Bus-Stop Control Strategies Based on Fuzzy Rules for the Operation of a Public Transport System

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***Abstract*—In the daily operation of a bus system, the movement of vehicles is affected by uncertain conditions as the day progresses, such as traffic congestion, unexpected delays, randomness in passenger demand, irregular vehicle dispatching times, and incidents. In a real-time setting, researchers have devoted significant effort to developing flexible control strategies, depending on the specific features of public transport systems. In this paper, we propose a control scheme for the operation of a bus system running along a linear corridor, based on expert rules and fuzzy logic. The parameters of the fuzzy controllers were tuned through a particle swarm optimization (PSO) algorithm. That is, the control strategies aim at keeping regular headways between consecutive buses, with the objective of reducing the total waiting time of passengers. The proposed control systems rely on measures of the position of each bus, which are easy to obtain and implement by means of emerging automatic vehicle location devices through Global Positioning System (GPS) technology. The utilized strategies are holding, stop-skipping, and the integration of both. After tuning the controller parameters, we conducted several simulation tests, obtaining promising results in terms of savings in waiting times with the implementation of the proposed rules, noting that the best performance occurred when fuzzy rules are included. The methodology has great impact, and it is easy to implement due to its simplicity.**

***Index Terms*—Bus-stop control strategies, expert systems, fuzzy control, public transport system.**

I. Introduction

In THE daily operation of public transport systems, mainly that of buses, the movement of vehicles is affected by different uncertain conditions as the day progresses, such as traffic congestion, unexpected delays, randomness in passenger demand, irregular vehicle-dispatching times, and incidents. At the planning level, the decision maker must decide the design key variables (i.e., route design, fleet size, capacity of buses, and frequency), which are based on the average values of spatial and temporal distribution of passenger demand and traffic congestion over certain periods of operation. However, following such a preplanned schedule could suggest inefficient operational schemes due to the inherent uncertainty of the system conditions. In a real-time setting, the variability of the headway between buses would considerably impact the users' level of service through the passengers' waiting time at bus stops. Note that the same average frequency (computed for a period of 2–3 h defined at an aggregate level) could result in quite different headway distributions; the typical phenomenon of bus bunching is an example of how the dynamic conditions can produce high variability in headways, which represents an undesired system behavior associated with long waiting times for passengers at bus stops.

As an attempt to reduce the negative effects of service disturbance, researchers have devoted significant effort to developing flexible control strategies, depending on the specific features of the problem. Particularly interesting are what the researchers call real-time control strategies that depend on the availability of real-time information through on-vehicle equipment such as automatic passenger counters and automatic vehicle location devices. These strategies are designed to allow the operator to dynamically react to system disturbances.

The most studied strategy in recent years is *holding*, in which vehicles are held at specific stations for a certain time and in most cases oriented to keep the headway between successive buses as close as possible to a predefined value. Moreover, to control vehicles by the opposite effect (speed up buses, instead of delaying them as in *holding*), we found strategies such as expressing or *stop-skipping*, where buses can skip some of the predefined stops according to what the operator decides in real time.

As field tests of hypothetical situations are, in general, quite expensive and hard to get implemented, some authors have developed simulations at the microscopic level of transit systems to evaluate, among other things, real-time control strategies such as *holding*, *stop-skipping*, and traffic signal priorities. For a deep review of such techniques and some illustrative applications, see [4] and [10].

In this paper, we propose control strategies designed to keep each group of three consecutive buses on a route equidistant, with the final objective of keeping regular headways between them to reduce the total waiting time of passengers. This is achieved through either *holding* buses at certain stops in certain situations or forcing buses to skip specific stops in others. In simple terms, the philosophy behind the expert control strategy

Manuscript received November 4, 2011; revised February 7, 2012; accepted February 8, 2012. Date of publication March 19, 2012; date of current version August 28, 2012. This work was supported in part by the ACT-32 Project, by Fondecyt Chile under Grant 1100239, and by the Millennium Institute "Complex Engineering Systems" (ICM: P-05-004-F, CONICYT: 522 FBO16). The Associate Editor for this paper was S. C. Wong.

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Digital Object Identifier 10.1109/TITS.2012.2188394

that we will propose consists of moving forward the central bus—if it is late—with respect to the central position of the trajectory between the bus immediately behind and the bus ahead, through *stop-skipping*; otherwise, the central bus is delayed through *holding*.

The control system relies on measures of the position of each bus, which is relatively easy to obtain nowadays by means of emerging Global Positioning System (GPS) technology. In our scheme, the strategies we include are *holding*, *stop-skipping* and the integration of both. These two strategies correspond to station-control-type according to the classification in [5]). Note that the proposed strategies (i.e., *holding* and *stop-skipping*) are able to either delay or advance the schedule of the buses, increasing the flexibility of the methodology for avoiding upstream chain effects.

In the literature, we find several *holding* models that rely on real-time vehicle location information [5], [7], [13], [22], and [28]. In [5]–[7], deterministic quadratic programs are proposed under a rolling horizon scheme, considering that the *holding* decision for a specific vehicle affects the operation of a specific subset of the precedent vehicles. On the one hand, they show that *holding* strategies could reduce the variance of the waiting time of passengers, as well as the expected value of both waiting and travel time. On the other, they concluded that having two or more *holding* stations over a corridor is not necessary. These conclusions are contradictory with the results in [22], where Sun and Hickman showed that *holding* vehicles in several control stations would be better than having a single *holding* stop. Most of the controllers are finally heuristically solved due to the mathematical complexity of the formulations. In [28], Zolfaghari *et al.* developed a mathematical control model for *holding* using real-time information of locations of buses along a specified route, which is solved with simulated annealing. In [26], Yu and Yang proposed a dynamic *holding* strategy where the on-time performance of the early bus operation at the next stop is considered and the *holding* times at stops are optimized. Genetic algorithms are proposed to optimize *holding* times.

In regard to *stop-skipping*, in [16], Khoat and Bernard concluded that this strategy would effectively reduce the in-vehicle travel time for passengers; however, the modeler has to be very careful due to the increment in waiting time of those passengers skipped by the buses at stops. As a real-time control strategy, *stop-skipping* has been studied in [5], [6], [11], [17], and [22]. The idea is to speed up buses by skipping stops (one or more) for vehicles to recover their preplanned schedule after a disruption or unexpected delay. This allows the total waiting time of passengers at stations to be reduced by considering the negative effects of extra waiting time for those passengers whose stop has been skipped. In general, a station-skipping decision is made before the buses depart from the terminal. Conversely, in [22], Sun and Hickman allowed the control action to be made once the vehicle is on route.

In [5], an integrated model encompassing *holding*, deadheading, and expressing is formulated. Cortés *et al.* [3] and Sáez *et al.* [21] designed and evaluated a predictive control strategy that also integrates the two strategies (i.e., *holding* and *stop-skipping*) to solve a real-time public transport control problem with uncertain passenger demand, relying on online information of the system behavior. Similarly to [22], in their model, the decision of skipping bus stops is made in real time, which makes their framework more adaptable and responsive to real-time delays.

With regard to fuzzy logic used in the field of transportation, in [24], Teodorovic and Lucic developed a model for the synchronization of a public transport system with transfers, where the number of passengers at the transfer station is assumed to be approximately known. The model is based on the fuzzy ant system that represents a combination of the ant colony system and fuzzy logic. Houng and Teodorovic [14] developed a decision support system framework for integrated emergency vehicle preemption and transit priority system investment planning. They proposed fuzzy set concepts and multiattribute decision-making methods to rank order transit signal priority strategy alternatives at the intersection level. In [20], Onifade *et al.* employed a fuzzy logic control technique to predict traffic conditions and proposed a flexible approach to allocate buses. The technique involves the fuzzification of input variables based on major traffic conditions, such as day of the week, time of the day, public holidays, weather conditions, and location. In addition, [19] presented a controller based on fuzzy logic, which assists the speed and distance vehicle control, offering driving strategies and actuation over the throttle of a car. In the context of traffic control applications, Gokulan and Srinivasan [12] proposed a fuzzy decision system to dynamically compute green times at each cycle of an intersection by using local data received from the sensors directly connected to the intersection and the communicated congestion data from the neighboring intersections connected to the outgoing.

We recognize two relevant sources of stochasticity commonly observed in transit systems such as the simple one-way loop route studied in this paper: 1) the demand at bus stops and 2) the traffic conditions. At this stage, we only present the developments associated with changes in demand patterns (passengers arriving at stops as detailed in [3] and [21]), which are reflected in the distances from the bus that triggers each event and the buses immediately before and after. The distances are affected due to the passenger transfer operations occurring at bus stops; if the demand is high, buses normally take more time at stops, increasing the total travel time through the system.

Thus, the demand is indirectly considered by the position of the vehicles, which changes depending on the intensity of passengers that arrive at each station during the modeling period. Therefore, as the position of the buses is the only input to feed the controllers, the required data from the system and the complexity of the online algorithms used are considerably simplified.

In this application, link speeds are assumed to be fixed, which is a reasonable assumption in the case of exclusive corridors of buses, where they are isolated from the rest of the traffic in most segments of the route.

Moreover, there is no explicit objective function defined in this scheme; however, the goal pursued by the operator is implicit in the logic of the control rules toward a regularization of headways, which positively affects passengers through the minimization of waiting times. The different schemes are tested in simulation.

In our research, even though we study a single line (no buses connections), we identified, as a major contribution of the proposed schemes, the simplicity and potential savings in computation resources and technology from the fact that this methodology is based on a set of straightforward control rules to decide the real-time behavior of vehicles. Moreover, the methods presented here do not depend on demand and traffic predictions, which are appropriate when neither future demand nor traffic information is available. The simplicity of the proposed methods in terms of applicability and computational complexity is a strong reason to justify the implementation of such schemes in real systems, which is feasible, even if just basic technology elements are available (GPS technology and online communication with the driver). Note in addition that the complexity of the proposed algorithms does not explode with the size of the problem; therefore, they can be applied to large-scale bus corridors.

More sophisticated strategies based on hybrid predictive control (HPC) have been developed by the same authors in previous works (see [3] and [21]), who formulated state-space models along with a detailed dynamic objective function and future demand prediction methodologies in a closed-loop control scheme. In these works, the formulation of the objective functions include terms of waiting and travel time incurred by the passengers in the system as a result of the application of *holding* and *stop-skipping*. The performance of such HPC schemes turned out to be better than the strategy based on fuzzy rules that we are proposing in this work, although the implementation of such complicated systems requires high-quality real-time measures of system variables; that implies sophisticated technological features, which are not available to most operators of transit systems in the world.

The major contribution of this paper is the development of an integrated real-time control scheme applied on a single bus corridor. We use the two aforementioned strategies *holding* and *stop-skipping*, which are both applied in the context of a control expert system and a control system based on fuzzy logic; in both controllers, the objective is to minimize the waiting time of passengers through the regularization of headways between consecutive buses. We establish expert rules for *holding* and *stop-skipping*, as well as fuzzy rules for *holding*, *stop-skipping*, and the integration of both.

In the next section, the proposed bus system is described, highlighting the way in which headway regularization is incorporated into the control scheme. In Section III, the different expert rules and fuzzy rules are described in detail, including a parameter tuning of fuzzy controllers. In Section IV, we develop simulation tests and discuss the results, ending in Section V with the conclusions and proposed further steps of this research topic.

II. Public Transport System

A. System Description

The methodology developed in this paper is applied on a simple network, corresponding to a linear public transport corridor that typically crosses a large urban region. The system can be represented by a one-way loop route (see Fig. 1), with N stations and N_b buses running along the loop, each with a capacity for carrying C passengers.

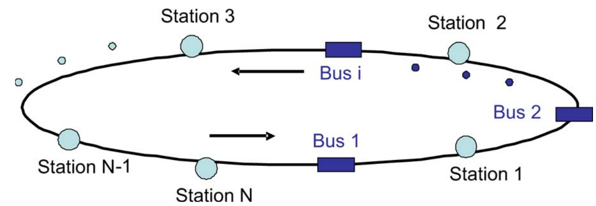

Fig. 1. Public transport system representation.

B. Design Assumptions

The corridor is assumed to be isolated from the rest of the traffic, and therefore, one can assume a fixed average speed for the buses v_0 on each stretch between consecutive stops. Moreover, from the number of buses running N_b, the total distance of the corridor, and the circulation speed, we can estimate the total cycle time of the buses (to complete the loop), which defines the design frequency f offered to the users. We assume that the operator has the technology to control the buses in real time. Station 1 is the terminal of the bus route, where all passengers must get off.

The operator determines the design variables (frequency f and fleet size N_b) based on historical demand information through a representative stop-to-stop demand matrix for each modeling period.

As mentioned in the previous section, one major advantage of the methods proposed here is simplicity, in the sense of defining rules only from bus position information, which means that the demand is not directly incorporated into the model but indirectly through the bus trajectories and how they change with the system evolution with time. The simulation assumes an uncertain demand, where passengers dynamically arrive at stations by following a Poisson process with different demand rates differentiated by station and period. The destination stop is randomly chosen among the stations ahead of the station from where the passenger gets on. The total time used for boarding and alighting is assumed to be proportional to the number of passengers and corresponds to the maximum time between boarding and alighting [21]. Finally, link speeds are assumed to be fixed in this model, which is a reasonable assumption in cases where buses are isolated through exclusive lanes/corridors.

In Section II-C, we define the available strategies the operator can rely on to regularize the headways in real time to recover the predefined design headway (which is inversely related to design frequency f) due to the natural distortion provoked by the time spent in transference of passengers, which is unknown beforehand because of the assumed uncertainty of the stop-to-stop demand.

C. Closed-Loop Strategy

The major goal of this work is to evaluate the performance of expert and fuzzy control strategies to regularize the headway

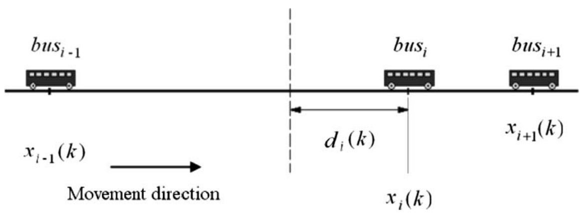

Fig. 2. Relative positions of three consecutive buses.

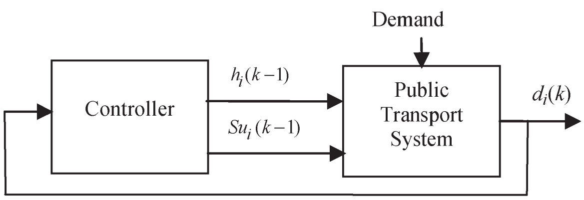

Fig. 3. Generic closed-loop control diagram.

between the arrivals of consecutive buses to stops. To achieve this objective, the strategies aim at keeping each group of three consecutive buses equidistant. Then, let us define a discrete event k as the bus arrives at any stop. In Fig. 2, we show the relative position of three consecutive buses $i-1$ (preceding bus), i (current bus), and $i+1$ (next bus). Let us define $x_{i-1}(k)$ as the position of the preceding bus$_{i-1}$, $x_i(k)$ as the position of the current bus$_i$, $x_{i+1}(k)$ as the position of the next bus$_{i+1}$, and k as the measurement data event. We define distance $d_i(k)$ for referring to the position of the middle bus with respect to the adjacent buses at the decision time. Therefore, $d_i(k)$ can also take negative values as it represents not only magnitude but direction with respect to the middle point, i.e.,

$$d_i(k) = x_i(k) - \left(\frac{x_{i-1}(k) + x_{i+1}(k)}{2}\right). \tag{1}$$

Fig. 3 shows a generic closed-loop diagram for a control strategy, in which the control actions are triggered when bus i reaches a stop (event k). The manipulated variables associated with event $k-1$ are *holding* $h_i\ (k)$ and *stop-skipping* $Su_i\ (k)$. In this application, we chose discrete values for the *holding* lapse $h_i(k)$: *Holding* action of bus i at instant k, where $h_i(k) = n_i\tau$, $n_i \in Z^+$, $\tau > 0$.

These expressions mean that the *holding* periods are multiples of a fixed step τ. This assumption is applied to simplify both the formulation and the application of the solution algorithm. In the numerical example, $\tau = 30$ [s], and $n_i \in \{0, 1, 2, 3\}$. The reason for choosing discrete *holding* lapses was first, from an operational standpoint, to facilitate the bus drivers to follow the instructions by the central dispatcher. Moreover, having differences of less than 30 s in *holding* values is not practical, mainly due to constraints given by real driving conditions (unexpected traffic, flexibility of the driver to start operating, communication with the central control station, etc.). It is worth mentioning that, in the simulator, we separately consider the *holding* action h_i and the dwell time [including the intervals for opening (OT_i) and closing (CT_i) the doors, plus boarding and alighting times (BT_i and AT_i, respectively)]. Therefore, the total stopping time TD_i of the ith bus is computed as $TD_i = h_i + OT_i + CT_i + \max\{BT_i, AT_i\}$ (assuming simultaneous boarding and alighting).

On the other hand, *stop-skipping* is defined as $Su_i\ (k) = 1$ when the bus skips the stop and $Su_i\ (k) = 0$ otherwise. Both manipulated variables are mutually exclusive at every bus stop; then, when *stop-skipping* is decided, the *holding* action cannot be applied, and *vice versa*.

Moreover, the discrete event control strategies, in which events are triggered by the arrival of buses at any stop, allow the proposed local approach to move close to the global solution, involving the entire fleet. This can be dynamically applied.

III. Control Strategies

A. *Expert Control Strategy*

In simple terms, the expert control strategy consists of moving bus$_i$ forward if it is late with respect to the central position of the trajectory between the precedent bus bus$_{i-1}$ and the next bus bus$_{i+1}$; otherwise, bus$_i$ is delayed.

Next, we define the expert controller as a set of rules. We assume that buses move at an average speed of $v = 25$ km/h, i.e., 6.94 m/s. Therefore, the product of speed v and the *holding* lapse β is the distance $v\beta$ that a bus refrains from traveling due to a *holding* control action that is equivalent to β, i.e., 208.2 m. As a consequence, if the bus is held at a lapse of 2β, it will refrain from traveling a distance of $2v\beta$. Similarly, if the bus is held at a lapse of 3β, it will refrain from traveling $3v\beta$.

Therefore, if the *holding* control action takes a value β, we can define a neighborhood ratio $v\beta/2$ around $d_i(k) = v\beta$ (i.e., $v\beta/2 < d_i(k) \leq 3v\beta/2$), where this control action will be applied.

Following the same reasoning, within the range $3v\beta/2 < d_i(k) \leq 5v\beta/2$, the *holding* control action will take a value 2β ($h_i(k) = 2\beta$), and for $5v\beta/2 < d_i\ (k)$, the *holding* control action will take a value $3\beta(h_i(k) = 3\beta)$. Instead, if $-v\beta/2 < d_i\ (k) \leq v\beta/2$, the *holding* and *stop-skipping* control actions are not necessary $(h_i(k) = 0, Su_i(k) = 0)$. Finally, if $d_i(k) \leq -v\beta/2$, the control action will be just *stop-skipping* $(h_i\ (k) = 0, Su_i\ (k) = 1)$.

Thus, adding the limit cases (equalities), we can formulate the expert control strategy (*holding* and *stop-skipping* based on rules) as the following five rules.

$$\text{If } d_i(k) \leq -v\beta/2, \text{ then } h_i(k) = 0,\ Su_i(k) = 1. \tag{2a}$$

$$\text{If } -v\beta/2 < d_i(k) \leq v\beta/2, \text{ then } h_i(k) = 0,\ Su_i(k) = 0. \tag{2b}$$

$$\text{If } v\beta/2 < d_i(k) \leq 3v\beta/2, \text{ then } h_i(k) = \beta,\ Su_i(k) = 0. \tag{2c}$$

$$\text{If } 3v\beta/2 < d_i(k) \leq 5v\beta/2, \text{ then } h_i(k) = 2\beta,\ Su_i(k) = 0. \tag{2d}$$

$$\text{If } 5v\beta/2 < d_i(k), \text{ then } h_i(k) = 3\beta,\ Su_i(k) = 0. \tag{2e}$$

Next, we will describe five control strategies based on the previous set of rules. The first two are particular cases of the expert control, whereas the remaining strategies correspond to fuzzy adaptations of the same rules.

1) Holding Based on Rules: In this case, the control strategy only computes holding actions; therefore, rule (2a) is discarded,

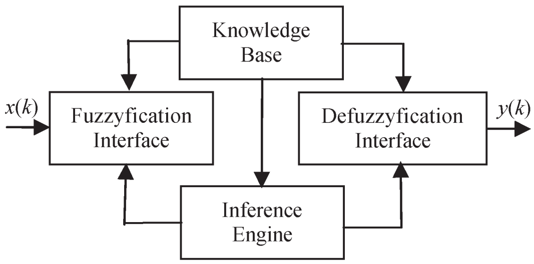

Fig. 4. Structure of a fuzzy system.

and rule (2b) is modified. Finally, we obtain the following set of rules.

$$\text{If } d_i(k) \leq v\beta/2, \text{ then } h_i(k)=0, \; Su_i(k)=0. \tag{3a}$$

$$\text{If } v\beta/2 < d_i(k) \leq 3v\beta/2, \text{ then } h_i(k)=\beta, \; Su_i(k)=0. \tag{3b}$$

$$\text{If } 3v\beta/2 < d_i(k) \leq 5v\beta/2, \text{ then } h_i(k)=2\beta, \; Su_i(k)=0. \tag{3c}$$

$$\text{If } 5v\beta/2 < d_i(k), \text{ then } h_i(k)=3\beta, \; Su_i(k)=0. \tag{3d}$$

2) Stop-Skipping Based on Rules: In this case, the control strategy only computes stop-skipping control actions. Thus, the set of rules becomes the following.

$$\text{If } d_i(k) \leq -v\beta/2, \text{ then } h_i(k) = 0, \; Su_i(k) = 1. \tag{4a}$$

$$\text{If } -v\beta/2 < d_i(k), \text{ then } h_i(k) = 0, \; Su_i(k) = 0. \tag{4b}$$

B. Control Strategies Based on Fuzzy Logic

In the aforementioned control strategies, the manipulated variables turn out to be a discontinuous function of position $d_i(k)$, which generates abrupt variations in *holding* decisions when position $d_i(k)$ is close to either the upper or the lower limits (i.e., $-v\beta/2, v\beta/2, 3v\beta/2$, or $5v\beta/2$). To avoid such discontinuous behavior associated with the expert control strategies, we propose to adapt the basic rules by using fuzzy logic techniques [1].

The fuzzy logic defines a fuzzy set A, whose elements belong to a discourse universe X as the set of tuples $A = [(x, \mu_A(x))/x \in X]$, where $\mu_A(x)$ denotes the membership function associated with fuzzy set A. The membership function assigns a membership degree between 0 and 1 to each the element of X. The most used membership functions are triangular, trapezoidal, and Gaussian in shape. Based on this definition, a fuzzy system corresponds to a set of rules of the following form:

$$\text{If } x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } A_2, \text{ then } y \text{ is } C \tag{5}$$

where A_1, A_2, and C are fuzzy sets.

Fig. 4 shows the structure of a fuzzy system used for control purposes, where $x(k)$ represents a vector of inputs $x_1(k)$ and $x_2(k)$, and $y(k)$ represents the output. The Knowledge Base contains the expert system rules, and the Inference Engine determines the output based on fuzzy logic operators. The Fuzzification Interface transforms the input variables (crisp or nonfuzzy values of the variables) into fuzzy variables. The Defuzzification Interface determines the output from the output fuzzy sets [18]. In this paper, we use the Zadeh logic defined for the Inference Engine and the gravity center method for defuzzification [1], [27].

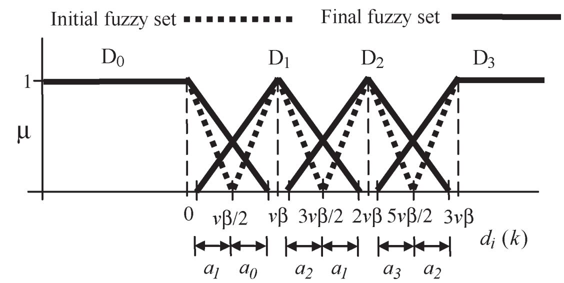

Fig. 5. Membership functions for position $d_i(k)$, with *holding* based on fuzzy rules.

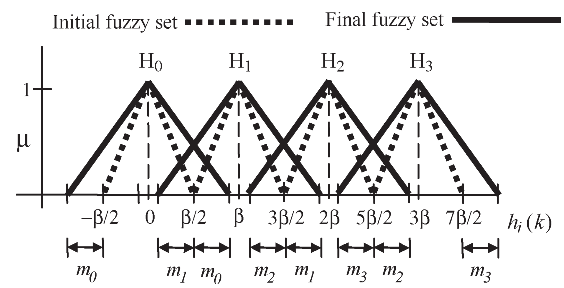

Fig. 6. Membership functions for *holding* $h_i(k)$, with *holding* based on fuzzy rules.

1) Holding Based on Fuzzy Rules: In this case, the rules associated with the fuzzy knowledge base are the same as those described in Section III-A1 but with variables $d_i(k)$ and $h_i(k)$ represented by fuzzy sets. Analytically, we have the following.

$$\text{If } d_i(k) \text{ is } \mathrm{D}_0, \text{ then } h_i(k) \text{ is } \mathrm{H}_0. \tag{6a}$$

$$\text{If } d_i(k) \text{ is } \mathrm{D}_1, \text{ then } h_i(k) \text{ is } \mathrm{H}_1. \tag{6b}$$

$$\text{If } d_i(k) \text{ is } \mathrm{D}_2, \text{ then } h_i(k) \text{ is } \mathrm{H}_2. \tag{6c}$$

$$\text{If } d_i(k) \text{ is } \mathrm{D}_3, \text{ then } h_i(k) \text{ is } \mathrm{H}_3. \tag{6d}$$

Fig. 5 and 6 show the membership functions of $d_i(k)$ and $h_i(k)$, respectively, for the triangular fuzzy sets D_0, D_1, D_2, D_3, and H_0, H_1, H_2, H_3 (with μ representing the membership degree of the corresponding variable). The lower limit of D_0 and the upper limit of D_3 were fixed to include all the values of the input variable. The membership functions are parameterized in terms of the average width corresponding to the base of the triangle. Parameters a_0, a_1, a_2, and a_3 are associated with the triangle bases for fuzzy sets of $d_i(k)$, and m_0, m_1, m_2, and m_3 are associated with the triangle bases for fuzzy sets of $h_i(k)$, as shown in Figs. 5 and 6. These figures show the initial shape of the membership functions in a dotted line and the adjusted shape in continuous line obtained from the parameter tuning described next in Section III-C.

Unlike the control strategy based on rules described in Section III-A1, *holding* takes any continuous value between $h_i(k) = 0$ and $h_i(k) = 3\beta$. Remember that the *stop-skipping* control actions are not considered in this controller $(Su_i(k) = 0)$.

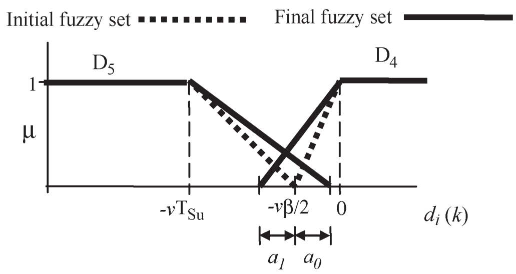

Fig. 7. Membership functions for distance $d_i(k)$, with *stop-skipping* based on fuzzy rules.

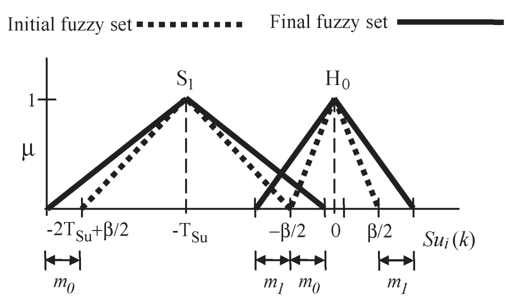

Fig. 8. Membership functions for *stop-skipping* $Su_i(k)$, with *stop-skipping* based on fuzzy rules.

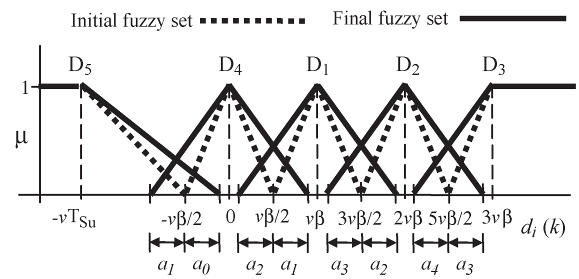

Fig. 9. Membership functions for distance $d_i(k)$, with *holding* and *stop-skipping* based on fuzzy rules.

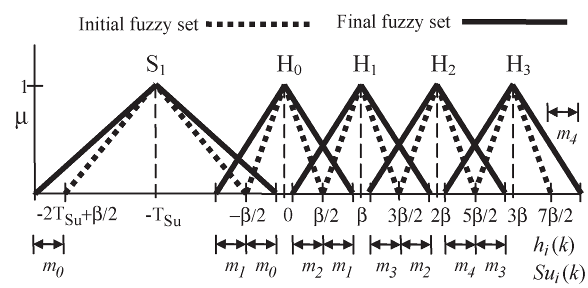

Fig. 10. Membership functions for output, with *holding* and *stop-skipping* based on fuzzy rules.

2) Stop-Skipping Based on Fuzzy Rules: This control strategy is derived from the scheme described in Section III-A2 but formulated as a fuzzy system.

$$\text{If } d_i(k) \text{ is } \mathrm{D}_5, \text{ then } Su_i(k) \text{ is } \mathrm{S}_1. \tag{7a}$$

$$\text{If } d_i(k) \text{ is } \mathrm{D}_6, \text{ then } Su_i(k) \text{ is } \mathrm{H}_0. \tag{7b}$$

The membership functions of variables $d_i(k)$ and $Su_i\ (k)$ for fuzzy sets D_5, D_6, S_1, and H_0 are shown in Figs. 7 and 8. Parameters a_0 and a_1 are associated with the triangle bases for the fuzzy sets of $d_i(k)$, and m_0 and m_1 are associated with the triangle bases for the fuzzy sets of $Su_i\ (k)$, as shown in Figs. 7 and 8. In addition, parameter T_{Su} represents the time saved by applying *stop-skipping*. These figures show the initial shape of the membership functions in a dotted line and the adjusted shape in continuous line obtained from the parameter tuning described next in Section III-C.

Note that, for this controller, the *holding* action is not considered $(h_i(k) = 0)$.

3) Holding and Stop-Skipping Based on Fuzzy Rules: In the integrated case, the holding and stop-skipping control actions are computed based on the following fuzzy rules.

$$\text{If } d_i(k) \text{ is } \mathrm{D}_1, \text{ then } h_i(k) \text{ is } \mathrm{H}_1. \tag{8a}$$

$$\text{If } d_i(k) \text{ is } \mathrm{D}_2, \text{ then } h_i(k) \text{ is } \mathrm{H}_2. \tag{8b}$$

$$\text{If } d_i(k) \text{ is } \mathrm{D}_3, \text{ then } h_i(k) \text{ is } \mathrm{H}_3. \tag{8c}$$

$$\text{If } d_i(k) \text{ is } \mathrm{D}_5, \text{ then } Su_i(k) \text{ is } \mathrm{S}_1. \tag{9a}$$

$$\text{If } d_i(k) \text{ is } \mathrm{D}_4, \text{ then } Su_i(k) \text{ is } \mathrm{H}_0. \tag{9b}$$

The membership functions of $d_i(k)$ associated with the fuzzy sets D_1, D_2, D_3, D_4, and D_5 are shown in Fig. 9. Moreover, the membership functions for the fuzzy sets H_0, H_1, H_2, H_3, and S_1 are shown in Fig. 10. Parameters a_0, a_1, a_2, a_3, and a_4 are associated with the triangle bases for fuzzy sets of $d_i(k)$, and m_0, m_1, m_2, m_3, and m_4 represent the parameters associated with the triangle bases for fuzzy sets of $h_i(k)$, as shown in Figs. 9 and 10.

In the next section, we describe the simulation scenarios for experimentation, and based on that, we tune the following parameters: β, T_{Su}, a_0, a_1, a_2, a_3, a_4, m_0, m_1, m_2, m_3, and m_4, involved in the fuzzy controllers (see Figs. 5–10). With the optimal parameters based on tuning, we conduct several experiments via simulation to quantify the benefits of the proposed methods with respect to the open-loop (OL, no control) scheme (see Section IV).

C. Parameter Tuning of Fuzzy Controller

Many approaches have been proposed for the generation of tuning fuzzy rules from numerical data. Most of them are based on evolutionary algorithms [8]. In this paper, evolutionary algorithms are used to select the relevant input variables to determine the number of membership functions on each input variable and to adjust the shape of each membership function. Thus, these methods can also determine the number and type of fuzzy rules, as well as the hierarchical structure of fuzzy rule-based systems. Cordon *et al.* [2] comprehensively reviewed various methods in this field, including the most representative genetic-algorithm-based methods for the design of fuzzy systems, which are mainly oriented to the generation of the number and distribution of the membership functions.

Another method for the design of tuning fuzzy rules is by using particle swarm optimization (PSO). This algorithm is based on a particle swarm that represents a population of candidate solutions [15]. The particles are randomly initialized, and then, they iteratively move within the search space to find

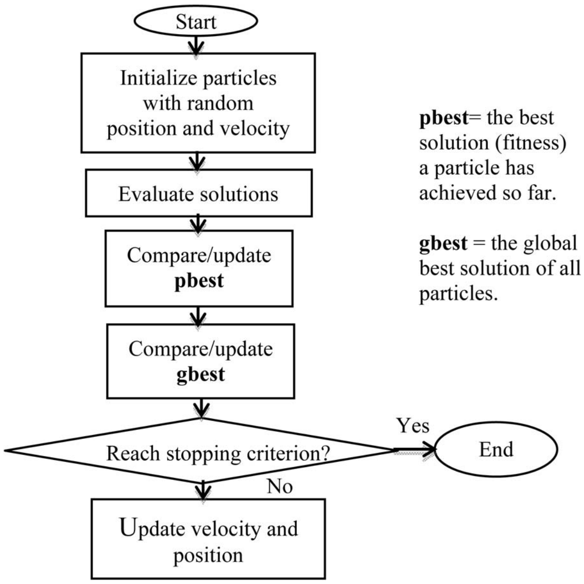

Fig. 11. Flow chart of traditional particle swarm optimization.

new solutions. The particles have a fitness associated with the solution quality, which is usually given by the objective function to be optimized. Each particle i is characterized by a position, a velocity, its best previous position, and the best position among all the particles belonging to the swarm. The particles are updated (they move) according to their cognitive and social behavior. An example of this is given in [9], where the PSO technique is employed to automatically tune the centers and the widths of the Gaussian membership functions at the input and output, resulting in the fuzzy membership function of a Mamdani type of fuzzy controller.

Considering the preceding discussion, we decided to tune the parameters of the fuzzy controller through the PSO technique. As previously described in Sections III-B1–B3, the *holding* based on fuzzy rules, *stop-skipping* based on fuzzy rules and their integration, and *holding* and *stop-skipping* based on fuzzy rules are parameterized (see Figs. 5–10). For those fuzzy controllers, β is the *holding* time; a_0, a_1, a_2, a_3, and a_4 are the parameters for the input fuzzy sets; m_0, m_1, m_2, m_3, and m_4 are the parameters for the output fuzzy sets; and Ts_u is the time saved by applying *stop-skipping*. Then, we design a set of experiments to determine the optimal values of the parameters mentioned in terms of the minimization of the average waiting time, which is highly correlated with the regularization of headways [5]–[7].

In Fig. 11, we show the traditional flow chart of PSO.

Since the original PSO algorithm does not guarantee convergence, in this work, we used a new PSO algorithm [25], which ensures local convergence and avoids the typical premature convergence found in the original algorithm.

In the original PSO algorithm reported in [15], if the current particle position is equal to both the current best position *pbest* and the global best position *gbest*, the velocity update will depend only on the value of the inertia weight and its current velocity. Thus, the particle will only move away from this point if its previous velocity and inertia weight are nonzero. If the previous velocities of all the particles in the population are very close to zero, then all of them will stop, and the algorithm will stop as well, which may lead to premature convergence of the original PSO algorithm. To solve this issue, we introduce a new update velocity algorithm to the PSO version proposed in [25]. The velocity for the particle *gbest* is updated based on three terms: 1) a reset of the particle position to the global best particle position; 2) a search direction (based on the value of the inertia weight and its current velocity); and 3) a random term for searching in the area surrounding the global best position *gbest*. The velocity of the remaining particles (different from *gbest*) is updated using the same velocity equation as in the original PSO. Then, the new PSO version ensures local convergence and avoids the premature convergence often observed in the original PSO implementations.

TABLE I
PARAMETERS OF THE PSO PROCESS

	β [s]	T_{Su} [s]	a_0 [m]	a_1 [m]	a_2 [m]	a_3 [m]	a_4 [m]
I	43	-	311	288	303	256	-
II	51	77	301	298	-	-	-
III	39	61	268	273	234	249	281
	m_0 [s]	m_1 [s]	m_2 [s]	m_3 [s]	m_4 [s]		
I	69	56	67	70	-		
II	65	76	-	-	-		
III	56	48	51	58	53		

I *Holding* based on fuzzy rules
II *Stop-skipping* based on fuzzy rules
III *Holding* and *stop-skipping* based on fuzzy rules

In our applications, we applied this new version of the PSO algorithm for tuning the parameters already described for the three fuzzy controllers proposed.

In Table I, the parameters that integrate the particles for each of the controllers designed along with their final values are shown. For example, for *holding* based on fuzzy rule control, the particles are summarized in the set $\{\beta, a_0, a_1, a_2, a_3, m_0, m_1, m_2, m_3\}$.

For tuning these fuzzy controllers, we consider $Nd = 30$ days with different demand patterns. The objective function used in the PSO algorithm refers to the minimization of $\sum_{d=1}^{Nd} \overline{T}_w(d)$, where $\overline{T}_w(d)$ is the mean waiting time of the stops on the route simulated by 2 h for each day d.

The epochs and particles for the population were chosen ad hoc to this particular problem (20 each), showing a reasonable convergence pattern to have consistent results. The other parameters were selected from similar studies in the PSO literature [23], i.e., particle velocity saturation of 50, two acceleration parameters, an initial inertia weight of 0.9, and a final inertia weight of 0.4.

IV. Simulation Experiments

The proposed control strategies were implemented in Matlab. The simulator of the transport public system corresponds to a discrete-event simulation platform. The rules for both fuzzy and expert controllers were also coded in Matlab.

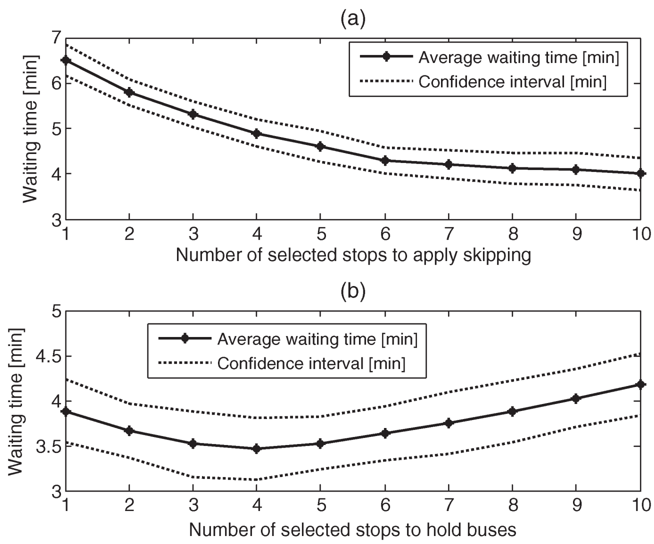

Fig. 12. Study for the number of selected control stations. (a) Stop-skipping based on rules. (b) Holding and stop-skipping based on rules.

The control strategies are applied to a bus corridor of 4000 m comprising ten stations evenly distributed over the whole route. The fleet comprises six buses with a capacity of 72 passengers each. The total simulation period was 2 h, with a warm-up time (discarded for statistics) of 15 min at the beginning and at the end of the simulation.

A. Selection of Stops to Apply Holding and Stop-Skipping Using an Expert Controller

First, we performed sensitivity studies to define the number of potential stops for applying *holding* and *stop-skipping*, which are both based on the expert controller. We conducted 30 replications for each combination of selected stops to either hold buses or skip stops. Thus, if, for example, five bus stops are tested for *holding*, 30 replications for each possible way to select five stops among the ten stops along the corridor were conducted.

In Fig. 12, we show the average waiting time and its 95% confidence interval for passengers, which are closely related to the variance of the distance between consecutive buses. Fig. 12(a) shows the sensitivity for *stop-skipping* in the context of the *stop-skipping* strategy based on rules only (see Section III-A2) to visualize the specific effect of such a strategy. As expected, in terms of waiting time, the tests showed that we should leave the option of applying this strategy in all the stops to minimize such an indicator. We then decided to apply potential skipping everywhere, which is reasonable as the conditions for applying skipping are quite restrictive. (Nobody should get off the bus.) Then, it does not make sense to add an additional constraint to the controller.

TABLE II
AVERAGE AND STANDARD DEVIATION OF WAITING TIME AND TRAVEL TIME PER PASSENGER AND COMPUTATION TIME

	Waiting time [min]			Travel time[min]		Computation time [s]
	Mean	Std.	B [%]	Mean	Std	
I	7.07	1.01	-	4.25	0.16	1.4
II	5.41	0.51	23.47	4.29	0.18	1.6
III	4.32	0.44	38.89	4.83	0.13	2.6
IV	3.90	0.36	44.83	4.39	0.10	1.6
V	3.82	0.29	45.96	4.40	0.11	2.6
VI	3.81	0.27	46.11	4.79	0.15	1.6
VII	3.32	0.25	53.04	4.20	0.16	2.8

I Open loop
II *Holding* based on rules
III *Holding* based on fuzzy rules
IV *Stop-skipping* based on rules
V *Stop-skipping* based on fuzzy rules
VI *Holding* and *stop-skipping* based on rules
VII *Holding* and *stop-skipping* based on fuzzy rules

After deciding to potentially being able to skip every stop, we analyze the number of stops to apply *holding*, in the context of the integrated *holding* and *stop-skipping* strategy based on rules (see Section III-A) in Fig. 12(b). As the figure shows, the best option is to apply *holding* at four stops. (The best combination was, effectively, stops 2, 3, 8, and 9.) The reduction of locations to hold vehicles also has a practical objective: The application of *holding* sometimes requires special infrastructure devices designed to accommodate vehicles, which makes sense on selected locations and not at every stop.

B. Simulation Results of the Proposed Strategies

With the considerations explained in Section IV-A, we ran 30 replications for each strategy explained in Section III. Table II shows the waiting time and travel time per passenger (on average and *std*), as well as the computation time taken to simulate the effective period of 2 h, applying the six proposed control schemes and the OL option, i.e., with no control at all.

In Table II, indicator B is also included, which quantifies the benefit associated with the performance of the proposed controller with respect to the OL system, which is computed through the expression shown at the bottom of the page.

From Table II, we observe that the travel times are similar in case of all the proposed controllers. The waiting times using only the *holding* action are reduced by 20% with respect to the OL strategy. A better performance of 40% savings of waiting time is obtained using the *stop-skipping* control action only,

$$B[\%] = \frac{[\text{Waiting Time Open Loop}] - [\text{Waiting time Control Strategy}]}{[\text{Waiting Time Open Loop}]} \cdot 100$$

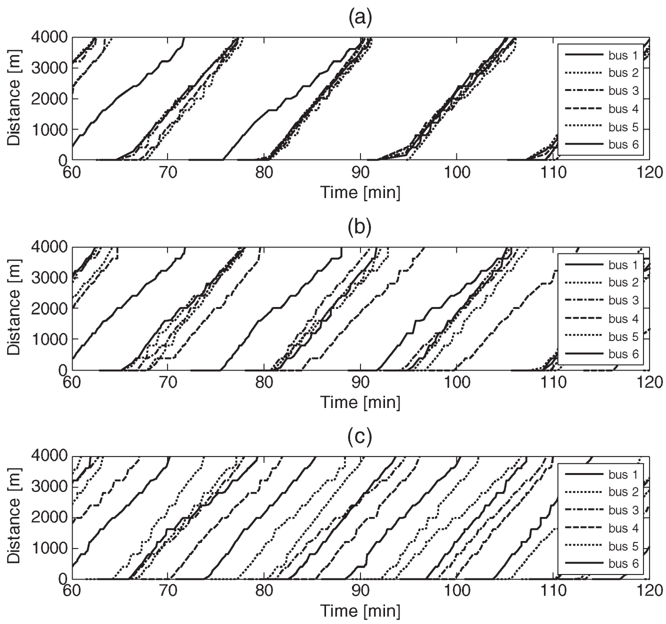

Fig. 13. Bus trajectories for different control strategies. (a) OL. (b) Holding and stop-skipping based on rules. (c) Holding and stop-skipping based on fuzzy rules.

with both deterministic rules and fuzzy rules. Finally, the best performance, with savings of about 47% in waiting time, is obtained when using a control strategy based on both *holding* and *stop-skipping* control actions. The maximum benefit is 53.04% when adding fuzzy rules, which was expected since the fuzzy implementation is more sophisticated and more sensitive than pure expert control and, therefore, in real applications (in this case through simulation), should report better performance than the deterministic set of rules. The reason for having almost no improvements in term of waiting time is basically because the rules are oriented to regularize frequencies, which has a direct impact on waiting time but not on travel times. The travel time over stretches between consecutive stops is quite stable since we consider a system running on exclusive corridors.

The considerable benefits of the strategies in all cases are also due to proper calibration of the parameters via sensitivity analysis, as shown in Section IV-A. This procedure has to be adapted to the particular conditions of the system to be controlled. Then, we can conclude that this preprocess (tuning of parameters) is very important to obtain a good performance of the set of rules to be implemented.

The objective of each set of rules implemented is to regularize bus headways. In Fig. 13, we can graphically see this effect, which ultimately generates benefits through savings in waiting times, as explained before. Each graph in Fig. 13 shows the trajectory of the buses (for a representative replication), considering (a) the OL, (b) expert control, and (c) fuzzy control and considering an integration of *holding* and *stop-skipping* [in (b) and (c)]. Fig. 13 shows only the final half of the simulation time.

From the figures, we can see how the trajectory of the buses is much more dispersed when the control actions are included, starting from a very condensed situation in Fig. 13(a) (which is an effect that is called bus bunching) and proceeding to a much better situation in terms of service regularity, mainly in Fig. 13(c). We observe that, in general, bunching conditions are reduced as the proposed control strategies are based on a local approach working toward the regularization of headways between arrivals of consecutive buses at stops. Finally, from empirical simulations, chain effects, bunching, and other kinds of problems were rarely observed.

V. Conclusion

In this paper, a scheme for the operation of a bus system running along a linear corridor has been proposed, which was based on expert rules and fuzzy logic. The control system relied on measures of the position of each bus, which was easy to obtain and implement by means of the emerging GPS technology. The strategies used were *holding*, *stop-skipping*, and their combination, with the final objective of keeping regular headways between buses to minimize the total waiting time of the passengers. The set of rules, in both deterministic and fuzzy cases, has been formulated in detail and then applied to a hypothetical linear corridor via simulation. The parameters of the fuzzy controllers have been tuned through a PSO algorithm.

The best controllers have been obtained for the combined strategy including *holding* and *stop-skipping*, considering fuzzy rules, which, for this application, reported savings in the average waiting times of 53.04% with respect to OL control, which represents the situation of keeping regular headways only at the terminal in one of the extremes of the corridor. Travel times on segments have been considered constant in the simulation, which means that the system only reported benefits in terms of waiting times. Sensitivity analyses have also shown that the optimal way to implement *holding* was just to hold buses in a selected number of stations, which, as mentioned before, is not a minor issue due to practical implementation issues (proper infrastructure to hold buses for example).

One important conclusion is the simplicity and potential savings in computation at resources and technology, which would result if this system were implemented. This is because the proposed methodologies are based on a set of straightforward control rules to decide the real-time behavior of buses.

As further research, we plan to extend the base of rules considering other inputs such as expected departure time from stops and expected bus occupancy. Moreover, other complex strategies can be included in the expert system, such as the additional injection of buses when the demand reaches a threshold due to online unexpected system conditions. In addition, we are testing models based on historical data for representing the behavioral patterns associated with variable speed in the fuzzy control design to incorporate both traffic congestion and demand disturbances.

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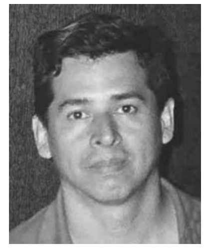

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