Advances in the Application of Stochastic Geometry in Robotics

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I. Introduction

Stochastic geometry is an established branch of mathematics that studies uncertainty in geometric structures [1], [2] and is, thus, a befitting framework for autonomous robotic mapping and the well known Simultaneous Localization and Mapping (SLAM) problem, where the fundamental concern is succinctly captured in the title of the 1988 seminal paper by Durrant-Whyte, "Uncertain geometry in robotics" [3]. The theory of random sets has long been used by statisticians in many diverse applications including agriculture, geology, and epidemiology [1], [2], [4]–[6]. In addition, there has been substantial work by probabilists and statisticians, in point process filtering, such as that by Ikoma et al [7]. Applications of this are random point pattern methods for multiple object recognition in image analysis [8], [9], and recent work based on random set analysis by Vo et al [10], which laid the foundations for set based multi-object visual tracking by Hoseinnezhad et al [11], [12]. The application of random sets in multi-target tracking has led to the development of Finite Set Statistics (FISST) which provides the basis for novel filters such as the Probability Hypothesis Density (PHD) filter [13]–[15] and the Cardinalized (C)-PHD filter [16] which recently attracted considerable research interest as well as deployment in commercial applications.

As noted in the field of multi-target filtering by Mahler ([17], page 571):

> "...having a good estimate of target number is half the battle in multi-target tracking. If one has 1,000 measurements but we know that roughly 900 of them are false alarms, then the problem of detecting the actual targets has been greatly simplified."

The articles in this special issue advocate that the same principle applies to feature detection and autonomous mapping in robotics, where instead of referring to the problem of target estimation, the problem of map feature or environmental object estimation are of concern. From here on, map features, targets and environmental objects of interest will simply be referred to as "features". In the case or robotic mapping and SLAM, realistic feature detection algorithms produce false alarms and missed detections and estimating the true number of map features is therefore central to these problems.

A philosophy often encountered within the SLAM community is that the number of estimated map features is not important in SLAM, provided that enough are estimated to provide successful robot location estimates. In response to this, the reader is referred to Figure 1 in which a human driver has clearly not estimated the correct number of objects within his/her environment. Unfortunate accidents aside, failing to correctly estimate the true number of objects or features, which have passed through the field(s) of view of a vehicle's sensor(s), can only detriment the location estimation performance of any SLAM algorithm. This special issue therefore addresses the concept of Bayes optimality for estimation with unknown feature number, by formulating autonomous mapping, SLAM and general tracking algorithms as Random Finite Set (RFS) estimation problems.

Fig. 1. *The importance of object number estimation in navigation tasks.*

An RFS is simply a finite-set-valued random variable. Similar to random vectors, the probability density (if it exists) is a very useful descriptor of an RFS, especially in filtering and estimation. However, the space of finite sets does not inherit the usual Euclidean notion of integration and density. Hence, standard tools for random vectors are not appropriate for random finite sets. Mahler's Finite Set Statistics (FISST) provide practical mathematical tools and principled approximations for dealing with RFSs [13], [17], based on a notion of integration and density that is consistent with point process theory [18].

So what are the *advances* in the applications of stochastic geometry, advocated in this special issue? In contrast to state of the art, vector based implementations of Bayesian filters, which require separate filters/routines to manage and associate measurements to features, the use of RFSs unifies the independent filters adopted by previous solutions through the recursive propagation of a distribution of an RFS of features. This allows the joint propagation of the estimated feature density to take place and, in the case of SLAM, leads to optimal map estimates in the presence of unknown map size, spurious measurements, feature detection and data association uncertainty. The proposed framework further allows for the

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joint treatment of error in feature number and location estimates as it jointly propagates both the estimate of the number of features and their corresponding states, and consequently eliminates the need for feature management and association algorithms.

II. A Brief Summary of Random Set Based Implementations in Robotics

In 2008, Mullane et al first applied the random set concept to the SLAM problem, in which a first order random set statistic - the Probability Hypothesis Density (PHD) was used, from which joint vehicle and feature based map estimates could be extracted [19]. Referred to as a "Brute force" approach, it demonstrated a viable RFS based SLAM solution. For environments with a significant number of features it is however computationally intractable, and hence a more elegant and computationally tractable RFS solution, based on Rao Blackwellization, was published in 2010 and further elaborated in 2011, in which the Cardinalized (C)-PHD and Multi-Target Multi-Bernoulli (MeMBer) SLAM filtering concepts were presented [20]–[22]. A simplified version of this work comprises the first article of this special issue.

In 2012, Lee et al addressed the SLAM problem with a single cluster PHD filter, which also utilized Rao Blackwellization, but generated a measurement likelihood function for trajectory weighting in a different manner. An extension of this work, applied to underwater SLAM, is the focus of the second article in this special issue [23]. Also in 2012, Moratuwage et al extended the RFS concept to multi-vehicle SLAM, providing demonstrations with two vehicles which collaborate to estimate a global map based on PHD filtering, along with their own trajectories [24]. An extension of this work comprises the fourth article in this special issue. Finally, also in 2012, Adams et al demonstrated Constant False Alarm Rate (CFAR) and scan integration feature detection techniques to provide principled detection statistics for short range radar in a Rao-Blackwellized PHD Filter SLAM framework [25].

To complement the multiple publications on the applications of FISST to robotic based problems, the first international workshop on *Stochastic Geometry in SLAM* was held at the 2012 IEEE International Conference on Robotics and Automation (ICRA 2012), in Minneapolis St. Paul, USA. This full day workshop was opened by the founder of FISST, Ronald Mahler who presented the foundations behind many of the FISST based filtering concepts. It also provided a forum for various FISST based robotic mapping, navigation and control presentations, some of which are extended in this special issue. During this workshop, many of the presenters and members of the audience indicated the importance of publicizing the recent advances in the application of stochastic geometry to robotic problems, which has instantiated this special issue.

III. Overview of the Special Issue

The first of the six articles in this special issue. *New Concepts in Map Estimation: Implementing PHD Filter SLAM*, focusses on a SLAM implementation which uses the most basic Bayesian set based estimator - the Probability Hypothesis Density Filter (PHD). The article first demonstrates the random nature of detections in sensing modalities as diverse as radar, laser range finders and vision. The information - referred to as features - provided by any feature detection algorithm and based on any sensing type, is prone to randomness both in the quantity of the detected features and their attributes such as range and bearing or image based quantities such as contrast levels. It shows that realistic measurement uncertainty comprises detection uncertainty in the form of false alarms and missed features as well as the usually considered spatial (e.g. range and bearing) uncertainty. The ability to account for all of these in a joint manner provides the motivation for re-modelling the SLAM problem as a set, rather than a vector, based framework. The concepts of RFSs are introduced and the implementation of the PHD filter, in the form of manipulating sums of Gaussians, is demonstrated through the use of simple block diagrams. A marine environment, in which an autonomous kayak estimates the number and location of objects on the sea surface as well as its own trajectory, provides the complex setting for SLAM trials. These are based on the presented sum of Gaussians PHD SLAM algorithm, with performance comparisons being made with state of the art Multiple Hypothesis (MH) FastSLAM.

The SLAM problem is treated as a single cluster (SC) process in the second article, entitled *SLAM with SC-PHD Filters: An Underwater Vehicle Application*. Here SLAM is defined as a particular type of cluster process in which the configuration of the map features is a daughter process, conditioned on the state of the vehicle, represented as a single parent process. The single cluster PHD filter approach can be separated into a parent and a conditional daughter term, allowing a hybrid particle filter and Gaussian mixture approach to be used for SLAM, in a manner similar to that proposed in the first article. However, in contrast to the first article, a single cluster process, rather than a Poisson process, is assumed on the prior map feature cardinality distribution. An implementation of the concept is demonstrated on an underwater robotic vehicle, the Girona 500, which utilizes stereo imagery and a speeded up robust feature (SURF) detector to detect key points in images for underwater SLAM.

The third article, *Playing Fetch With Your Robot* is based on a Segway robotic platform and again uses vision for the detection of an unknown number of objects, this time based on shape and colour matching. These are scattered about an environment for the robot to locate, collect and return to the user. The work uses a grid with cells containing occupancy probability values. An RFS is used to represent a set of labels of occupied cells together with a further set, which contains every combination of the RFSs from zero to the maximum number of objects which can be tracked. In contrast to the first two articles, a Bayesian filter iterates over the distribution of the RFSs to estimate the varying number of object locations. This information is then used to automatically instruct the robot to move to maximize its immediate information gain - a technique referred to as "information surfing". The robot is controlled to move in the maximum gradient of mutual information between the sensor readings and the cell based object position estimates based on a quantity known as the Rényi divergence. This moves the robot and the camera's field of view into the direction of objects to be fetched.

The focus of the fourth article, *RFS Collaborative Multi-Vehicle SLAM in Dynamic High Clutter Environments*, is Collaborative Multi-vehicle (CM) SLAM under an RFS framework. Formulated for two vehicles, which collaborate to build a single global map and estimates of both trajectories, this article introduces the concept of the general multi-sensor PHD update. This update requires the union of the set based measurements from each vehicle to be partitioned into binary subsets. The article high-lights the computational problem which results in the case of many robots, as all possible combinations of these subsets would be necessary to form the Bayes optimal CM-SLAM measurement update. To test the concepts with two robots, simulations demonstrate the robustness of the RFS based solution under varying degrees of clutter (false alarms). In a real experiment, a car park with moving people provides the scene for the dynamic environment where the proposed RFS based multi-vehicle SLAM solution is compared against a state of the art CM SLAM solution which depends on external feature association and management routines.

The fifth article entitled *A New Efficient Topological Approach to Map Merging Based on a Probabilistic Generalized Voronoi Diagram*, again addresses multi-robot applications and focusses on grid based map fusion. Occupancy grid maps from multiple vehicles are merged without prior knowledge of their relative transformations. This is achieved through graph matching, in which the graph is a topological representation of the map and is based on a Generalized Voronoi Diagram (GVD). Referred to as map fusion, the approach demonstrated in this article exploits the uniqueness of GVDs to combine large maps. The confidence values associated with certain areas of each robot's map is further encoded into the topological structure, by building a Probabilistic (P)GVD. Map matching takes place via a 2D correlation to match laser range finder based edges. The probabilistic nature of the PGVDs allows areas of the maps with higher certainty to be preferentially matched. The technique is verified through four experiments: First, based on a publicly available data set; second, based on two real indoor vehicles communicating map information between them; third, based on three vehicles operating in a larger indoor environment and finally based on a simulated, highly cluttered environment.

The estimation of the location and number of objects/features which can each generate multiple sensor detections, due to their large size and/or occlusions, is the subject of the final article *Random Set Methods for Multiple Extended Object Estimation*. This article demonstrates how the assumptions of the earlier articles, in which single objects are assumed to yield single detections, can be relaxed allowing for multiple detections per object. This scenario naturally lends itself to the RFS concept, since a subset of all of the detections can now result from each object. This in turn requires a partitioning of the full set of detections, into subsets, the union of which comprises the full detection set. During the PHD Filter measurement update, each partition requires a likelihood, corresponding to how probable that group of measurements stems from a single object. There are many ways in which such partitions can be formed, all of which should theoretically be considered for Bayes optimality. This article provides algorithms for limiting the number of partitions to computationally manageable levels without sacrificing estimation performance. By considering different alternatives for the measurement model, based for example on assumed geometric extended object shapes, a multi-object tracking PHD Filter implementation is demonstrated. The experimental results demonstrate the ability of the filter to estimate the quantity and location of an unknown number of pedestrians, based on laser range finder data, even when pedestrians occlude each other. This article models the probability of detection as non-homogeneous in the sensor surveillance region, based on object location estimates, yielding good estimates of pedestrian number, even when they are completely occluded.

IV. Looking Forward

Stochastic geometry has been applied in diverse engineering fields for many decades, but only in the last decade have the tools of Finite Set Statistics (FISST) become available for set based estimation applications. This special issue is largely a collection of robotic applications based on these recently formulated tools. Within the field of robotics, many avenues exist for further research, based on FISST, including improved sensor models which take into account object occlusions, generalized mapping concepts such as semantic maps and active navigation in which vehicles are autonomously commanded to maximize their information gain. We hope that this special issue provides motivation for further advances in the use of stochastic geometry in SLAM and general robotic applications.

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