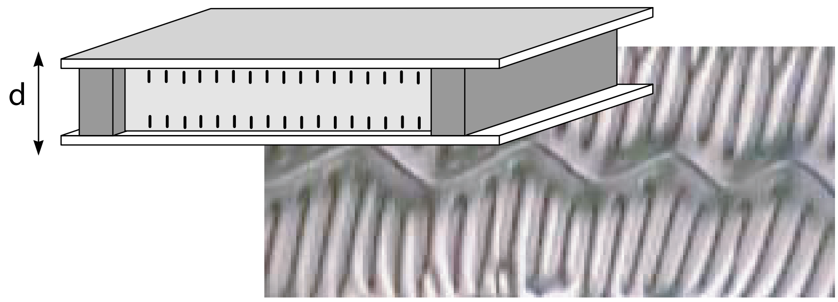
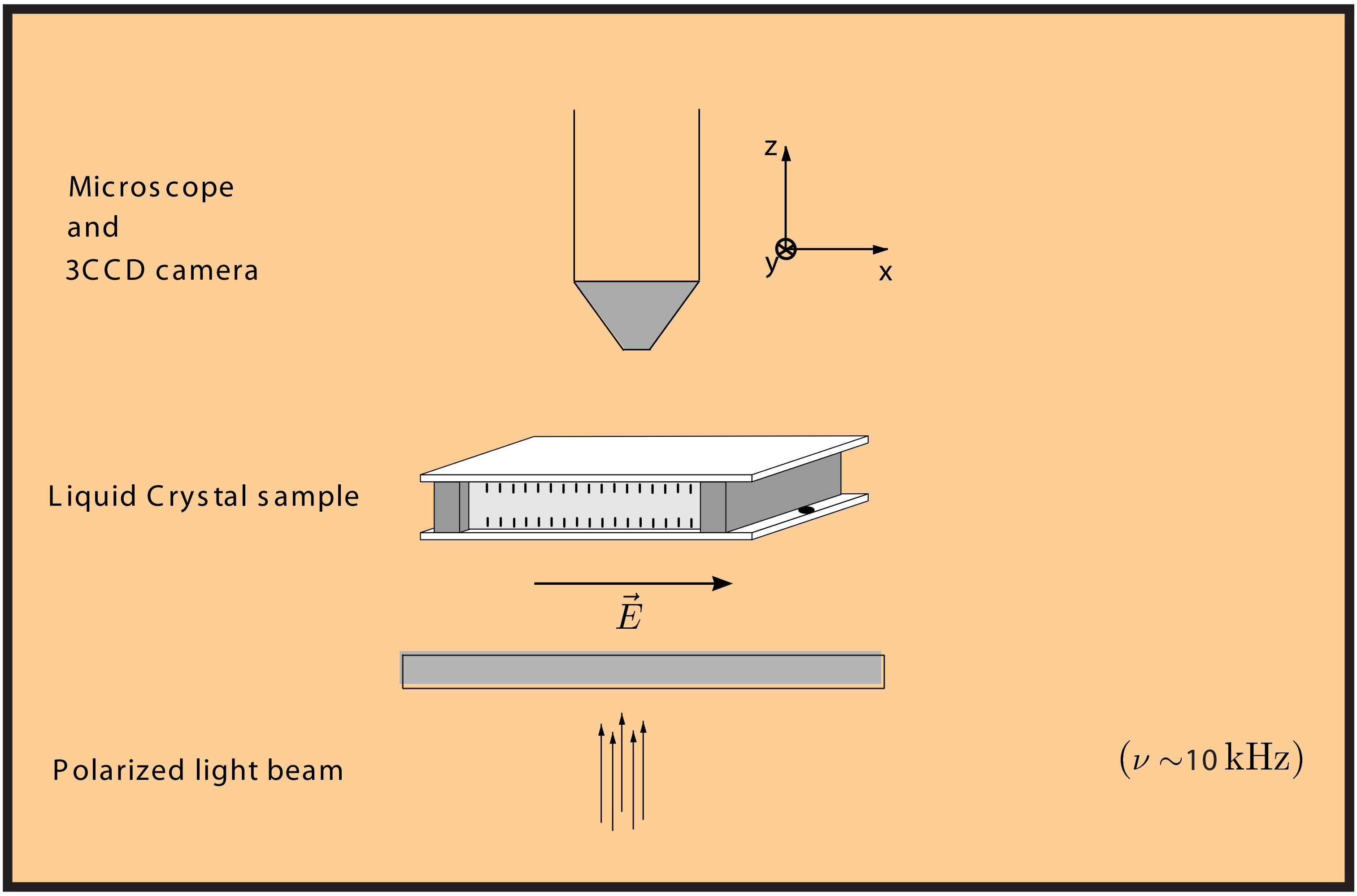
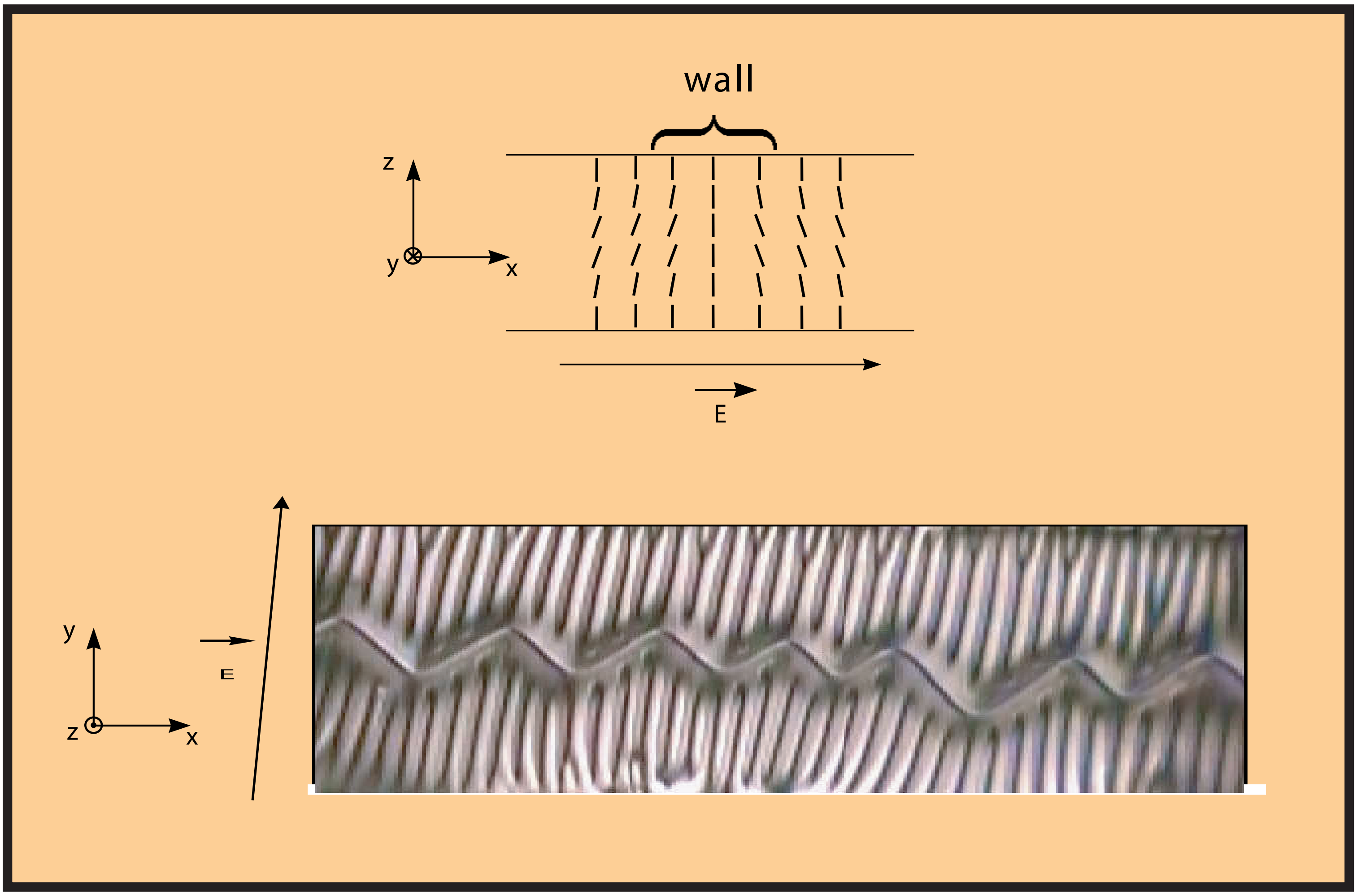
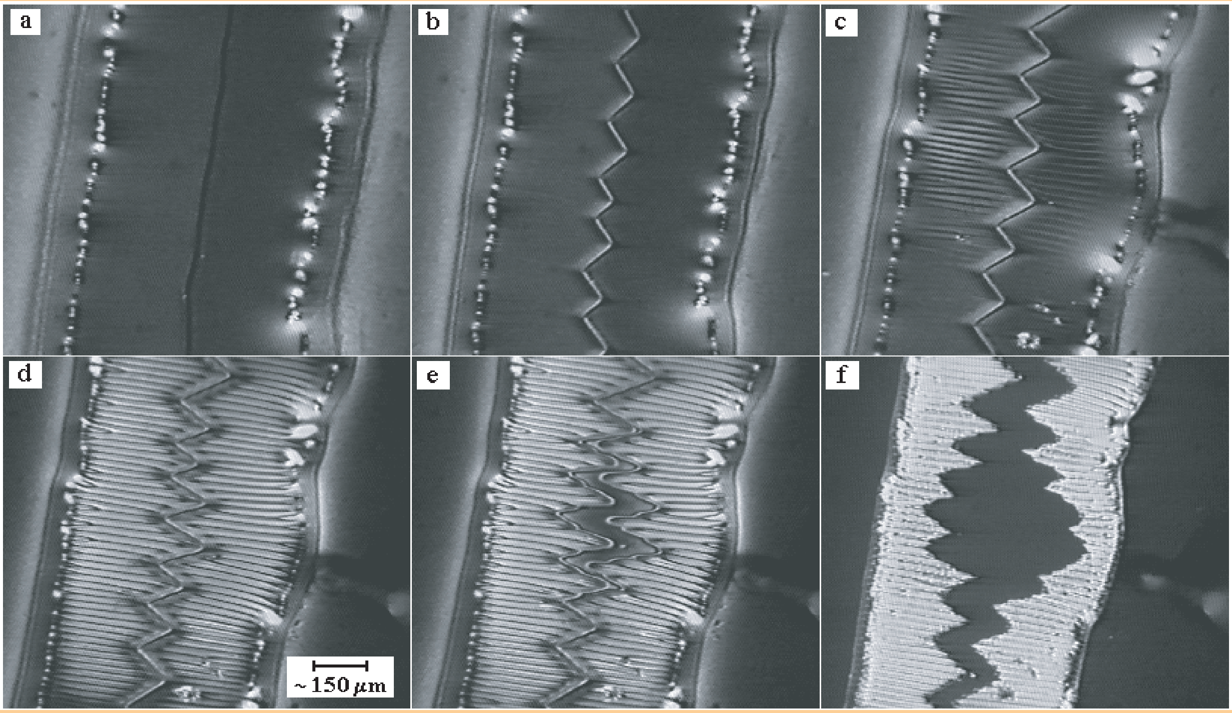

d

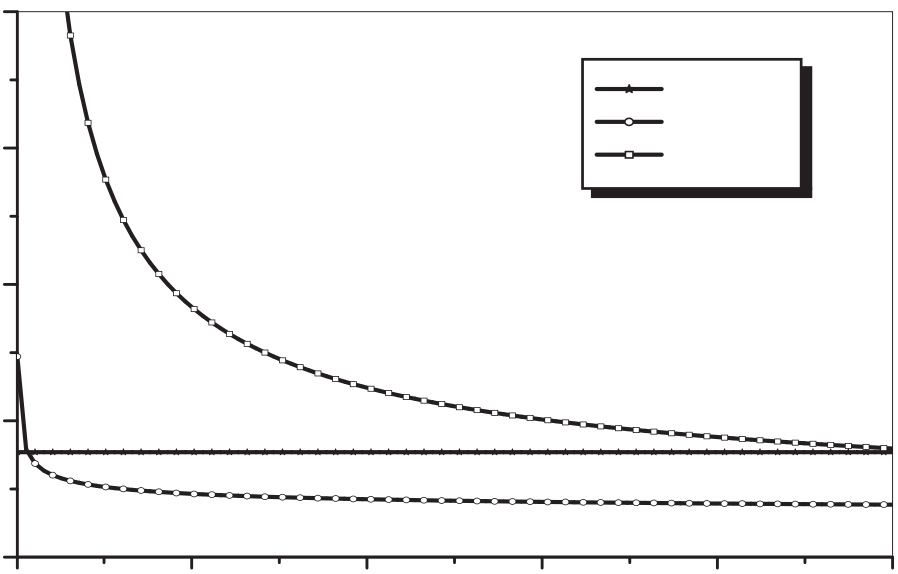

Microscope
and
3CCD camera
z
y
x
Liquid Crystal sample
$\vec{E}$
Polarized light beam
$(\nu \sim 10\,\mathrm{kHz})$


wall
z
y
x
E
y
z
x
E



Experimental observations, when the temperature is changing!.

Elastic constants close to Nem-SmeticA Transition.



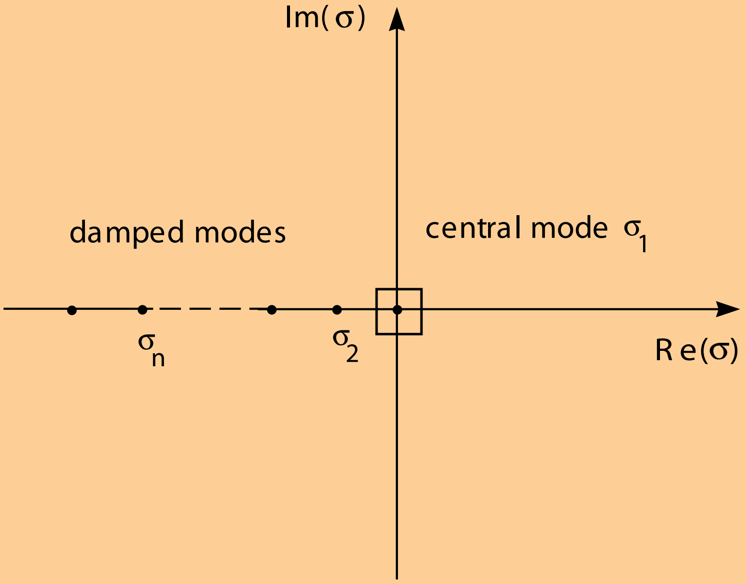
The Frank free energy

$$\frac{\partial \vec{n}}{\partial t} = -\frac{\delta \mathcal{F}}{\delta \vec{n}}, \qquad \vec{n}\cdot\vec{n} = 1,$$

where

$$\mathcal{F} = \int \frac{1}{2}\Big[K_1(\vec{\nabla}\vec{n})^2 + K_2(\vec{n}.(\vec{\nabla}\times\vec{n}))^2 + K_3(\vec{n}\times(\vec{\nabla}\times\vec{n}))^2 - \epsilon_a(\vec{n}.\vec{E})^2 - \chi_a(\vec{n}.\vec{H})^2\Big]\, d\tau$$

For not too high elastic anisotropy, the electric field induces Freedericksz transition is characterized by a unique marginal mode which is the first Fourier mode along the z-axis.



$$\begin{cases} n_x = Y\cos\left(\frac{\pi z}{d}\right) \\ n_y = 0, \\ n_z = 1 - \frac{Y^2}{2}\cos\left(\frac{\pi z}{d}\right)^2 \end{cases}$$

The order parameter that describes the bifurcation is a scalar one (y(x,y,t)).

Using the following change of variable

$$\begin{pmatrix} X \\ Y \end{pmatrix} = Z \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{K_1 - K_2}{2\gamma_c} \partial_{xy} Z \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{K_1 - K_2}{4\gamma_c^2} \left(K_2 \partial_x^2 + K_1 \partial_y^2\right) \partial_{xy} Z \begin{pmatrix} 0 \\ 1 \end{pmatrix} + h.o.t.$$

one get the dynamical equation for the scalar order parameter Z near the Freedericksz transition threshold (Landau Eq.) :

$$\boxed{\gamma_1 \partial_t Z = \epsilon \ Z - a Z^3 + (K_1 \partial_x^2 + K_2 \partial_y^2) Z}$$

where

$$\epsilon \quad = \epsilon_a E^2 - K_3 \frac{\pi^2}{d^2} \ , \qquad\qquad a \quad = \quad \frac{1}{2}(K_1 - \frac{3}{2} K_3) \frac{\pi^2}{d^2} + \frac{3}{4} \epsilon_a E^2$$

In the anysotropic case $(K_2 \ << \ K_1)$

$$\boxed{\begin{aligned} \partial_t Z = \epsilon Z - a Z^3 + K_1 \partial_x^2 \ Z + K_2 \partial_y^2 \ Z + \frac{3}{4}\left(Z(\partial_y Z)^2 - \frac{Z^2}{2} \partial_{y^2} Z \right) \\ + \frac{(K_1 - K_2)^2}{2\gamma_c} \partial_{x^2 y^2} Z + \cancel{\frac{(K_1 - K_2)^2}{4\gamma_c^2}} K_1 \partial_{x^2 y^4} Z \end{aligned}}$$

where $\gamma_c \quad = \quad \frac{1}{2} \chi_a H_c^2 \qquad (\epsilon = 0 \quad \text{for} \quad H = H_c \ )$

K_1 , K_2 , K_3 are the splay, twist and bend elastic constants

γ_1 is the rotational viscosity.

Interface dynamics

$$Z_o = \frac{\epsilon}{\sqrt{a}} \tanh\left(\frac{\epsilon}{\sqrt{2}}(x-P)\right)$$

$$\partial_t P = D_2 P_{yy} + D_3 P_y^2 P_{yy} - D_4 P_{4y}$$

where

$$\begin{cases} D_2 = K_2 - \frac{2 K_1 \epsilon}{5\ \gamma_c} + \frac{3\ K_3\ \epsilon}{40b} \\ D_3 = \frac{48\,\epsilon}{7{\gamma_c}^2} {K_1}^2 \\ D_4 = \frac{2\ \epsilon}{5\ \gamma_c^2} K_1^2 \end{cases}$$

- $\partial_t P = -\frac{\delta \mathcal{F}[P]}{\delta P}$ with $\mathcal{F}[P] = \int \left[\frac{\epsilon}{2} P_y^2 + \frac{1}{12} P_y^4 + \frac{1}{2} P_{yy}^2\right] dy$

  $\Rightarrow$ relaxational dynamics

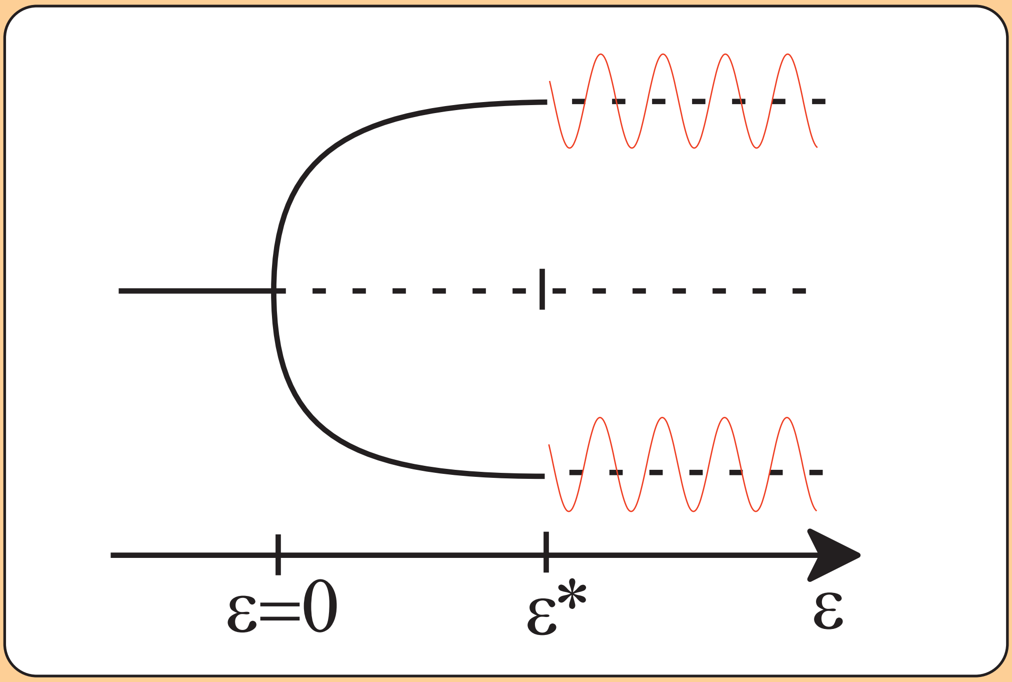
- The preceding equation is a continuity equation :

$$\partial_t P = \partial_y (\epsilon P_y + \frac{1}{3} P_y^3 - P_{3y})$$

In a infinite medium, this corresponds to the conservation of the global quantity

$$\int P\ dy$$

Bifurcation Diagram



Numerical simulations