Energy bursts in vibrated shallow granular systems

N. Rivas*, D. Risso†, R. Soto* and P. Cordero*

*Departamento de Física, FCFM, Universidad de Chile, Santiago, Chile
†Departamento de Física, Universidad del BioBio, Concepción, Chile

Abstract. In a mixture of two species of inelastic spheres of equal size but different mass, placed in a vertically vibrated shallow box (large horizontal dimensions and height comparable to the grains' size), there is spontaneous segregation. Once the system is at least partly segregated energy bursts recurrently take place: the horizontal kinetic energy of the heavy particles, that normally is small, suddenly increases an order of magnitude. An explanation of these events is provided based on the existence of a fixed point for an isolated particle bouncing with only vertical motion between the top and bottom plates. Energy bursts occur when clusters of heavy particles start a chain reaction of collisions that transfer vertical energy to horizontal energy producing an expansion of the cluster.

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INTRODUCTION

Granular systems, like sand, are systems formed by hard macroscopic particles. They are intrinsically dissipative systems, since colliding grains transfer rotational and translational energy into their internal degrees of freedom. The temperature does not play a role in the dynamics of the system because the energy needed to considerably move a grain is much larger than the temperature energy scale $k_B T$. If no energy is injected into the system, the dissipative collisions will make all particles come to rest—as when we fill a sugar bowl. In this static state the top surface will most probably not be horizontal but rather the top layers will be forming a heap. But when energy is injected, the granular system may behave like a fluid, even though there are important differences with molecular fluids because of the energy dissipation at grain collisions [1]. In a wide variety of situations the presence of any interstitial fluid, such as air, can be neglected. When the system has two species of grains—that differ in their size, shape, density or inelasticity—segregation generically takes place. Depending on the forcing parameters, segregation can be partial or total with different driving mechanisms [2].

Granular systems found in nature normally are irregular in shape and with a diversity of sizes. However, it became evident that the main features of granular media are determined by their stiffness and the energy dissipation at collisions. For this reason and in order to make quantitative and reproducible predictions, the study of these systems are usually focused on systems consisting of *spherical particles* and quite often the systems are monodisperse or, perhaps, bidisperse. It is clear now that these systems present an immensely rich variety of behaviors.

The study of granular systems in a vibrated shallow box—a box with large horizontal

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dimensions and height comparable to the particle's diameter— allows both to study the collective behaviour of the system as well as that of individual particles [3, 4, 5, 6]. As it should be expected, the system variables behave quite anisotropically to the extent that, for example, the horizontal mean kinetic energy is quite different from the vertical one. Placing monodisperse inelastic spheres in a vertically vibrated shallow box of height comparable to the size of the particles, a particular phase separation takes place: regions appear with quite different densities and granular temperatures [3]. Namely, grains form solid-like regions surrounded by fluid-like regions. In this phase separation waves develop, driven by negative compressibility of the effective pressure [4].

In this article we study a mixture of two species of inelastic spheres of equal size but different mass, placed in a vertically vibrated shallow box when there is spontaneous segregation. Once the system is at least partly segregated energy bursts take place. An explanation of these events is provided based on the existence of a fixed point for an isolated particle bouncing with only vertical motion between the top and bottom plates.

SYSTEM CONFIGURATION

Two species of spheres of the same diameter σ but different mass are placed in a square shallow box as described below. The lighter/heavier particles will be called L and H respectively. In the simulations we use the so-called Inelastic Hard Sphere model, in which collisions are instantaneous and are characterized by restitution and friction coefficients (static and dynamic), which we take to be the same for the L's and H's in all their collisions. The spheres have translation and rotation degrees of freedom; their collision rules, including those with the walls, can be found in [7].

The shallow box, with horizontal periodic boundary conditions, oscillates with amplitude A and frequency ω. The simulations make use of an event driven algorithm [8] with the following parameters: mass ratio $m_H/m_L = 10$, box height $L_z/\sigma = 1.82$, normal and tangential restitution coefficients $r = 0.8$, static and dynamic friction coefficients $\mu_s = 0.3$ and $\mu_d = 0.15$, the angular frequency and amplitude of vibration are kept fixed at $\omega\sqrt{\sigma/g} = 7.0$ and $A/\sigma = 0.15$ so that the dimensionless acceleration is $\Gamma \equiv A\omega^2/g = 7.35$. Simulations are reported using two systems: (a) a small system $N_H = 500$, $N_L = 1000$ and $L_x/\sigma = L_y/\sigma = 40$ and (b) a large system $N_H = 2000$, $N_L = 4000$ and $L_x/\sigma = L_y/\sigma = 80$. Both simulations have the same area fraction $\rho = \pi\sigma^2(N_H+N_L)/(4L_xL_y) = 0.74$ and number ratio $N_H : N_L = 1 : 2$, and only differ in their size.

The appearance of the energy bursts is related to the existence of a fixed point in the dynamics of a single particle in the shallow box. The friction with the plates damps the horizontal and rotational motion, hence the fixed point is characterized by a vertical dynamics in which the particle collides alternatively with the bottom and top plates with the frequency ω of the box. For a wide range of the parameters this periodic trajectory is unique, and thus aligns all particles the vibrating box—in the form of a moving horizontal layer—even if there is no interaction between them.

For the parameters used in the simulation the fixed point is unique and stable. When the system is initialized, the particles approach the fixed point but the grain-grain collisions may prevent to reach it, with different results for the H and L particles. The H's reach the fixed point keeping a small horizontal energy due to the collisions with the L's while, due to the mass contrast, the L particles are continuously taken off the fixed point and, as a result, their average horizontal energy is significant

The mass difference produces segregation of the species but the underlying driving mechanism is not yet known. Such mechanism is not the focus of this article, but rather a phenomenon that takes place after segregation has occurred. Shortly after starting from an initial random configuration many small dense clusters of H's appear. Later on there is a slow dynamics in which the clusters coalesce. The clusters tend to have some L's in their bulk. Because of the fixed point, from a top view the H's appear as if they were standing still, while the L's outside the clusters show a significant horizontal agitation. The external pressure exerted by the light particles leads the heavy ones to form denser clusters. As the fixed point is unique, the clusters move in phase as practically one solid layer.

HE ENERGY BURSTS

As described in [9], once there is at least one cluster, the segregation process is repeatedly interrupted by sudden bursts of the horizontal kinetic energy of the H particles implying a fast expansion of the cluster. Figure 1 shows the evolution of the horizontal energy for the two simulated systems. Two regimes are clearly observed: the small system shows bursts that are irregular in intensity and time lags, while the bursts in the larger system are roughly periodically spaced with a rather well defined amplitude. Later we put forward an explanation of the observed periodicity.

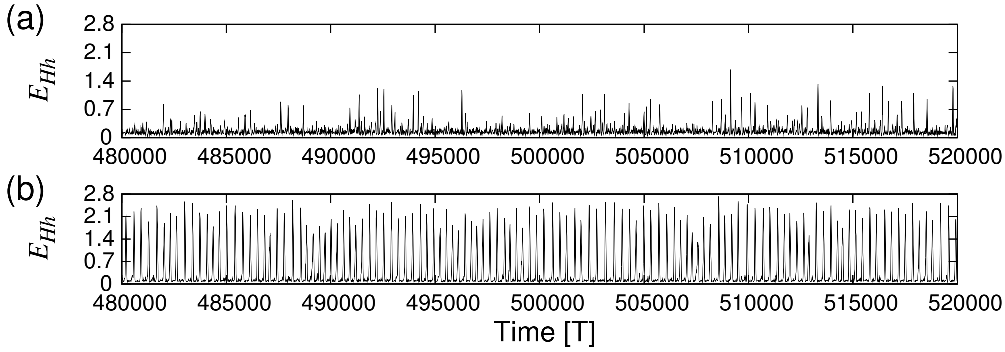

FIGURE 1. Horizontal kinetic energy of the H particles $E_{Hh}(t)$ obtained in the simulations for the (a) small and (b) large systems.

Each burst begins as an abrupt increase of the agitation of the H's in a small region in one of the clusters, implying a local expansion followed by a fast propagation of the horizontal agitation to a much larger zone. Next the L-H collisions slowly compress the cluster again, eventually recovering the original density. Two configuration sequences, one for each system size, are presented in Fig. 2. The main difference between the two systems is that in the large one the agitation always covers the complete cluster, while in the small one only part of the cluster is normally involved.

To analyze the bursts we show, in Fig. 3, the evolution of the average (per particle) horizontal and vertical kinetic energy of particles H, E_{Hh}, E_{Hv}. The standard deviation

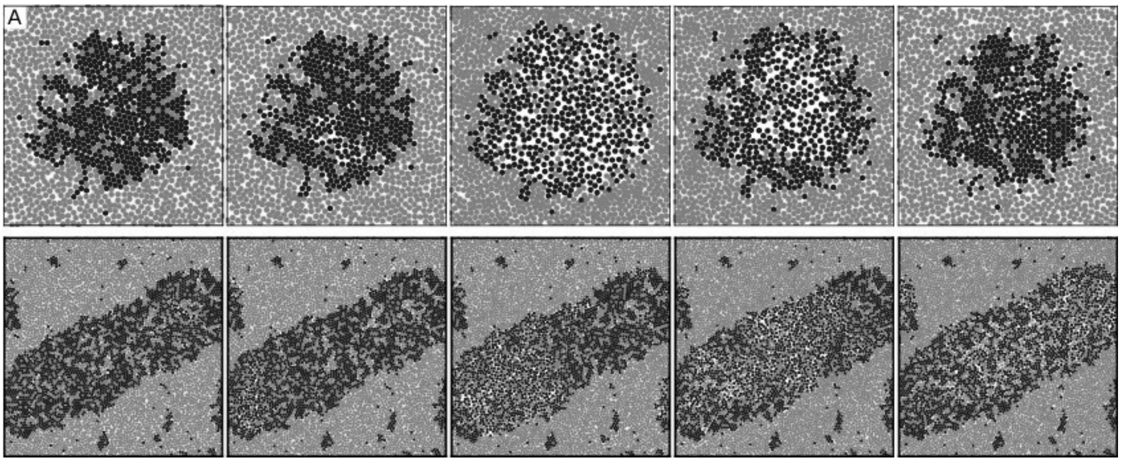

FIGURE 2. Sequence of top view configurations of the system showing an explosion for the small system (top) and the large system (bottom). The heavy particles are presented in black, while the light ones in gray.

σ_{Hz} of the stroboscopic height of the H's—when the box is at its lowest position—is also shown. Previous to the energy burst σ_{Hz} is small, showing that the H's move coherently in phase. It is seen that during the burst σ_{Hz} jumps an order of magnitude and it recovers its typical small value as soon as the burst has finished, namely the H's are again moving in phase. It is this coherent motion that is destroyed at every energy burst.

The picture at the particle level is that eventually two heavy particles, not exactly in phase, collide. This is a collision between two energetic particles which, as a result, all of a sudden, get a comparably large horizontal energy, triggering a chain reaction of collisions among neighboring H's rapidly propagating within the cluster. The chain reaction rapidly transforms the vertical kinetic energy E_{Hv} into horizontal energy E_{Hh}, phenomenon that is observed in Fig. 3.

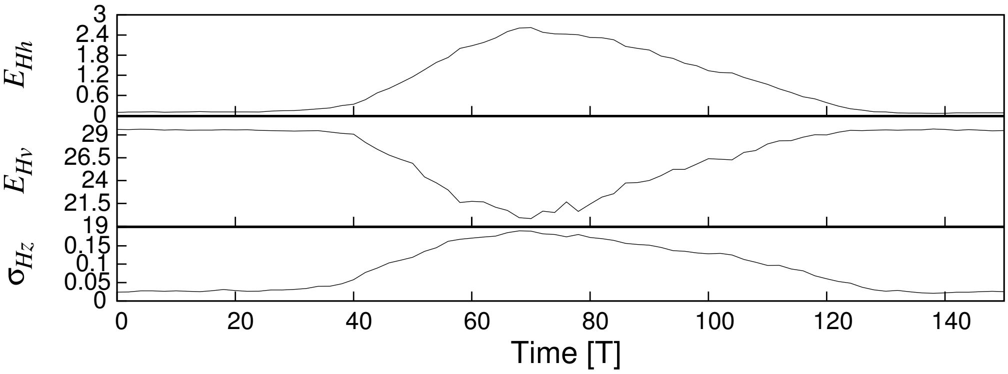

FIGURE 3. Evolution of a single energy burst. The average horizontal and vertical energies of the heavy particles E_{Hh}, E_{Hv}, and the standard deviation of the set of heights of particles H, σ_{Hz} taken stroboscopically when the box is at its lowest position.

THE REGULARITY OF THE EVENTS

The burst waiting times and intensities show different degrees of regularity and for the large system they even look almost as if they were periodic.

After observing many different realizations there appears to be an explanation for the different degrees of regularity. For bursts to take place, a high enough density of the cluster is needed. A burst produces an expansion and no other burst can take place in that region until the density increases again. This implies a necessary waiting time between bursts.

For the waiting times to be regular several conditions have to be met: (i) there must be a unique cluster; (ii) the cluster has to be sufficiently convex so that a burst does not destroy the connectivity of it; (iii) the concentration of L's inside the cluster must not be too large so that they do not block the burst propagating front allowing the existence of several alternative paths for the burst propagation.

If a burst does not propagate through all the cluster then the time of compression is highly variable as it depends on the amount of particles that were involved in it. Even more, other bursts can take place while the cluster is compressing in those parts of it that remain dense.

CONCLUSIONS

Numerical simulations of a granular mixture of two types of grains —in a vibrated shallow box— which only differ in their mass show energy bursts characterized by the rapid conversion of vertical energy into horizontal energy; these bursts are preceded by the segregation of the species. In the segregated state the massive grains approach a fixed point characterized by a vanishing horizontal energy and a vertical motion in phase with the box oscillations. The light grains cannot reach the fixed point because collisions with the heavy ones take them easily off, remaining with an important horizontal energy. The heavy grains trapped about the fixed point move in phase and collide most of the time with a tiny momentum transfer in the horizontal direction. As a result a densely packed cluster of the heavy grains develops. Eventually, however, a small dephasing between heavy grains or an energetic collision with a lighter one, allows a heavy grain to acquire significant horizontal momentum leaving the fixed point. The subsequent collisions with neighboring heavy grains transfer a high amount of energy to the horizontal agitation in the form of a chain reaction, generating an energy burst.

When there is a unique cluster different regimes may be reached depending on the relative number of light particles inside the cluster of the heavy ones. When there are too many light particles in the cluster the bursts are localized, and have a broad distribution of intensities and waiting times, otherwise the bursts propagate through all the cluster and show characteristic intensities and waiting times.

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