

# **ADAPTING PRODUCTIVITY THEORY TO THE QUADRATIC COST FUNCTION. An application to the Spanish electric sector**

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### ***Abstract***

In this article we have adapted productivity analysis to the case of a cost model using a quadratic cost function and discrete data. The main theoretical result is a productivity index that can be decomposed into modified versions of the contribution of technical change and the effect of the variations in the scale of production. This framework has been applied to the study of the Spanish electric sector from 1985 to 1996, during which relevant regulatory changes were introduced in order to increase productivity. For this, a normalized quadratic cost function was estimated. The results show important productivity gains with both technical change and scale effect playing important roles.

**Key words: productivity, quadratic cost function, technical change, electricity sector, multiple products**

## 1. Introduction

Most of the empirical work dealing with productivity measurement and technical change using cost functions, have used the decomposition proposed by Denny, Fuss and Waverman (1981). Such decomposition was originally applied for translog cost functions and adapted to discrete data, but is not directly applicable to other flexible forms as the quadratic.

As the translogarithmic cost function is not defined for zero values of the right hand side variables, the quadratic has been of help particularly when dealing with multiproduct activities. In this case, pooled data usually contains some zero outputs for some firms at some point in time. The quadratic cost function has been used indeed to deal with this problem, e.g. Röller (1990), who proposed a CES quadratic function, and Pulley and Braunstein (1992), who used a composite cost function.

On the other hand, although well defined for zero production levels, the quadratic cost function in its simplest form (Taylor expansion of order two), does not fulfill an important property like homogeneity of order one in factor prices. This can be imposed as constraints on the parameters in the translog function, but it can not be imposed in the quadratic without destroying its flexibility (Caves, Christensen and Tretheway, 1980). Normalization (using one factor price as a numeraire dividing cost and the other factor prices)<sup>1</sup> has been used as a procedure to avoid this unpleasant property (see for example Halvorsen, 1991).

The objective of this article is to adapt productivity theory and technical change analysis to the quadratic functional form and to discrete data variations. To do this, we have applied the quadratic approximation Lemma (Diewert, 1976). Then the resulting approach is applied to the electric sector in Spain using a normalized quadratic cost function for the period 1985-1996. Along these years, important regulatory changes took place in the

Spanish electric sector, and most of the analysis made indicates that there were efficiency gains due to the reforms.

In section 2 we develop the productivity model. In section 3 the Spanish electric sector is described and the econometric model is presented. The results are analyzed in section 4 and the most important conclusions are presented in the final section.

## 2. The productivity model and the quadratic cost function.

Let us try to find an appropriate productivity index for the quadratic cost function. Let us begin by decomposing a change in costs in all the different components. Consider a cost function

$$C=C(w, q, t) \quad [1]$$

where  $C$  is the minimum expenditure to produce output vector  $q$  at factor prices  $w$ . If  $C$  is represented through a quadratic cost function, then the *quadratic approximation lemma* (Diewert, 1976), which establishes that

$$f(z^1) - f(z^0) = \frac{1}{2} [\nabla_z f(z^1) + \nabla_z f(z^0)] (z^1 - z^0) \quad [2]$$

gives an **exact** value for the difference between the cost function evaluated at two points.

Applying [2] to [1] for a cost change between periods  $t_1$  and  $t_0$  one gets

$$\begin{aligned}
C_{t1} - C_{t0} = & \\
& \frac{1}{2} \left[ \sum_{i=1}^n \left( \left( \frac{\partial C}{\partial w_i} \right)_{t1} + \left( \frac{\partial C}{\partial w_i} \right)_{t0} \right) (w_{i,t1} - w_{i,t0}) \right] \\
& + \frac{1}{2} \left[ \sum_{j=1}^m \left( \left( \frac{\partial C}{\partial q_j} \right)_{t1} + \left( \frac{\partial C}{\partial q_j} \right)_{t0} \right) (q_{j,t1} - q_{j,t0}) \right] \quad [3] \\
& + \frac{1}{2} \left[ \left( \frac{\partial C}{\partial t} \right)_{t1} + \left( \frac{\partial C}{\partial t} \right)_{t0} \right] (t_1 - t_0)
\end{aligned}$$

Dividing [3] into  $C_{t0}$  and taking into account that  $\frac{\partial C}{\partial w_i} = x_i$  and that  $\frac{\partial C}{\partial q_j} = m_j$  are the  $i$ th

optimal factor demand and the  $j$ th marginal cost respectively, the rate at which cost changes can be approximated as

$$\begin{aligned}
\frac{C_{t1} - C_{t0}}{C_{t0}} = & \\
& \frac{1}{2C_{t0}} \left[ \sum_{i=1}^n (x_{i,t1} + x_{i,t0}) (w_{i,t1} - w_{i,t0}) \right] \\
& + \frac{1}{2C_{t0}} \left[ \sum_{j=1}^m (m_{j,t1} + m_{j,t0}) (q_{j,t1} - q_{j,t0}) \right] \quad [4] \\
& + \frac{1}{2C_{t0}} \left[ \left( \frac{\partial C}{\partial t} \right)_{t1} + \left( \frac{\partial C}{\partial t} \right)_{t0} \right] (t_1 - t_0)
\end{aligned}$$

where the first term is the contribution of factor price changes, the second is the contribution of product variations and the third is the consequence of a change of the cost function itself in time.

Now we will follow Denny, Fuss and Waverman (1981), but applied to discrete changes in

variables. Let us call  $\overset{\circ}{A}$ ,  $\overset{\circ}{B}$  and  $\overset{\circ}{D}$  the three elements in equation [4]. Let us decompose the first one as

$$\begin{aligned} \overset{\circ}{A} &= \frac{1}{2C_{t0}} \left[ \sum_{i=1}^n (x_{i,t1} + x_{i,t0}) \Delta w_i \right] = \frac{1}{2C_{t0}} \sum_{i=1}^n x_{i,t1} \Delta w_i + \frac{1}{2C_{t0}} \sum_{i=1}^n x_{i,t0} \Delta w_i \\ &= \frac{C_{t1}}{2C_{t0}} \sum_{i=1}^n \frac{w_{i0}}{w_{i1}} s_{i,t1} \frac{\Delta w_i}{w_{i0}} + \frac{1}{2} \sum_{i=1}^n s_{i,t0} \frac{\Delta w_i}{w_{i0}} = \frac{1}{2} \sum_{i=1}^n \frac{C_{t1}}{C_{t0}} \frac{w_{i0}}{w_{i1}} s_{i,t1} \overset{\circ}{w}_i + \frac{1}{2} \sum_{i=1}^n s_{i,t0} \overset{\circ}{w}_i \end{aligned} \quad [5]$$

where  $s_i$  is factor  $i$ 's cost share. For short, the rate of change in cost due to variations in factor prices have been decomposed into two terms that depend on the rate of change in factor prices,  $\overset{\circ}{w}_i$ , and on factor shares in both periods.

The second term in equation [4] can be transformed as

$$\overset{\circ}{B} = \frac{1}{2C_{t0}} \left[ \sum_{j=1}^m m_{j,t1} \Delta q_j + m_{j,t0} \Delta q_j \right] = \frac{1}{2} \sum_{j=1}^m \frac{C_{t1}}{C_{t0}} \frac{q_{j0}}{q_{j1}} \mathbf{e}_{c,qj1} \overset{\circ}{q}_j + \frac{1}{2} \sum_{j=1}^m \mathbf{e}_{c,qj0} \overset{\circ}{q}_j \quad [6]$$

where  $\mathbf{e}_{c,qj}$  is the product elasticity of cost, and  $\overset{\circ}{q}_j$  the rate of change in output  $j$ .

Finally, the third term can be easily manipulated to obtain

$$\overset{\circ}{D} = \frac{1}{2C_{t0}} \left[ \left( \frac{\partial C}{\partial t} \right)_{t1} + \left( \frac{\partial C}{\partial t} \right)_{t0} \right] (t_1 - t_0) = \left[ \frac{C_{t1}}{2C_{t0}} T_{t1} + \frac{1}{2} T_{t0} \right] (t_1 - t_0) \quad [7]$$

where  $T = \frac{1}{C} \frac{\partial C}{\partial t}$  is the technical change. This establishes that changes in the cost function itself can be expressed as half the change in  $t_0$  plus half the change in  $t_1$  weighted by the cost relation, such that a cost increase would make technical change in  $t_1$  weight more than in  $t_0$  and the contrary would happen for a cost reduction.

On the other hand, cost in  $t_1$  can be expressed as

$$C_{t1} = \sum_{i=1}^n (w_{i0} + \Delta w_i)(x_{i0} + \Delta x_i) = \sum_{i=1}^n w_{i0}x_{i0} + \sum_{i=1}^n w_{i0}\Delta x_i + \sum_{i=1}^n \Delta w_i x_{i0} + \sum_{i=1}^n \Delta w_i \Delta x_i \quad [8]$$

such that using  $w_i$  as a common factor in the last two terms and converting into rates

$$\frac{C_{t1} - C_{t0}}{C_{t0}} = \sum_{i=1}^n \frac{w_{i0}x_{i0}}{C_{t0}} \frac{\Delta x_i}{x_{i0}} + \sum_{i=1}^n \frac{w_{i0}C_{t1}}{w_{i1}C_{t0}} \frac{w_{i1}x_{i1}}{C_{t1}} \frac{\Delta w_i}{w_{i0}} = \sum_{i=1}^n s_{i0} \overset{\circ}{x}_i + \sum_{i=1}^n \frac{w_{i0}C_{t1}}{w_{i1}C_{t0}} s_{i1} \overset{\circ}{w}_i \quad [9]$$

that is

$$\frac{1}{2} \sum_{i=1}^n \frac{w_{i0}C_{t1}}{w_{i1}C_{t0}} s_{i1} \overset{\circ}{w}_i = \frac{1}{2} \overset{\circ}{C} - \frac{1}{2} \sum_{i=1}^n s_{i0} \overset{\circ}{x}_i \quad [10]$$

where  $\overset{\circ}{x}_i$  is the rate of change of factor  $i$ . Analogously, from [8] and using  $x_i$  as a common factor between the second and fourth term, one gets

$$\frac{1}{2} \sum_{i=1}^n s_{i0} \overset{\circ}{w}_i = \frac{1}{2} \overset{\circ}{C} - \frac{1}{2} \sum_{i=1}^n \frac{C_{t1}x_{i0}}{C_{t0}x_{i1}} s_{i1} \overset{\circ}{x}_i \quad [11]$$

Adding [10] and [11]

$$A = \overset{\circ}{C} - \sum_{i=1}^n \frac{1}{2} \left( \frac{C_{t1}x_{i0}}{C_{t0}x_{i1}} s_{i1} + s_{i0} \right) \overset{\circ}{x}_i \quad [12]$$

Solving for  $-D$  in [4], and using [6] and [12] one obtains

$$-D = \frac{1}{2} \sum_{j=1}^m \left( \frac{C_{t1}q_{j0}}{C_{t0}q_{j1}} \mathbf{e}_{C,q_{j1}} + \mathbf{e}_{C,q_{j0}} \right) q_j \overset{\circ}{q}_j - \frac{1}{2} \sum_{i=1}^n \left( \frac{C_{t1}x_{i0}}{C_{t0}x_{i1}} s_{i1} + s_{i0} \right) \overset{\circ}{x}_i \quad [13]$$

where the second term in the right hand side is an index of factor change between  $t_1$  and  $t_0$ ,

which we will call  $\overset{\circ}{I}$ . Let us define  $\overset{\circ}{M}$  as an index of product change between periods  $t_1$  and  $t_0$ , namely

$$\overset{\circ}{M} = \frac{\sum_{j=1}^m \left( \frac{C_{t1}q_{j0}}{C_{t0}q_{j1}} \mathbf{e}_{C,q_{j1}} + \mathbf{e}_{C,q_{j0}} \right) \overset{\circ}{q}_j}{\sum_{j=1}^m \left( \frac{C_{t1}q_{j0}}{C_{t0}q_{j1}} \mathbf{e}_{C,q_{j1}} + \mathbf{e}_{C,q_{j0}} \right)} \quad [14]$$

Introducing [14] into [13], adding and subtracting  $\overset{\circ}{M}$  one gets

$$-\overset{\circ}{D} = \overset{\circ}{M} - I + \overset{\circ}{M} \left[ \frac{1}{2} \sum_{j=1}^m \left( \frac{C_{t1}q_{j0}}{C_{t0}q_{j1}} \mathbf{e}_{C,q_{j1}} + \mathbf{e}_{C,q_{j0}} \right) - 1 \right] \quad [15]$$

Finally, define  $\overset{\circ}{P} = \overset{\circ}{M} - I$  as the difference between the product and factor indices previously defined, such that  $\overset{\circ}{P}$  represents the productivity index adapted to the quadratic form and discrete changes in the cost function variables, i.e.

$$\overset{\circ}{P} = -\overset{\circ}{D} + \overset{\circ}{M} \left[ 1 - \frac{1}{2} \sum_{j=1}^m \left( \frac{C_{t1}q_{j0}}{C_{t0}q_{j1}} \mathbf{e}_{C,q_{j1}} + \mathbf{e}_{C,q_{j0}} \right) \right] \quad [16]$$

If radial (proportional) variations of the product vector are considered and the technology exhibits constant returns to scale, then the expression in parenthesis vanish, in which case the productivity index coincides with the index of technical change. Under increasing (decreasing) returns the productivity index is larger (smaller) than the index of technical change (for radial variations of products). These results are equivalent to those derived by Denny, Fuss and Waverman (1981).

### 3. An application to the Spanish electric sector.

In this section we will apply the theoretical development presented above to the Spanish electric sector for the period 1985-1996, when Spain experienced an interesting regulation change, going from a traditional cost plus regulation to a system based upon incentives to

productivity. In the following section we will review the structure and regulation of the Spanish electric sector. Later on we present the estimation of a multiproduct cost function adapted to the special characteristics of the representative Spanish firm along this period.

### **3.1.- Structure and regulation of the Spanish electric sector during the period 1985-1996**

Until 1996, the Spanish electric sector worked as an integrated system<sup>2</sup>. In the short run, both the transmission phase and dispatching were in the hands of an independent entity called Red Eléctrica de España (Spanish Electric Network, SEN). In the long run, generation needs were globally defined through the National Energy Plans. Distribution was mostly managed by large firms, vertically integrated with generation, which were assigned exclusive responsibility on specific geographical zones. The particular feature of the Spanish electric sector in the period 1985-1996 has been that transmission is independent of both generation and distribution phases.

The LSF was designed from 1983 to 1987, and begun to be applied since January 1988<sup>3</sup>. Although the whole mechanism was rather complex, the basic idea behind the LSF was as follows. A firm involved in generation and/or distribution of electricity would receive a payment equal to its standard costs. The standard costs correspond to a valuation procedure (common to all firms) of fixed and variable costs of generation and distribution including an adequate retribution of capital investment.

According to the LSF methodology, revenues should cover system wide costs and a single tariff holds for the whole territory. On the other hand, the firms present different equipment and different market structures that translate into different distribution expenses and different revenue per kwh sold. As actual revenue does not correspond to actual sales to

customers but to the calculated standard costs, a crossed-firms compensation system is needed in order to balance the financial result of each firm. The basic elements of the LSF methodology can be summarized as follows (Rodríguez and Castro, 1994):

- a) The central administration determines the standard cost SC for each firm according to equipment (generation) and distribution structure.
- b) Each firm produces according to its (unified) plan incurring a cost C and receiving a revenue R, making a nominal net revenue  $NR=R-C$ .
- c) Each firm receives a compensation (T) equal to the difference between SC and R. (pays if negative):  $T = SC-R$ .
- d) Thus, the actual net revenue of each firm happens to be  $ANR = NR+T = SC-C$

This form of regulation pushes each firm towards the maximization of the difference between the standard and the real costs. It favors cost efficiency in production at a firm level, because this works towards increasing firm specific profits. The LSF has been labeled as a case of *yardstick competition* in which the price of the regulated firm is set as a function of the average cost of the remaining firms. Rodríguez and Castro (1994) consider that setting the level of the standard cost is an *ad hoc* procedure that evolves through an explicit price index. Thus, the standard cost should be interpreted as a price cap that is periodically updated independently of the evolution of average efficiency in the sector.

Different authors agree that the LSF has produced the appropriate incentives for cost reduction, although some point out possible shortcomings as well. Kühn and Regibeau (1998) praise the approach but also identify a series of elements that could have produced unwanted firm behavior. First, cost reduction incentives are not applied equally to all type

of costs. Second, actual profits, i.e. the difference between SC and C, could be enlarged through direct negotiation between the government and the firms.

Crampes and Laffont (1995) analyzed the effect of the reward system built within the LSF on efficient behavior of firms, using the framework of incentive theory. For them, LSF regulation mimics a system of *yardstick competition* where the standard cost is the reference once it is taken by each firm as exogenous, at least in the short run. The compensation mechanism together with some correction elements induces efficiency although some factors related with the LSF could produce certain bias in investment decisions.

On the other hand, using non-parametric techniques Arocena and Rodríguez (1998) found productivity improvements in thermal coal plants in the period 1988-1995. According to their results, annual productivity increased in average by 3,2% during the period, mostly explained by management improvements that increased efficiency, a direct effect of the LSF according to the authors.

### **3.2 The Normalized Quadratic Cost Function**

In this work we will use a normalized quadratic cost function (NCQF), which is a well-defined flexible functional form that permits zero values for the right hand side variables. It is flexible because no restrictions are imposed on the sign nor values of the first and second derivatives. Thus, data is allowed to show freely the relations between factors and/or between products. As known, the cost function should be homogeneous of degree one in factor prices, monotonic in factor prices and in products, and concave in factor prices. The first property is imposed by simply normalizing total cost and all factor prices by one factor

price. Both monotonicity and concavity should be verified through the Jacobian and the Hessian of the estimated function.

If the normalizing factor is the price of input  $n$ , the NCQF can be written as

$$\begin{aligned}
 C = & \hat{a}_0 + \sum_{j=1}^m \hat{a}_j q_j + \sum_{i=1}^{n-1} \hat{a}_i w_i + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \hat{a}_{ij} q_i q_j + \frac{1}{2} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \hat{a}_{ij} w_i w_j + \sum_{i=1}^{n-1} \sum_{j=1}^m \hat{a}_{ij} w_i q_j \\
 & + \mathbf{j} t + \sum_{j=1}^m \hat{e}_j q_j t + \sum_{i=1}^{n-1} \hat{i}_i w_i t + \frac{1}{2} \delta t^2 + \sum_{i=1}^{N-1} \hat{u}_i D_i
 \end{aligned} \tag{17}$$

where  $C$  is the normalized cost,  $m$  is the number of products,  $n$  is the number of factors,  $w_i$  are the normalized factor prices,  $q_j$  are product quantities,  $t$  is a time trend,  $D_i$  are the firm specific dummies,  $\mathbf{w}_i$  are the firm specific effects and  $N$  is the number of firms. The time trend captures how the cost function changes over time, from which technical change can be detected and quantified. This variable has been crossed with both factor prices and products, such that non-neutral technical change and the contribution by product and phase can be detected as well.

The firm specific effects are designed to capture the differences among firms that are not explained by the rest of the variables. Although the model assumes that all firms have access to the same technology, they operate with different cost levels. In other words,  $\mathbf{w}_i$  permits a correction at the origin. This means that the error term can be looked at as

$$U_{it} = \mathbf{w}_i + \mathbf{e}_t$$

where  $U_{it}$  is the sum of a firm specific term  $\mathbf{w}_i$  that captures non-observed heterogeneity at a firm level including individual inefficiency, and a purely random term  $\mathbf{e}_t$ .

Factor demands can be obtained from [17] applying Shephard's Lemma, i.e.

$$x_i = \frac{\partial C}{\partial w_i} = \mathbf{b}_i + \sum_{j=1}^{n-1} \mathbf{g}_j w_j + \sum_{j=1}^m \mathbf{r}_{ij} q_j + \mathbf{m}_i t \quad [18]$$

Equations [18] add no additional parameters and increases significantly the degrees of freedom of the econometric model by including new information (factor demands). This is why the joint estimation of equations [17] and [18] make the parameter estimates more efficient.

From the estimated cost function, all the information necessary to calculate the scale effect of production on productivity, namely the cost-product elasticities and the estimated costs. Furthermore, to obtain the technical change index presented in [7] the derivative of  $C$  with respect to  $t$  is needed, which is

$$\frac{\partial C}{\partial t} = \mathbf{j} + \mathbf{p} + \sum_{i=1}^n \mathbf{m}_i w_i + \sum_{j=1}^m \mathbf{l}_j q_j \quad [19]$$

Note that  $\dot{\lambda}_i$  is the derivative of  $x_i$  with respect to  $t$ , which means that the demand for input  $i$  changes in time at a constant rate  $\dot{\lambda}_i$ , independently of the rest of the variables (a purely technical effect). Analogously,  $\dot{e}_j$  is the derivative of product  $j$  marginal cost with respect to time, and has a similar interpretation.

Dividing into  $C$ , this procedure results in the decomposition of technical change into three terms (Baltagi and Griffin, 1988).

$$\text{Effect due to pure technical change: } \frac{1}{C}(\mathbf{j} + 2\mathbf{p})$$

$$\text{Effects due to non-neutral technical change: } \frac{1}{C} \sum_i^n \mathbf{m}_i w_i$$

Effects due to scale-augmenting technical change:  $\frac{1}{C} \sum_i^m I_i q_i$

We will use this decomposition in the empirical analysis presented in the next section

### 3.3 The variables and Data

Equation (17) is intended to explain the economic cost of generation and distribution. Total costs are the sum of all expenses with the exception of purchased power (1996 millions pesetas). Distribution costs represent only those derived from energy transmission (circulation) and network maintenance as well as those associated with final delivery to customers, which are all independent on the origin of the energy (self generated or acquired). This is the reason why we did not consider the expenses on purchased power in total costs. Accordingly, its price was not included in the right hand side<sup>4</sup>. Firms use fuel<sup>5</sup>, labor, capital and intermediate inputs. Appendix 1 contains factor prices calculation.

We distinguish four generation products, namely thermal coal (gtc), thermal fuel (gtf), hydroelectric (gh), and nuclear (gn), as well as one distribution output. The gigawatt-hour unit (million kilowatt-hour) was used for all products in the model. Thus, the product vector is

$$Q = (gtc, gtf, gh, gn, di)$$

The unit of observation is a firm a given year, not a plant as both cost minimization and the regulatory incentives are firm oriented. Since we have specified a multiproduct technology, we do not required to limit the sample according technical similarity. We gathered a data pool of twelve firms observed annually during the period 1985 to 1996, i.e. during the LSF period. The precise information was obtained directly from the yearbooks (memorias) of

the firms. As not all years were available for all firms, the *pool* is unbalanced and contains 106 observations. All important Spanish firms were included with the exception of ENDESA, as their data included mining activities mixed with electric generation.

Firms considered are: Unión Eléctrica Fenosa (FENOSA), Compañía Sevillana de Electricidad (SEVILLANA), Fuerzas Eléctricas de Cataluña (FECSA), Empresa Nacional Hidroeléctrica del Ribagorzana (ENHER), Hicroeléctrica del Cantábrico (HC), Electra de Viesgo (VIESGO), Hidroeléctrica de Cataluña (HEC), Eléctricas Reunidas de Zaragoza (ERZ), Empresa Nacional de Córdoba (ENECO) (which is the only one that just generates), IBERDUERO, and Hidroeléctrica Española (HE). These two last ones merged by 1992 to create IBERDROLA. Altogether, these firms represent 81% of energy consumption and nearly 50% of electricity production in Spain. It is relevant to note that three firms produce only one type of generation product and that five of them produce all four generation types. Appendix 2 contains two tables summarizing costs and production of all firms.

### **3.4 Results**

We have estimated the system of equations (17) and (18) with the variables deviated with respect to the sample mean, which permits an immediate interpretation of the parameters. The price of labor was used as the normalizing factor. The system was estimated using Zellner's (1962) seemingly unrelated regression procedure applied to the data described above. In order to estimate the system, one of the firm specific dummies had to be eliminated to avoid multicollinearity. The chosen firm was ENECO. Results are presented in Appendix 3.

The estimated cost function is homogeneous of degree one by construction. Monotonicity in factor prices and products was verified for 99% of the observations. The Hessian shows

that the function is concave in factor prices. Thus, the estimated cost function fulfills all conditions and can be taken as a good representation of the (dual) underlying technology.

The coefficients in general are intuitively appealing in terms of sign and value of those with immediate meaning. The independent term (C1) plus the average of the dummies (C49 to C59) yields a value very close to the average of the normalized cost, as expected. Normalized marginal costs at the mean (parameters C2 to C6) are all positive and have appealing relative magnitudes. Parameters  $\beta_i$  (C7 to C9) replicate factor demands at the mean. The first order time trend is negative (C10), which means that costs diminish with time, everything else being constant. The positive sign of the second order term (C48) shows that this effect is decreasing. There are no firms with a statistically significant specific effect less than zero, which means that no firm operates below the cost of the base firm ENECO for the same level of the rest of the variables. Although not all of the cost excess can be associated with inefficiency, permanent inefficiency within the period is indeed part of the cause of a significant positive value. This is the case for ENHER, ERZ, HEC, and VIESGO. Those firms whose specific effect is not significant have the same costs as the one used for reference (FECSA, FENOSA, HC, HE, IBERDUERO, IBERDROLA and SEVILLANA).

#### **4. Productivity analysis.**

As shown in equation [16] the rate of change of productivity can be separated into the evolution of technical change and the effect of the change of production levels when economies of scale are not constant. To obtain these we need the estimated cost and the cost product elasticities. Table 1 contains the results evaluated at each annual mean and at the mean of the whole period.

**Table 1. Cost-product elasticities and economies of scale**

<b>YEAR</b>	<b>COAL</b>	<b>FUEL</b>	<b>HIDROEL</b>	<b>NUCLEAR</b>	<b>DISTRIB</b>	<b>S</b>
<b>1985</b>	0.31	0.009	0.08	0.04	0.31	1.33
<b>1986</b>	0.31	0.002	0.04	0.05	0.37	1.28
<b>1987</b>	0.29	0.008	0.07	0.11	0.29	1.32
<b>1988</b>	0.21	0.015	0.16	0.26	0.13	1.29
<b>1989</b>	0.26	0.019	0.05	0.21	0.30	1.18
<b>1990</b>	0.33	0.017	0.04	0.13	0.31	1.21
<b>1991</b>	0.33	0.041	0.06	0.14	0.27	1.19
<b>1992</b>	0.28	0.101	0.08	0.21	0.26	1.07
<b>1993</b>	0.29	0.036	0.08	0.18	0.38	1.03
<b>1994</b>	0.30	0.033	0.10	0.17	0.37	1.02
<b>1995</b>	0.31	0.076	0.09	0.16	0.35	1.01
<b>1996</b>	0.27	0.030	0.22	0.22	0.19	1.08
<b>AVERAGE</b>	0.28	0.035	0.10	0.18	0.29	1.13

The largest elasticities within the generation phase are for coal and nuclear, but the distribution product exhibits an even larger elasticity. The multiproduct degree of economies of scale indicates decreasing average radial costs, getting close to constant by the end of the period.

In table 2 we present the estimated variations of the productivity index together with its decomposition into the technical change index and the economies of scale term in equation [16]. In global terms, the productivity index has increased annually by 5,3% in average, with 2,4% corresponding to the technical change index and the rest to the scale effect. The technical change index is quite stable during the period, varying from figures around 3% during the first seven years to 2% in the last four. The scale effect shows larger variability depending on production, as the industry faces increasing returns during the whole period.

Thus, the most important scale effects occur within the period 1989-1992, and the effect is negative from 1993 on as the production index decreases.

**Table 2. Rate of variation of productivity, technical change and scale effect.**

	86/85	87/86	88/87	89/88	90/89	91/90	92/91	93/92	94/93	95/94	96/95
<b>TCI</b>	2.926	3.143	2.984	2.822	2.976	3.255	2.647	2.052	2.358	2.308	1.998
<b>SEI</b>	0.906	0.655	2.874	11.812	2.750	4.887	5.66	-3.41	-1.081	7.405	-1.711
<b>PI</b>	3.83	3.80	5.86	14.63	5.73	8.14	8.31	-1.36	1.27	9.71	0.29

TCI: Technical Change Index. SEI: Scale Effect Index. PI: Productivity Index

In order to further analyze the technical change, it is convenient to decompose it taking advantage of the richness of the estimated cost function, which is what we do next.

**Table 3. Decomposition of technical change**

YEAR	TCI	PURE TCI	SCALE AUGMENTING	NON NEUTRAL EFFECT
86/85	2.926	3.61	-0.62	-0.06
87/86	3.143	3.53	-0.3	-0.08
88/87	2.984	3.69	-0.61	-0.09
89/88	2.822	3.10	-0.21	-0.06
90/89	2.976	2.84	0.17	-0.03
91/90	3.255	3.29	-0.05	0.01
92/91	2.647	3.00	-0.40	0.06
93/92	2.052	1.98	0.04	0.03
94/93	2.358	1.71	0.58	0.06
95/94	2.308	1.69	0.54	0.08
96/95	1.998	1.64	0.31	0.05
<b>AVERAGE</b>	2,67	2.73	-0.05	-0.003

The results show that most of the technical change is a pure effect, while the non-neutral is practically negligible and the scale augmenting effect has some relevance. Pure technical change represents better use of resources that can not be associated with inputs nor products. The decomposition in table 3 permits a quantitative investigation of the causes

behind the relatively stable contribution of technical change. During the first four years the non-neutral and the scale augmenting effects are negative while in the last four years both contribute to productivity improvements. On the other hand, pure technical change during the first period practically doubles that during the last period, which results in a relatively smooth overall effect. Note that the scale augmenting effect explains up to 25% of the total technical change (period 93-94), with fuel generation and distribution having the largest contribution. During the first years, fuel generation adds positively (and significantly) while distribution shows a negative effect at the beginning and a positive one by the end of the period. From 1992 on, coal generation has a positive contribution to technical change.

It is worth noting that the overall results obtained here, are similar to those obtained by Arozena and Rodríguez (1998). However, the analysis behind the aggregate figures is richer in our case, as the whole range of production has been specified, including distribution.

## **5. Conclusions**

In this article we have adapted productivity analysis to the case of a cost model using a (normalized) quadratic cost function and discrete data. The main theoretical result is a productivity index that can be decomposed into modified versions of the contribution of technical change and the effect of the change in the scale of production. Qualitatively, these are similar to the results obtained with the translog functional form.

This framework has been applied to the Spanish electric sector along the period 1985-1996, during which relevant regulatory changes were introduced in order to increase productivity. A normalized quadratic cost function was estimated, and the results show an average annual productivity improvement of 5,3%, where 45% corresponds to technical change and

55% to the scale effect. The decomposition of technical change shows that the pure technical effect is the most relevant, while the scale augmenting effect is relatively important during some years only; the non-neutral effect is practically negligible. Further decomposition of the scale augmenting effect shows that fuel generation and distribution were the products that contributed the most to technical change. These results confirm important productivity gains during the period 1985-96. In agreement with the opinion of all authors cited here that have studied the Spanish electric sector, this can be attributed to the incentives provoked by the LSF.

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### **Notes.**

1. The normalization of quadratic functions has been also applied to profit functions as proposed by Lau (1976) and most recently applied by Bhattacharyya and Glover (1993).
2. For an exhaustive description and analysis of this period see Ramos, Martínez-Budría and Jara-Díaz (2002).
3. Although the LSF was officially approved on January 1988, the compensation system designed to induce efficiency incentives was applied since 1983.
4. This procedure was first introduced by Gilsdorf (1994, 1995) who claimed that acquiring energy constitutes just a transfer between whoever generated and the consumer. The cost function without purchased power would be incorrectly specified if there had been an incentive to generate as a consequence of double marginalization. We believe that this was not the case during the LSF period because increasing power capacity was already strictly regulated before this period. In addition, we have to point out the following facts. First, the ratio between distribution and generation (D/G) increased by 6.5% for all firms. Second, the utilization of a particular type of installed power was not an arbitrary decision by the firm, as it was done according to a strict merit order governed by variable cost, preventing strategic behavior regarding the generation variable. Third, the correlation between D/G and time was less than 0.02 in absolute value. Therefore, we conclude that either regulation prevented double marginalization and/or there was no simultaneous incentive to generate and distribute.
5. The derived demand for fuel deserved a special treatment. First, fuel consumption depends only on the amount of coal and fuel generation (thermal). Thus, fuel price has not been crossed with the other products.

Second, the required amount of fuel depends only on the technical relation it has with thermal generation, which means that it can not be substituted and, therefore, other factor prices do not explain fuel demand either. Thus, fuel price was not crossed with other prices. Lastly, as thermal generation is imposed by central dispatching rules, fuel price affects production costs but do not influence the amount of fuel to be used. As a conclusion, fuel consumption depends only on the amount of coal and fuel generation and time. For short,  $X_C^* = X_C^*(gtc, gtf, t)$  and fuel price has to be crossed with this variables only by virtue of Shephard's lemma. This holds only for this factor price.

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## Appendix 1. Factor Prices

As factors are in fact aggregates (capital, labor, fuel and intermediate input), we have to construct indices for factor prices, which requires the corresponding expenditures and a proxy measure for each factor. Thus, the calculation of a single labor price (pl) index is straightforward and units are million annual pesetas per worker. We use a fuel price (pc) variable obtained from the cost of an equivalent ton of coal that represents the cost of fossils fuels, obtaining (10<sup>3</sup> pts/ton). We do not consider the fuel factor in the case of nuclear energy. The annual consumption of uranium is included as depreciation for the same year (i.e. part of the cost of capital).

An index for the price of capital for each firm was obtained as

$$p_{kt} = \frac{A_t + r_t * FP_t}{IMNE_t}$$

where  $p_{kt}$  is the price of capital in year t,  $A_t$  is the amortization in year t,  $r_t$  is the average rate of return in the electric sector in year t,  $FP_t$  is stockholders' equity in year t and  $IMNE_t$  are the net tangible fixed assets used during year t. The price of capital thus defined is a relative rate that takes into account the depreciation charges of each year and the return on own funds as a proxy of capital expenditures. We use as the measure of capital the net tangible fixed assets currently used. We use as the return on own funds ( $r_t$ ) the average financial returns (net profit before taxes/own funds before taxes) of the firms which are members of UNESA.

Expenditures in intermediate inputs are related with operating expenses, excluding labor costs and procurements (purchased power and fuel). It is a quite heterogeneous aggregate that is very much short run oriented. Therefore, to obtain a price index (pi), the corresponding expenses were divided into net revenues, subtracting those from purchased power.

## Appendix 2

Table A.2.1. MEAN PRODUCTION BY FIRM IN THE PERIOD 1985-1996 (million kwh).

	G. Coal	G. fuel	G. hyd.	G. nuc.	G. total	DIST.	G+D	G/D
Average	2706	197	2176	3160	8239	11350	19589	0.72
Variation coef	1.16	2.35	1.48	1.69	1.16	1.05	---	---
ENECO	2042	0	0	0	2042	0	2042	---
ENHER	0	0	2296	0	2296	8572	10868	0.27
ERZ	0	0	503	0	503	3745	4248	0.13
FECOSA	710	328	1033	6633	8704	11630	20334	0.75
FENOSA	9178	273	3540	4621	17613	18867	36480	0.93
H.C.	5188	0	641	529	6358	5481	11839	1.16
H.E.	0	585	4111	13439	18135	21887	40022	0.83
H.E.C.	0	0	535	702	1237	3548	4785	0.35
IBERDUERO	2625	133	9636	4264	16658	22607	39264	0.74
IBERDROLA	5483	1163	11035	22813	40494	53753	94248	0.75
SEVILLANA	4777	865	429	4876	10948	18472	29419	0.59
VIESGO	957	0	624	0	1581	3109	4690	0.51

Source: firm released data.

Table A.2.2. MEAN EXPENDITURE AND INPUT PRICES BY FIRM IN THE PERIOD 1985-1996.

	Total C	Labor	Fuel	Int.Inp	Cap.	MK	P. Labor	P.Cap.	P. fuel	P. I.I.
	Millions pesetas 1996								pts/ kg	
Average	107717	24963	17099	18588	47067	459886	7.28	0.12	8.28	0.148
Variat. Coef.	1.07	1.09	1.15	1.22	1.17	1.11	0.12	0.47	1.04	0.29
ENECO	19625	1469	11988	1200	4967	21446	6.46	0.254	8.06	0.054
ENHER	52097	16005	0	12930	23162	227436	7.68	0.107	---	0.193
ERZ	19863	7050	0	4905	7908	71233	6.70	0.11	---	0.201
FECOSA	117829	27779	7219	18947	63883	708167	7.61	0.093	8.10	0.141
FENOSA	213259	43689	57790	29970	81810	975772	7.68	0.087	8.57	0.134
H.C.	66199	8393	30569	7943	19294	211605	7.57	0.092	8.57	0.159
H.E.	229728	51072	5871	43975	128810	1279873	7.70	0.101	7.79	0.146
H.E.C	27299	8479	0	4619	14202	140967	7.79	0.101	---	0.139
IBERDUERO	209836	56958	18983	38318	95577	1004093	7.95	0.094	9.15	0.146
IBERDROLA	501024	119234	37369	103601	240820	2023161	7.94	0.119	7.94	0.155
SEVILLANA	151195	37176	27204	23976	62839	434135	6.25	0.146	7.97	0.125
VIESGO	27102	6683	5722	4281	10417	92513	6.51	0.113	7.97	0.167

MK: capital measure.

Source: firm released data.

### Appendix 3. ESTIMATION RESULTS

VARIABLE	PARAMETER	VALUE	T-STUDENT.
Constant	$\alpha$ (C1)	12192.2	8.03
Gtc	$\alpha$ gtc (C2)	1.598	9.90
Gtf	$\alpha$ gtf (C3)	2.702	4.42
Gh	$\alpha$ gh (C4)	0.666	4.89
Gn	$\alpha$ gn (C5)	0.858	5.67
Di	$\alpha$ di (C6)	0.384	3.52
Wc	$\beta$ Wc (C7)	279.48	60.35
Wi	$\beta$ Wi (C8)	17091.5	73.17
Wk	$\beta$ Wk (C9)	61516.3	35.97
T	$\varphi$ T (C10)	-384.72	-7.48
Gtc-gtc	$\delta$ gtc-gtc (C11)	-0.109 E-3	-5.42
Gtc-gtf	$\delta$ gtc-gtf (C12)	0.146 E-3	1.57
Gtc-gh	$\delta$ gtc-gh (C13)	0.149 E-4	0.48
Gtc-gn	$\delta$ gtc-gn (C14)	-0.236 E-4	-0.84
Gtc-di	$\delta$ gtc-da (C15)	0.375 E-4	2.47
Gtc-Wc	$\rho$ gtc-Wc (C16)	0.0919	58.33
Gtc-Wi	$\rho$ gtc-Wi (C17)	0.315	3.12
Gtc-Wk	$\rho$ gtc-Wk (C18)	4.228	6.237
Gtc-T	$\lambda$ gtc-T (C19)	-0.013	-1.82
Gtf-gtf	$\delta$ gtf-gtf (C20)	0.180 E-3	0.31
Gtf-gh	$\delta$ gtf-gh (C21)	0.868	3.07
Gtf-gn	$\delta$ gtf-gn (C22)	0.316	1.77
Gtf-Di	$\delta$ gtf-Di (C23)	-0.302	-2.25
Gtf-Wc	$\rho$ gtf-Wc (C24)	0.086	8.06
Gtf-Wi	$\rho$ gtf-Wi (C25)	2.458	2.96
Gtf-Wk	$\rho$ gtf-Wk (C26)	15.31	2.58
Gtf-T	$\lambda$ gtf-T (C27)	0.392	2.28
Gh-gh	$\delta$ gh-gh (C28)	0.101E-3	3.75
Gh-gn	$\delta$ gh-gn (C29)	0.152E-3	3.99
Gh-Di	$\delta$ gh-Wi (C30)	-0.132E-3	-3.94
Gh-Wi	$\rho$ gh-Wi (C31)	0.300	1.72
Gh-Wk	$\rho$ gh-Wk (C32)	6.158	5.20
Gh-T	$\lambda$ gh-T (C33)	0.070	2.84
Gn-gn	$\delta$ gn-gn (C34)	0.865	2.85
Gn-Di	$\delta$ gn-di (C35)	-0.103	-3.24
Gn-Wi	$\rho$ gn-Wi (C36)	0.674	5.08
Gn-Wk	$\rho$ gn-Wk (C37)	7.104	8.10
Gn-T	$\lambda$ gn-T (C38)	-0.134E-2	-0.84
Di-Di	$\delta$ Di-Di (C39)	0.382	3.22
Di-Wi	$\rho$ Di-Wi (C40)	1.174	11.97
Di-Wk	$\rho$ Di-Wk (C41)	0	-
Di-T	$\lambda$ Di-T (C42)	-0.032	-3.74
Wc-T	$\mu$ Wc-T (C43)	-7.764	-5.41
Wi-Wi	$\gamma$ Wi-Wi (C44)	-53611.4	-3.06
Wi-Wk	$\gamma$ Wi-Wk (C45)	80307.3	3.24

Wi-T	$\mu$ Wi-T (C46)	-563.23	-7.01
Wk-T	$\mu$ Wk-T (C47)	-3193.4	-5.64
T-T	$\pi$ T-T (C48)	12.282	2.65
Dummy enh	$\omega$ enh (C49)	3656	2.84
Dummy erz	$\omega$ erz (C50)	2425	3.17
Dummy fec	$\omega$ fec (C51)	3402	1.67
Dummy fen	$\omega$ fen (C52)	2342	1.11
Dummy hc	$\omega$ hc (C53)	604	0.65
Dummy he	$\omega$ he (C54)	3554	0.77
Dummy hec	$\omega$ hec (C55)	2584	3.54
Dummy ibo	$\omega$ ibo (C56)	4960	1.74
Dummy iba	$\omega$ iba (C57)	4748	0.60
Dummy sev	$\omega$ sev (C58)	2840	1.35
Dummy vi	$\omega$ vi (C59)	1931	3.38

<b>EQUATION 1. COST FUNCTION</b>		
Average C: 14,400	R squared: 0.995	Corrected R.squared: 0.983
<b>EQUATION 2. FUEL</b>		
Average $X_F$ : 279.11	R squared: 0.975	Corrected R.squared: 0.969
<b>EQUATION 3. INTERMEDIATE INPUT</b>		
Average $X_{II}$ : 17,063	R squared: 0.984	Corrected R.squared: 0.978
<b>EQUATION 4. CAPITAL</b>		
Average $X_K$ : 61267.2	R squared: 0.927	Corrected R.squared: 0.916