

Sudden Chain Energy Transfer Events in Vibrated Granular Media

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In a mixture of two species of grains of equal size but different mass, placed in a vertically vibrated shallow box, there is spontaneous segregation. Once the system is at least partly segregated and clusters of the heavy particles have formed, there are sudden peaks of the horizontal kinetic energy of the heavy particles, that is otherwise small. Together with the energy peaks the clusters rapidly expand and the segregation is partially lost. The process repeats once segregation has taken place again, either randomly or with some regularity in time depending on the experimental or numerical parameters. An explanation for these events is provided based on the existence of a fixed point for an isolated particle bouncing with only vertical motion. The horizontal energy peaks occur when the energy stored in the vertical motion is partly transferred into horizontal energy through a chain reaction of collisions between heavy particles.

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Introduction.—Granular media, when externally excited, can have a fluidlike behavior showing many flow regimes resembling those of molecular fluids even though there are strong differences due to the energy dissipation at grain collisions [1]. Fluidized granular media remain in a far from equilibrium regime, presenting phenomena that makes difficult the development of hydrodynamic models with quantitative predictive power. Among other specific phenomena, we remark absence of scale separation [2], lack of energy equipartition [3], violation of the fluctuation-dissipation relations [4], nonthermodynamic fluxes [5], spontaneous development of inhomogeneities [6], and segregation [7]. A careful description of these and other phenomena is crucial to develop models that describe the collective behavior of granular fluids.

Fluidized granular media in a shallow geometry (quasi-two dimensional) has attracted attention because it allows for a detailed analysis of both the collective behavior and the motion of individual grains [8–11]. The possibility of quantifying the system's dynamics at both scales may help in building a mathematical model for the collective dynamics of granular media.

In this Letter we report an experimental and numerical study of a phenomenon that takes place when two particle species of equal size but different mass are put in a vertically vibrated shallow box. The focus of the present Letter is not on segregation, but on a quite peculiar phenomenon that takes place once the species have segregated: the horizontal energy of the system has sudden and brief peaks, together with fast expansions of the regions rich in heavy particles. This is what we call chain energy transfer events. The characterization of these events shows a direct link between the individual particle dynamics and the observed collective behavior.

In experiments, the two species do not only differ in their mass but, being made of different materials, they also differ in their mechanical properties. Numerical simulations allow us then to identify the key elements that generate this phenomenon. Simulating the simple hard sphere model it becomes evident that the mass difference is the key parameter that controls the occurrence of energy peaks and sudden expansions; the differences in mechanical properties only change them quantitatively. Simulations give information on the particle's vertical motion, helping to complete the description of the phenomenon.

System configuration.—Two species of spherical grains of the same diameter σ but different mass are placed in a square shallow box as described below. The lighter (heavier) particles will be called L (H).

In the experiments, we use a mixture of $N_H = 170$ stainless steel (density 7.8 g/cm^3) and $N_L = 291$ polyamide (density 1.14 g/cm^3) particles, both of $\sigma = 3 \text{ mm}$. The shallow box has dimensions $L_x/\sigma = L_y/\sigma = 33.33 \pm 0.03$ and $L_z/\sigma = 1.813 \pm 0.004$. The box consists of two 10 mm thick indium tin oxide coated glass plates separated by a square steel frame. It is fixed to a base, which supports an array of high intensity light-emitting diodes. This whole setup is forced sinusoidally with an electromechanical vibrator. An accelerometer is fixed to the base of the cell, which allows the measurement of the imposed forcing amplitude. Top view images are obtained with a high speed camera. Particle positions and velocities can be determined at subpixel accuracy. The vibration frequency is fixed to $f = \omega/2\pi = 1/T = 100 \text{ Hz}$. Two values of the vibration amplitude are used, $A/\sigma = 0.036$ and $A/\sigma = 0.045$.

In the simulations, we use instantaneous collisions that are characterized by restitution and friction coefficients, which are velocity independent and different for both kinds

of particles, but the same for particle-particle and particle-wall collisions. An event driven algorithm is used [12]. The shallow box oscillates vertically with a biparabolic waveform. All simulations start with a random initial mixing.

Here, we report two types of simulations: The first type (S1) corresponds to simulations performed at higher densities than the experiment. Its purpose is to show that the chain energy transfer events are robust. In these simulations, the box has flat bottom and top walls and horizontal periodic boundary conditions. We use the following parameters: mass ratio between a heavy and a light particle $m_H/m_L = 10$, box size $L_x/\sigma = L_y/\sigma = 40$, $L_z/\sigma = 1.82$, normal and tangential restitution coefficients $\alpha_L = \alpha_H = 0.8$, static and dynamic friction coefficients $\mu_s = 0.3$ and $\mu_d = 0.15$, amplitude and frequency of vibration $A/\sigma = 0.15$, $\omega\sqrt{\sigma/g} = 7$. The number of particles are $N_L = 1000$ and $N_H = 500$, giving the filling fraction $\rho = N\pi\sigma^2/4L_xL_y = 0.736$. Other box sizes, filling fractions and heavy particle concentration were simulated with results similar to the ones we are presenting.

The second type of simulations (S2) are performed to find which experimental imperfections are relevant for the particular observations (for example, the specific time and energy scales). Possible imperfections are the plate surface roughness and small curvature. The latter turns out to be more important than the former. In order to consider this, we use a slightly parabolic bottom plate, with a variation of height between the bottom and flat top plates of only 1%. In these simulations we use lateral hard walls. Comparing S2 with S1 we learn which are the essential ingredients of the phenomenon and which aspects only change quantitatively the observations. For S2 simulations, we use the following parameters: $m_H/m_L = 7.8$, $L_x/\sigma = L_y/\sigma = 33$, $L_z/\sigma = 1.78$, $\alpha_L = 0.97$, $\alpha_H = 0.94$, $\mu_s = 0.1$ and $\mu_d = 0.05$, $\omega\sqrt{\sigma/g} = 11.0$ and $A/\sigma = 0.055$ or $A/\sigma = 0.062$. The number of L and H particles are the same as in experiments, with $\rho = N\pi\sigma^2/4L_xL_y = 0.44$.

Unless explicitly stated, the following descriptions refer to both simulations and experiments. Shortly after starting from a homogeneous configuration segregation between the two species takes place and many small dense clusters of H 's appear. Later on, the clusters coalesce and tend to have some L 's in their bulk. From a top view the H 's appear as if they were standing still, while the L 's outside the clusters show a significant horizontal agitation. The external pressure exerted by the L 's leads the H 's to form dense clusters [13]. The coalescence process, that is normally slow if the box is perfectly flat (S1), is sped up if there is a small curvature of the plates, coming from slight imperfections of the experimental setup, or imposed explicitly in a numerical simulation (S2), with the resulting cluster located at the center. In either case, the subsequent description of the phenomenon is independent of any small curvature. It only depends on the density of the cluster of the H 's and on the number and distribution of light particles which remained trapped inside it.

The chain events.—Once there is at least one cluster there is a sudden phenomenon of short duration. Figure 1 shows typical temporal sequences of the grain positions when an event takes place, for simulations S1, S2, and experiments. A chain reaction begins as an abrupt increase in the horizontal agitation of the H 's in a small region of one of the clusters, implying a local expansion followed by a fast propagation of the horizontal agitation to a larger zone. After a short while the horizontal agitation of the H 's quickly decays, recovering its original value. Next the L - H collisions slowly compress the cluster again, eventually recovering the original density. Both cluster size and horizontal agitation, computed as mean quadratic radius δ and the average (per particle) horizontal energy of the H 's, E_{Hh} , respectively, are shown in Fig. 2 for a sequence of several chain events in experiments and S2 simulations. Together with the fast expansions of the cluster there are energy peaks. Later, δ slowly decreases until the next energy burst takes place. Indeed, after the expansion, the dynamics of δ is much slower than that of the energies, because the former is related to a conserved quantity (the mass), while energy is not conserved and evolves in fast temporal scales.

Simulations show that the system is highly anisotropic, with vertical energies much larger than the horizontal ones for both species. Before an energy peak the horizontal agitation of the L 's is higher than for the H 's. The reason

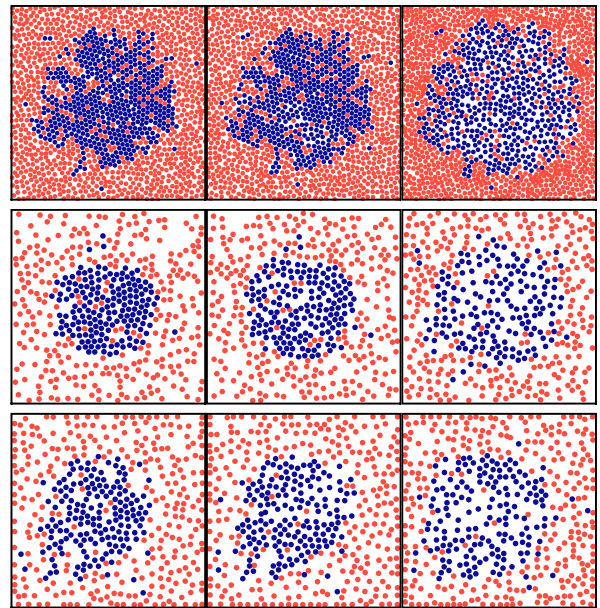


FIG. 1 (color online). Sequence of top view configurations for simulations S1 (top), S2 (middle) and experiments (bottom), showing the development of an energy-peak. Heavy (H) particles are presented in blue and light (L) ones in red. First, the system reaches partial segregation. Then the beginning of the event is seen as a small region of lower density within the cluster. Next, the low density region has propagated to practically all the cluster. Later (not shown) the system comes back to a state similar to the one before the expansion.

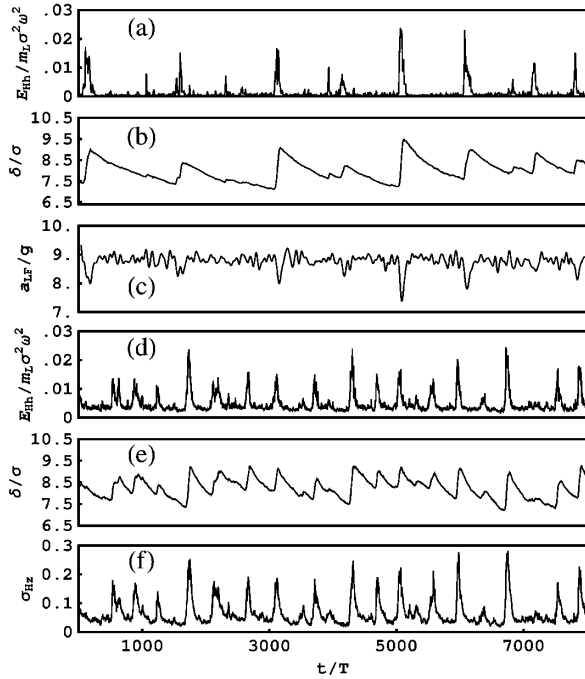


FIG. 2. Time evolution of E_{Hh} (a), δ (b) and a_{LF} (c) for an experiment with $A/\sigma = 0.045$. E_{Hh} (d), δ (e) and σ_{Hz} (f) for a simulation S1 and $A/\sigma = 0.062$.

is that the L 's are having collisions with particles of both types and, in particular, the collisions with the H 's keep them excited both vertically and horizontally. On the other hand most H 's are at their fixed point, with almost all their energy concentrated in the vertical motion. The abrupt change of E_{Hh} is accompanied by simultaneous but weaker changes in the other computed energies. This behavior is generic.

Under certain conditions, a single grain bouncing in a shallow box vibrating at a frequency ω evolves towards a fixed point: the bouncing movement becomes periodic with the same frequency ω and, because of the friction with the horizontal walls, the particle does not move horizontally neither does it rotate. Namely all its kinetic energy is vertical energy. The role of the fixed point in the behavior of the whole system is evident by looking at side view videos of experiments and simulations [13]. It is seen that the H 's are bouncing in phase as practically one solid layer: they are collectively trapped at the fixed point very close to each other. In fact, the fixed point can be easily computed in the absence of gravity (a good approximation in this case) with a resulting vertical energy for the heavy particles in the simulations of $0.32 m_L (\sigma\omega)^2$, that fully agrees with the value of E_{Hv} before and after the energy-peak event, confirming that most H 's are close to the fixed point. Hence, the collisions among heavy particles create almost no horizontal agitation. It is this coherent movement that is destroyed when an energy-peak event takes place.

To check that the picture described above is correct we have measured in simulations the standard deviation σ_{Hz} of

the stroboscopic height of the H 's, in units of σ , when the box is at its lowest position, shown in Fig. 2(f). σ_{Hz} is almost zero before the energy burst and it jumps an order of magnitude during the event. It recovers its typical small value together with E_{Hh} , indicating that the H 's are again moving in phase. The cross correlation function between E_{Hh} and σ_{Hz} shows a clear maximum centered at null time delay, confirming that the energy bursts are accompanied by a massive desynchronization of the H 's. Experimentally, this sudden desynchronization is observed through a_{LF} , which is the envelope of the acceleration signal including particle-plate collisions, obtained by low-pass frequency filtering. The main contribution to a_{LF} comes from the simultaneous H particle-plate collisions, which therefore exert a large force on the plate. The negative peaks, that occur at the same instants than the energy peaks [see Fig. 2(c)], correspond to the absence of coherent collisions due to the dephasing of the H 's and the decrease in their vertical kinetic energy.

In summary, at the start of the dynamics the horizontal energy E_{Hh} of the H 's begins to decrease, reaching a small asymptotic value because most of the H 's are approaching the fixed point. Particles L produce only small perturbations to the heavier ones but, due to the mass difference, the H 's can easily take the L 's off their fixed point getting significant horizontal energy. As a result, the L 's very seldom reach their fixed point and if they do so, it is for a short time. The permanent collisions with the L 's that some of the H 's suffer, allow for a residual nonvanishing horizontal energy E_{Hh} . The clusters of H 's slowly evolve to higher densities. At one moment, an H may collide with an L in such a way that the H gets a large horizontal velocity. The heavy particle then hits a neighboring energetic H triggering a chain reaction of collisions among neighboring H 's rapidly propagating within the cluster implying a sharp peak in E_{Hh} . The main energy source for the increase of E_{Hh} is the vertical energy E_{Hv} which correspondingly decreases during the energy burst. The chain energy transfer induces that the E_{Lh} and E_{Lv} follow a similar pattern to a smaller extent. This sudden chain energy transfer from the vertical to the horizontal motion is responsible for the observed fast increase of the cluster radius δ .

Statistical analysis.—The time lapses between successive E_{Hh} peaks and their intensities show different degrees of regularity; in some cases the peaks seem almost periodic. Figure 3 presents the power spectral densities of δ/σ for both experiments and simulations S2. Two distinct regimes are observed: for low amplitude there is no periodicity in the signal, while for $A/\sigma = 0.045$ for experiments ($A/\sigma = 0.062$ for S2 simulations) a clear maximum, although broad, is observed. It is centered at a period $\approx 1000 T$ ($\approx 400 T$ for simulations), which is consistent with the time series in Figs. 2(b) and 2(e). Depending on the system and forcing parameters, simulations S1 also show almost regular regimes. In some cases these are much more regular than those obtained in S2 simulations.

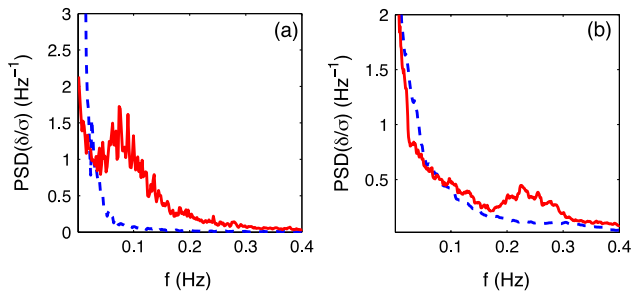


FIG. 3 (color online). Power spectrum density of δ/σ . (a) Experimental results for $A/\sigma = 0.036$ and $A/\sigma = 0.045$ (blue and red curves, respectively); (b) Simulational results (S2) for $A/\sigma = 0.055$ and $A/\sigma = 0.062$ (blue and red curves, respectively) represented using $\sigma = 3$ mm.

After exploring many different system parameters there appears to be an explanation for the observed periodicity under certain conditions. We observe that in systems where the region of horizontally excited H 's always propagate through most of a unique cluster, the peaks are more regular. This is so because for the chain reaction to take place, a high enough density of the cluster is needed, and it takes a characteristic time for the cluster to reach this density after such an event. If the chain reaction does not propagate through all the cluster then the time of compression is highly variable as it depends on the amount of particles that were involved in it, and also other similar events can take place while the cluster is compressing in those parts of the cluster that remained dense. In order for the explosion to cover most of the H cluster, several conditions must be fulfilled, particularly that its density is large enough to allow for an uninterrupted propagation of the horizontal energy. Moreover, the concentration of L 's inside the cluster needs to be small so that they do not block the propagating front. These conditions can be achieved either by changing the control parameters or by having small geometrical defects such as a small curvature of the box to help the H 's migrate toward the center. Indeed, in S2 simulations, an artificial small curvature was added and the resulting events are more regular compared to the same simulations done without the small curvature.

Conclusions.—Experiments and numerical simulations of a granular mixture of grains that differ in their mass density—in a vibrated shallow box—show horizontal energy peaks characterized by the rapid conversion of vertical kinetic energy into a horizontal one; these events are preceded by the segregation of the species. In the segregated state the massive grains approach collectively a fixed point characterized by a vanishing horizontal energy and vertical motion in phase with the walls' oscillation. When there is a unique cluster different regimes are reached depending on the density of the cluster of heavy particles and the relative concentration of light particles inside it. When there are too many light particles in the cluster the

horizontal agitation does not involve the entire cluster and the corresponding chain energy transfer events show no characteristic time or energy scales. Otherwise, explosions propagate through all of the cluster and show a characteristic intensity and time lapse between successive events.

Simulations and experiments differ in the collision details and, consequently, they do not give the same quantitative results. Although the fixed point exists only for spherical particles, small nonsphericity, as present in the experiments and artificially put in simulations, results in a small horizontal agitation but the energy bursts still take place. The independence of the phenomenon on the precise details shows that it is robust, and requires only a large enough mass contrast. This phenomenon shows that to correctly describe the collective dynamics in a confined geometry, the dynamics of individual grains in the vertical direction is crucial. The revealed link between the small and large scale dynamics should help in building hydrodynamiclike models of confined granular media.

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