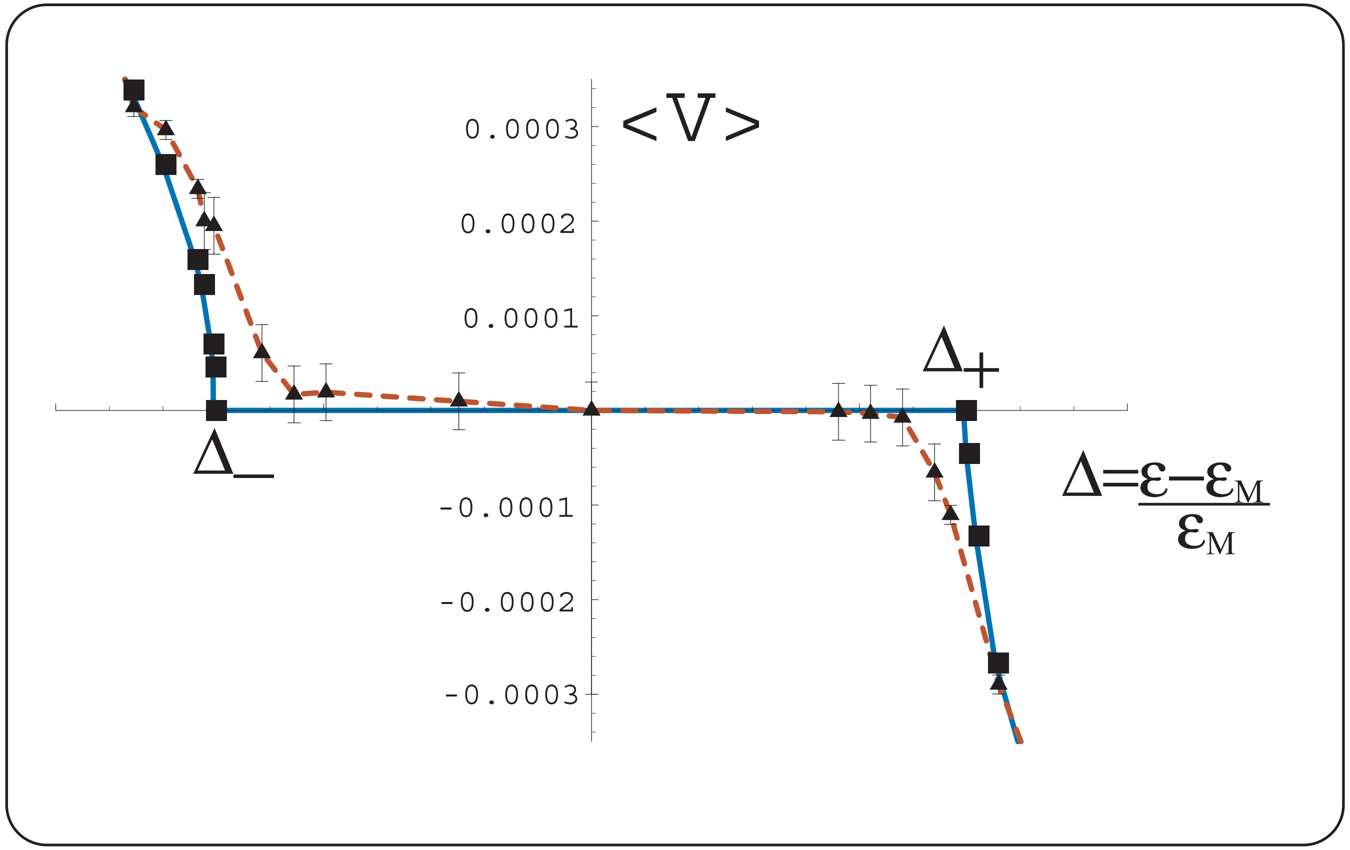
Front stochastic speed

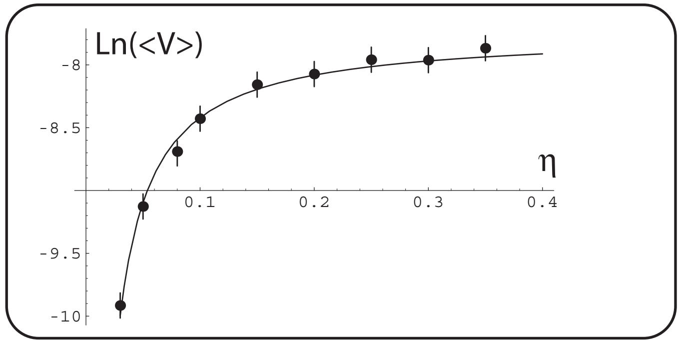
- Stochastic speed

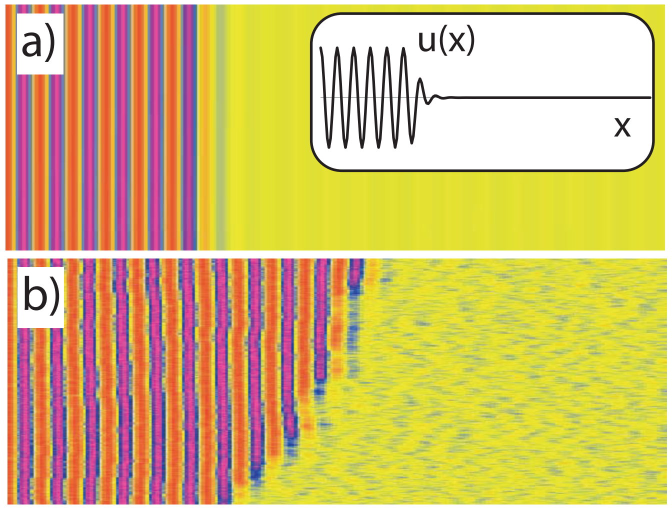
$$\langle v\rangle = \frac{\pi\sqrt{|\epsilon|}}{qa}\left(\frac{1}{\tau_+}-\frac{1}{\tau_-}\right)$$

- In the limit of weak noise

$$\langle v\rangle \;=\; \frac{2\sqrt{|\epsilon|}}{qa\sqrt{\partial_{yy}U\left(a'\right)\left|\partial_{yy}U\left(c'\right)\right|}}e^{-\frac{(U(c')-U(a'))}{\theta}}\left(1-\sqrt{\frac{\left|\partial_{yy}U\left(c'\right)\right|}{\left|\partial_{yy}U\left(b'\right)\right|}}e^{-\frac{(U(b')-U(c'))}{\theta}}\right).$$







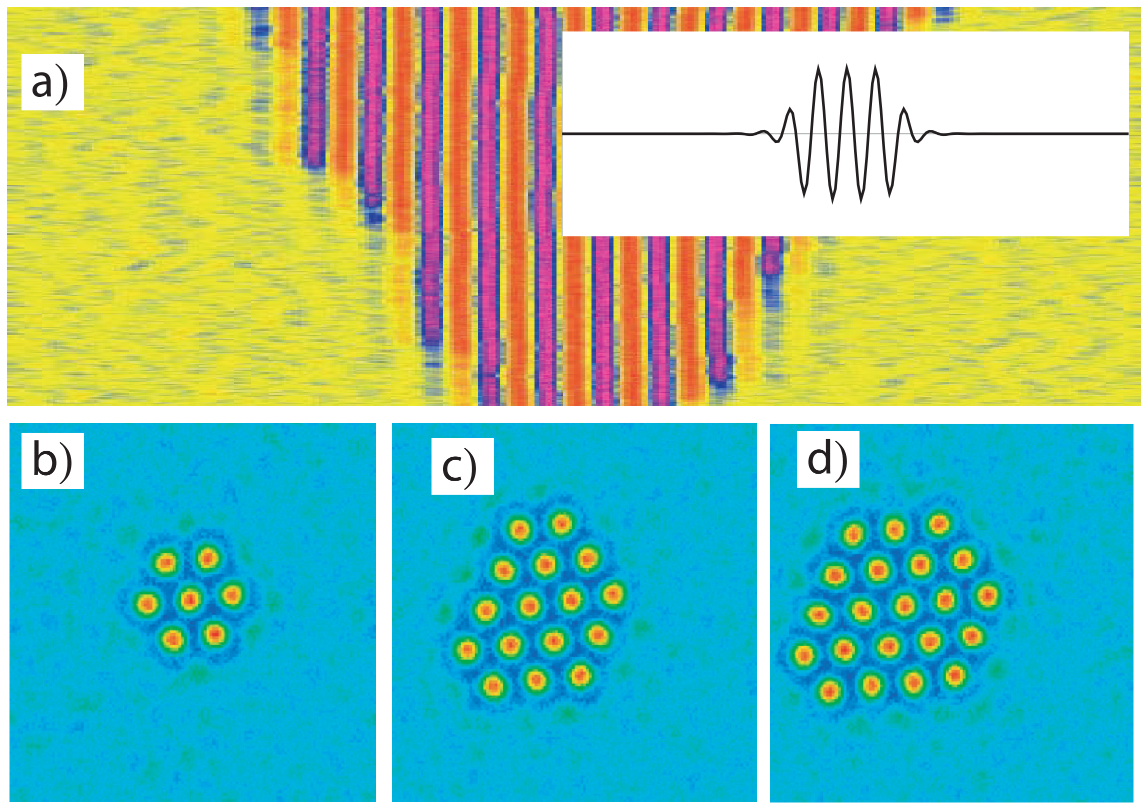
Generalization

- A system that exhibits a front between a patterns and homogenoeus state, symmetry arguments $\{x \to -x,\ A \to \bar{A}\}$ $\{x \to x + x_o,\ A \to Ae^{iqx_o}\}$
  The envelope equation satisfies

$$\partial_T A = f\left(|A|^2\right) A + \partial_{XX} A + \sum_{m,n} g_{mn} A^m \bar{A}^n e^{iq(1+n-m)x}$$

  One has analogous arguments.

- Localized patterns

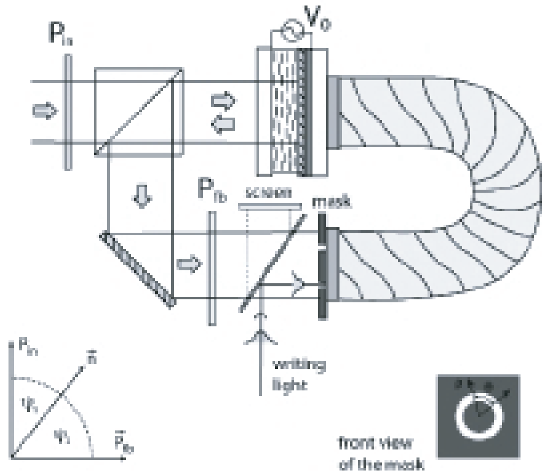


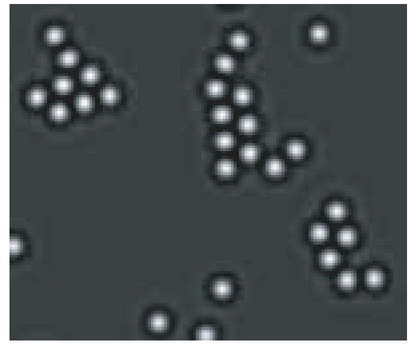
Conclusions

- The effect of noise in a motionless front between a periodic spatial state and an homogeneous one is studied.
- Numerical simulations show that noise induces front propagation.
- From the subcritical Swift-Hohenberg equation with noise, we deduce an adequate equation for the envelope and the core of the front.
- The conversion of random fluctuations into direct motion of front core is responsible of the propagation.
- We obtain an analytical expression for the velocity of the front, which is in good agreement with numerical simulations.

Outlook

- To study experimentally the noises induces front propagation To





- To study the localized patterns