A Random Set Approach to SLAM

John Mullane
Ba-Ngu Vo[♮]
Martin Adams
Wijerupage Sardha Wijesoma

School of Electrical & Electronic Engineering
Nanyang Technological University
Singapore

[♮]School of Electrical & Electronic Engineering
University of Melbourne
Australia

Presentation Outline

Presentation Outline

SLAM: Overview

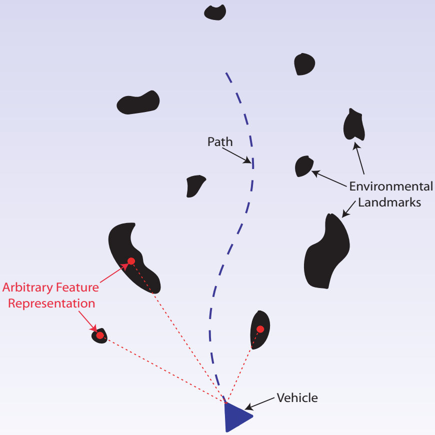


Figure: *2D Feature-SLAM Illustration.*

SLAM: New Concepts

- What is the estimation problem?
  - Unknown vehicle trajectory and the feature map
  - Uncertain vehicle pose, feature location *and* feature number

    [Thrun, 2002],[Dissanayake *et al.*, '01], [Leonard, Durrant-Whyte and Cox, '92], [Smith et. al. '90]

SLAM: New Concepts

- What is the estimation problem?
  - Unknown vehicle trajectory and the feature map
  - Uncertain vehicle pose, feature location *and* feature number

    [Thrun, 2002],[Dissanayake *et al.*, '01], [Leonard, Durrant-Whyte and Cox, '92], [Smith et. al. '90]

- What is the true joint SLAM state?
  - Vehicle trajectory: A vector of vehicle poses at each time.
  - The feature map: A *set* of features representing the map.

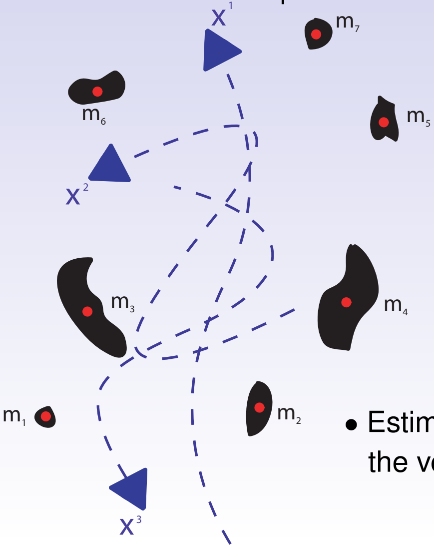
SLAM: New Concepts

- What is the estimation problem?
  - Unknown vehicle trajectory and the feature map
  - Uncertain vehicle pose, feature location *and* feature number

    [Thrun, 2002],[Dissanayake *et al.*, '01], [Leonard, Durrant-Whyte and Cox, '92], [Smith et. al. '90]
- What is the true joint SLAM state?
  - Vehicle trajectory: A vector of vehicle poses at each time.
  - The feature map: A *set* of features representing the map.
- SLAM Estimation Error vs. Ground Truth?
  - Vehicle estimate evaluation: RMSE
  - Map estimate evaluation: Joint error in feature number and location.

The Feature Map: What is the true state ?

✔ The feature map as a vector: Feature order is rigid.



Given X^1:

$M = [m_1, m_2, m_3, m_4, m_5, m_6, m_7]$

Given X^2:

$M = [m_4, m_2, m_3, m_1, m_5, m_7, m_6]$

Given X^3:

$M = [m_6, m_7, m_5, m_4, m_3, m_2, m_1]$

- Estimated map vector depends on the vehicle trajectory ?

The Feature Map: Finite Set Representation

$$\underbrace{\begin{array}{c} M = [m_1, m_2, m_3, m_4, m_5, m_6, m_7] \\ M = [m_4, m_2, m_3, m_1, m_5, m_7, m_6] \\ \vdots \\ M = [m_7, m_6, m_5, m_4, m_3, m_2, m_1] \end{array}}$$

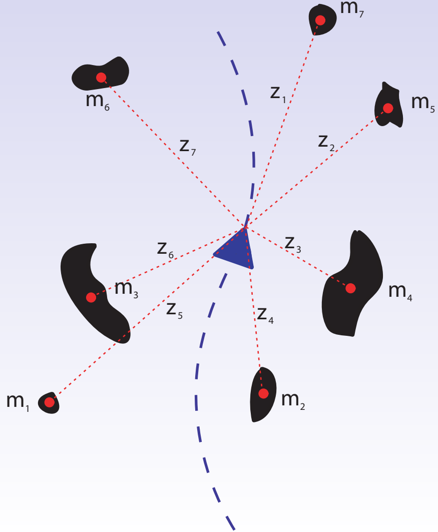
$$M = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7\}$$

- ✔ Order of features *cannot* be significant
- ✔ Finite set representation naturally encapsulates all possible permutations of the features in map

Presentation Outline

Data Association

- The Measurement-State assignment problem:



$$Z = [z_1, z_2, z_3, z_4, z_5, z_6, z_7]$$

?

$$M = [m_1, m_2, m_3, m_4, m_5, m_6, m_7]$$

Data Association

- ✔ Inherent problem in SLAM (even for an ideal sensor)
- ✘ Current vector-valued formulations *requires* it to be solved prior to Bayesian (Kalman) update: [Lochana et. al, 2006], [Niera and Tardos, 2001], [Makarsov and Durrant-Whyte, 1995]
  - Why ?: Features and measurements are rigidly ordered in a finite-vector-valued map state
- ✔ Proposed approach *does not require* it to be solved:
  - Why ?: Features and measurements are represented by finite-valued-sets. No distinct order assumed

The Feature Measurement

- Classical Measurement Model:

$$Z = h(M, X) + \text{Noise}$$

$$\begin{aligned}
M &= [m_1,m_2,m_3,m_4,m_5,m_6,m_7] \Rightarrow & Z &= [z_1,z_2,z_3,z_4,z_5,z_6,z_7] \\
M &= [m_4,m_2,m_3,m_1,m_5,m_7,m_6] \Rightarrow & Z &= [z_4,z_2,z_3,z_1,z_5,z_7,z_6] \\
M &= [m_6,m_7,m_5,m_4,m_3,m_2,m_1] \Rightarrow & Z &= [z_6,z_7,z_5,z_4,z_3,z_2,z_1]
\end{aligned}$$

✔ No physical significance to the order of measurements

✘ Vector approaches require measurement order to match the feature order in the map state

✘ Vector approaches require the data association problem to be solved

Finite Set Representation

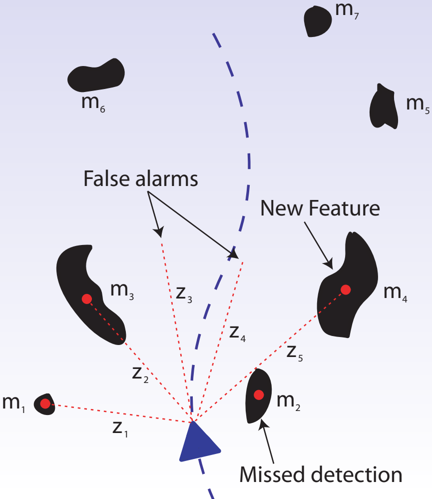
$$
\begin{array}{cc}
M = [m_1, m_2, m_3, m_4, m_5, m_6, m_7] & Z = [z_1, z_2, z_3, z_4, z_5, z_6, z_7] \\
M = [m_4, m_2, m_3, m_1, m_5, m_7, m_6] & Z = [z_4, z_2, z_3, z_1, z_5, z_7, z_6] \\
\vdots & \vdots \\
\underbrace{M = [m_7, m_6, m_5, m_4, m_3, m_2, m_1]} & \underbrace{Z = [z_7, z_6, z_5, z_4, z_3, z_2, z_1]} \\
M = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7\} & Z = \{z_1, z_2, z_3, z_4, z_5, z_6, z_7\}
\end{array}
$$

- ✔ Finite set representation naturally encapsulates all possible permutations of the feature map and measurement
- ✔ No rigid ordering of states. Data association assignment does not have to be addressed

Presentation Outline

The Map Management Problem

- Number of features is *a priori* unknown. The Map size grows monotonically:



Vector map transition:

$$M_{k-1} = [m_1, m_2, m_3]$$

$$M_{k|k-1} \stackrel{?}{=} [m_1, m_2, m_3] \text{``}+\text{''} [m_4]$$

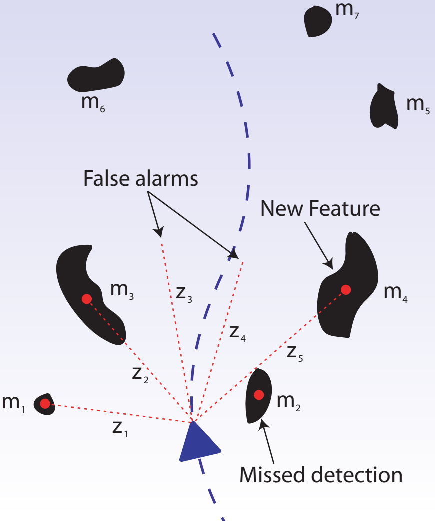
Set map transition:

$$M_{k-1} = \{m_1, m_2, m_3\}$$

$$M_{k|k-1} = \{m_1, m_2, m_3\} \cup \{m_4\}$$

The Map Management Problem

- Imperfect sensor:



Vector measurement:

$$Z_k = h([\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3, \mathrm{m}_4], X_k) + \text{Noise}$$

$$[\mathrm{z}_1, \mathrm{z}_2, \mathrm{z}_3, \mathrm{z}_4, \mathrm{z}_5] \stackrel{?}{=} h([\mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3, \mathrm{m}_4], X_k) + \text{Noise}$$

Set Measurement:

$$Z_k = \bigcup_{m \in M_{k|k-1}} \Theta(m, X_k) \cup C_k$$

The Map Management Problem

- Feature-based SLAM is commonly phrased as *"a state estimation problem involving a variable number of dimensions (features)"* [Thrun, 2002],[Dissanayake *et al.*, '01], [Leonard, Durrant-Whyte and Cox, '92], [Smith et. al. '90]

✘ Random vector representation does not model uncertainty in number

✘ Post-processing feature number filters required [Montemerlo et. al., '03], [Lochana et. al., '06], [Dissanayake et. al, '01]

✔ Random finite set (RFS) models uncertainty in state values *and* number

Presentation Outline

Classical vector-valued Bayesian Approach

- Time update: $p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) =$

$$\int f(X^k, M_k | X^{k-1}, M_{k-1}, u_{k-1}) p(X^{k-1}, M_{k-1} | Z^{k-1}, u^{k-2}, X_0) dX_{k-1}$$

Classical vector-valued Bayesian Approach

- Time update: $p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) =$

$$\int f(X^k, M_k | X^{k-1}, M_{k-1}, u_{k-1}) p(X^{k-1}, M_{k-1} | Z^{k-1}, u^{k-2}, X_0) dX_{k-1}$$

- Acquire measurement, Z_k

Independent: Data Association

- Measurement update:

$$p(X^k, M_k | Z^k, u^{k-1}, X_0) = \frac{g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0)}{\int \int g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) dX_k dM_k}$$

Classical vector-valued Bayesian Approach

- Time update: $p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) =$

$$\int f(X^k, M_k | X^{k-1}, M_{k-1}, u_{k-1}) p(X^{k-1}, M_{k-1} | Z^{k-1}, u^{k-2}, X_0) dX_{k-1}$$

- Acquire measurement, Z_k

Independent: Data Association

- Measurement update:

$$p(X^k, M_k | Z^k, u^{k-1}, X_0) = \frac{g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0)}{\int \int g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) dX_k dM_k}$$

Independent: Map Management

Classical vector-valued Bayesian Approach

- Time update: $p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) =$

$$\int f(X^k, M_k | X^{k-1}, M_{k-1}, u_{k-1}) p(X^{k-1}, M_{k-1} | Z^{k-1}, u^{k-2}, X_0) dX_{k-1}$$

- Acquire measurement, Z_k

Independent: Data Association

- Measurement update:

Must be of equal dimension

$$p(X^k, M_k | Z^k, u^{k-1}, X_0) = \frac{g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0)}{\int \int g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) dX_k dM_k}$$

Independent: Map Management

✘ Feature state and number are not jointly propagated or estimated

Proposed RFS Bayesian Approach

- Time update: $p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) =$

$$\int f(X^k, M_k | X^{k-1}, M_{k-1}, u_{k-1}) p(X^{k-1}, M_{k-1} | Z^{k-1}, u^{k-2}, X_0) dX_{k-1}$$

Proposed RFS Bayesian Approach

- Time update: $p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) =$

$$\int f(X^k, M_k | X^{k-1}, M_{k-1}, u_{k-1}) p(X^{k-1}, M_{k-1} | Z^{k-1}, u^{k-2}, X_0) dX_{k-1}$$

- Acquire measurement, Z_k
- Measurement update:

$$p(X^k, M_k | Z^k, u^{k-1}, X_0) = \frac{g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0)}{\int \int g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) dX_k dM_k}$$

Proposed RFS Bayesian Approach

- Time update: $p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) =$

$$\int f(X^k, M_k | X^{k-1}, M_{k-1}, u_{k-1}) p(X^{k-1}, M_{k-1} | Z^{k-1}, u^{k-2}, X_0) dX_{k-1}$$

- Acquire measurement, Z_k
- Measurement update:

Can be of differing dimension

$$p(X^k, M_k | Z^k, u^{k-1}, X_0) = \frac{g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0)}{\int \int g(Z_k | M_k, X_k) p(X^k, M_k | Z^{k-1}, u^{k-1}, X_0) dX_k dM_k}$$

✔ Jointly estimate feature state, number and vehicle pose

Presentation Outline

The Poisson RFS Feature Map

- The RFS Map:

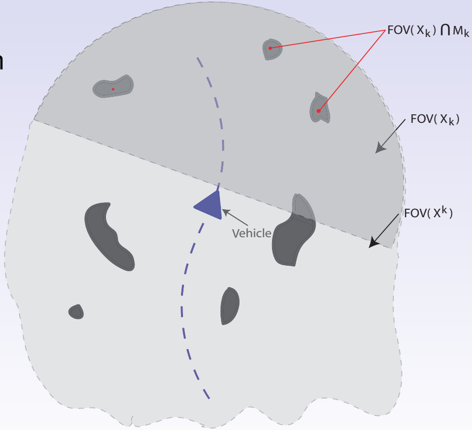
  $|M_k|$ increases monotonically

  However, $FOV(X_k) \cap M_k$ can be assumed Poisson
  [Makarsov, 1995]

- The RFS Measurement:

  $$Z_k = \bigcup_{m \in M_k} \Theta(m, X_k) \cup C_k(\cdot)$$

  Clutter measurements, C_k, also assumed Poisson



RFS Process Model

- The RFS Map:

  The map state at time $k-1$,

  $M_{k-1} = M \cap FOV(X^{k-1})$ where

  $FOV(X^{k-1}) = FOV(X_0) \cup FOV(X_1) \cup \cdots \cup FOV(X_{k-1})$

- The RFS Process Model

  $$M_{k|k-1} = M_{k-1} \cup \left(FOV(X^k) \cap \bar{M}_{k-1} \right)$$

RFS Process Model

- The RFS Map:

  The map state at time $k-1$,

  $M_{k-1} = M \cap FOV(X^{k-1})$ where

  $FOV(X^{k-1}) = FOV(X_0) \cup FOV(X_1) \cup \cdots \cup FOV(X_{k-1})$

- The RFS Process Model

  $$M_{k|k-1} = M_{k-1} \cup \left(FOV(X^k) \cap \bar{M}_{k-1} \right)$$

- New feature RFS
  - No *a priori* knowledge of the map
  - Use Z_{k-1} to form the RFS

RFS Measurement Model

- RFS Measurement Model

$$Z_k = \bigcup_{m \in M_k} \Theta(m, X_k) \cup C_k(X_k)$$

- where,

$$\Theta(m, X_k) = \begin{cases} \{z\} & \text{with density } p_D(m, X_k) g(z|m, X_k) \\ \{\emptyset\} & \text{with probability } 1 - p_D(m, X_k) \end{cases}$$

- and $C_k(X_k)$ is the (perhaps vehicle state dependent) RFS of the spurious measurements

Presentation Outline

First Order Recursion

$$p_{k|k}(M_k = \{m_1, \ldots, m_{q(k)}\} | Z^k, X^k, u^{k-1}, X_0) = \frac{g_k(Z_k|M_k, X_k) p_{k|k-1}(M_k|Z^{k-1}, X^k, u^{k-1}, X_0)}{\int g_k(Z_k|M_k, X_k) p_{k|k-1}(M_k|Z^{k-1}, u^{k-1}, X_0) dM_k}$$

- Multiple integrals render the recursion intractable
- Propagate the first-order moment, the intensity function: v_k [Mahler '03, Vo '06]

$$v_{k|k-1}(M_k|X_k) = v_{k-1}(M_{k-1}|X_{k-1}) + b(M_k|\cdot)$$

$$v_k(M|X_k) = v_{k|k-1}(M|X_k) \Bigg[1 - P_D(M|X_k) + \sum_{z \in Z_k} \frac{P_D(M|X_k) g(z|M, X_k)}{c_k(z) + \int P_D(\zeta|X_k) g_k(z|\zeta, X_k) v_{k|k-1}(\zeta|X_k) d\zeta} \Bigg]$$

GMM Implementation

- Prior:

$$v_{k-1}(M_{k-1}|X_{k-1}) = \sum_{j=1}^{J_{k-1}} w_{k-1}^{(j)} \mathcal{N}(m; \mu_{k-1}^{(j)}, P_{k-1}^{(j)})$$

$$b(M_k|Z_{k-1}, X_{k-1}) = \sum_{j=1}^{J_{b,k}} w_{b,k}^{(j)} \mathcal{N}(m; \mu_{b,k}^{(j)}, P_{b,k}^{(j)})$$

- Prediction:

$$v_{k|k-1}(M_k|X_k) = \sum_{j=1}^{J_{k|k-1}} w_{k|k-1}^{(j)} \mathcal{N}(m; \mu_{k|k-1}^{(j)}, P_{k|k-1}^{(j)})$$

- Update:

$$v_k(M_k|X_k) = v_{k|k-1}(M_k|X_k)\Bigg[1 - P_D(m|X_k) + \sum_{z \in Z_k} \sum_{j=1}^{J_{k|k-1}} v_{G,k}^{(j)}(z, m|X_k)\Bigg]$$

The GMM Intensity Function

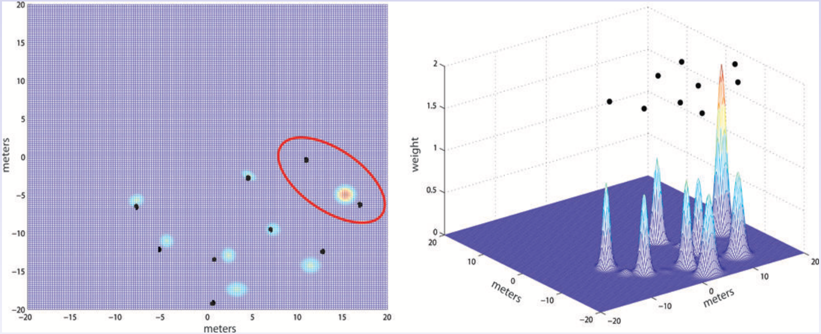


Figure: *Gaussian mixture representation of the intensity function, showing peaks at feature locations, with 2 features represented by a single peak with weight 2 as highlighted. Black dots show the true feature locations within sensor range.*

The GMM Intensity Function

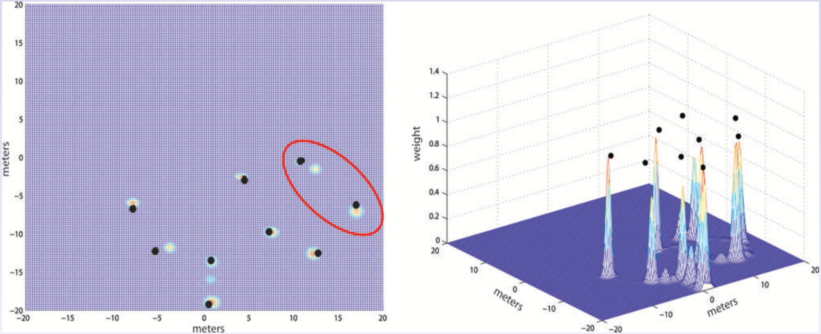
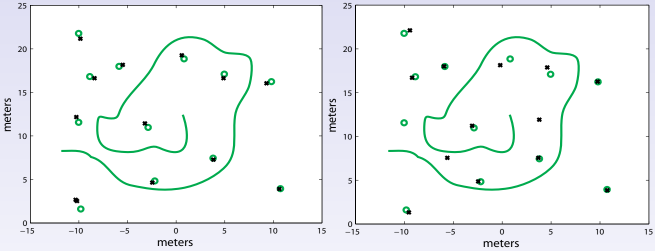


Figure: *Gaussian mixture representation of the intensity function, showing peaks at feature locations, with all features correctly resolved. The new Gaussians of the mixture have unity weights, with some smaller components also evident of small weight.*

Presentation Outline

FBRM: Error Quantification

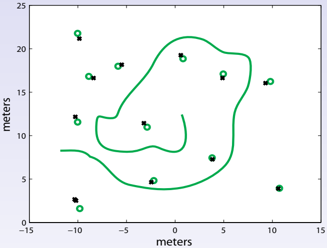
- Current error analysis:
  - ✘ Independently analyses each map estimate
  - ✘ Disregards the dimensionality estimation problem

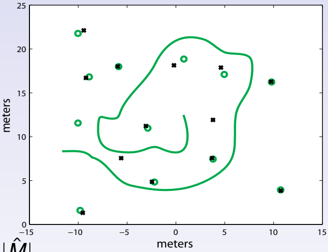


FBRM: Error Quantification

- Current error analysis:
  - ✘ Independently analyses each map estimate
  - ✘ Disregards the dimensionality estimation problem

$\bar{d}^{(5)}_{(2)} = 1.48$





$\bar{d}^{(5)}_{(2)} = 2.02$

$$\bar{d}_p^{(c)}(\hat{M}, M) := \left(\frac{1}{|M|}\left(\min_{j\in\{1,\dots,|M|\}} \sum_{i=1}^{|\hat{M}|} d^{(c)}(\hat{m}_i, m_j)^p + c^p(|M| - |\hat{M}|)\right)\right)^{1/p}$$

where,

$$d^{(c)}(\hat{m}_i, m_j) = \min(c, ||\hat{m}_i - m_j||)$$

[Schuhmacher, '08]

FBRM: Simulated Environment

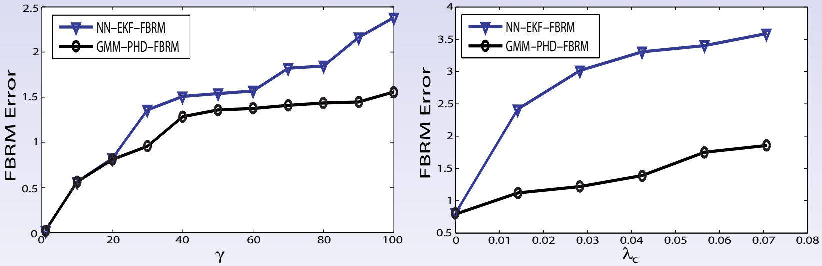


Figure: *Comparison of FB mapping error vs. measurement noise (left) and clutter rate (right) for the proposed filter and the classical EKF solution.*

FBRM: Simulated Environment

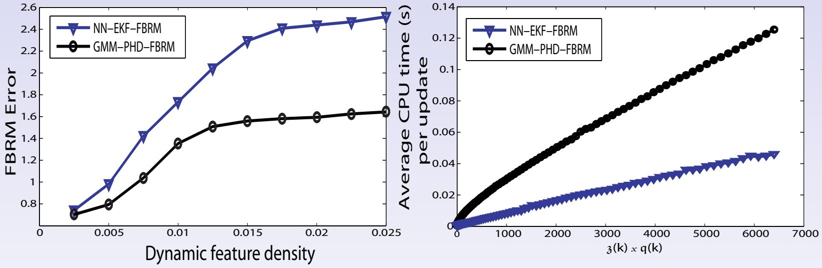


Figure: *Comparison of the map estimation error in the presence of increasing densities of moving features (left) and computational analysis (right).*

FBRM: Real Environment

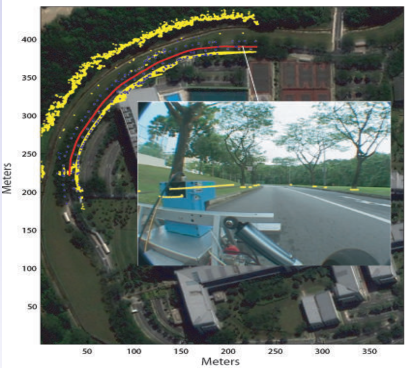


Figure: *Overview of campus dataset segment for PHD-FBRM filter testing.*

FBRM: Real Environment

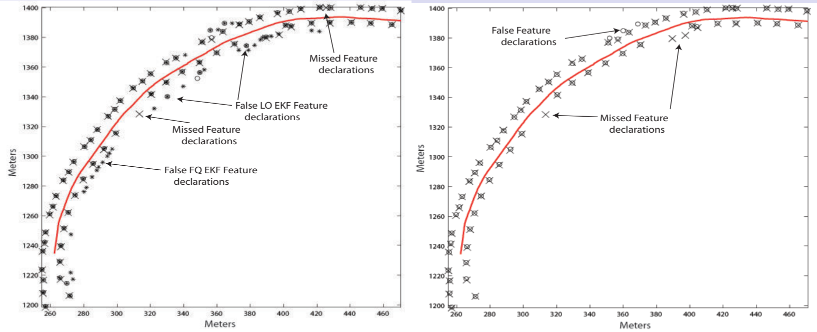


Figure: *FBRM map estimate comparison.*

FBRM: Real Environment

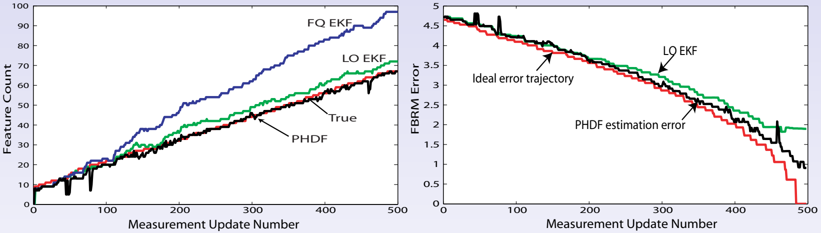


Figure: *Cardinality and FBRM Error plots of various FBRM filters in campus dataset segment.*

Presentation Outline

The FB-SLAM Problem

$$p_{k|k}(X^k, M_k = \{m_1, \ldots, m_{q(k)}\} | Z^k, u^{k-1}, X_0) = \frac{g_k(Z_k|M_k, X_k) p_{k|k-1}(X^k, M_k|Z^{k-1}, u^{k-1}, X_0)}{\int\int g_k(Z_k|M_k, X_k) p_{k|k-1}(X^k, M_k|Z^{k-1}, u^{k-1}, X_0) dX_k dM_k}$$

- Bayesian recursive approach
- Measurement uncertainty
- Map feature number and spatial uncertainty
- Vehicle pose uncertainty

FB-SLAM requires the *joint* propagation of the map dimensional and spatial uncertainty, as well as the vehicle pose uncertainty.

The RFS FB-SLAM Recursion

- The vehicle trajectory is represented by the vector,

$$X^k = [X_1, X_2, \cdots, X_k]$$

since k (the dimensionality) is known and the order is significant.

The RFS FB-SLAM Recursion

- The vehicle trajectory is represented by the vector,

$$X^k = [X_1, X_2, \cdots, X_k]$$

since k (the dimensionality) is known and the order is significant.

- The feature map is represented by a finite set,

$$M_k = \left\{m_1, m_2, \cdots, m_{q(k)}\right\}$$

$q(k)$ is the unknown number of observed map features over the vehicle trajectory, X^k

The RFS FB-SLAM Recursion

- Vehicle state evolves in time according to a standard Markov vehicle model,

$$X_{k|k-1} = f_v(X_{k-1}, u_k + Q_k)$$

- A static (but increasing in cardinality) map evolves according to,

$$M_{k|k-1} = M_{k-1} \cup B_k(Z_{k-1})$$

where $B_k(\cdot)$ is the RFS of the new features which have entered the map.

The RFS FB-SLAM Recursion

- The RFS Bayesian FB-SLAM recursion can then be written,
  - $p_k(X^k, M_k|Z^k, u^{k-1}, X_0) =$

$$\frac{g_k(Z_k|M_k, X_k)p_{k|k-1}(X^k, M_k|Z^{k-1}, u^{k-1}, X_0)}{\int\int g_k(Z_k|M_k, X_k)p_{k|k-1}(X^k, M_k|Z^{k-1}, u^{k-1}, X_0)dX_k\mu(dM_k)}$$

  - where, $p_{k|k-1}(X^k, M_k|Z^{k-1}, u^{k-1}, X_0) =$

$$\int f_k(X^k, M_k|X_{k-1}, M_{k-1}, u_{k-1})\times$$
$$p_{k-1}(X^{k-1}, M_{k-1}|Z^{k-1}, u^{k-2}, X_0)dX_{k-1}\mu(dM_{k-1})$$

The RFS FB-SLAM Recursion

- The RFS Bayesian FB-SLAM recursion can then be written,
  - $p_k(X^k, M_k | Z^k, u^{k-1}, X_0) =$

$$\frac{g_k(Z_k|M_k, X_k)p_{k|k-1}(X^k, M_k|Z^{k-1}, u^{k-1}, X_0)}{\int\int g_k(Z_k|M_k, X_k)p_{k|k-1}(X^k, M_k|Z^{k-1}, u^{k-1}, X_0)dX_k\mu(dM_k)}$$

  - where, $p_{k|k-1}(X^k, M_k|Z^{k-1}, u^{k-1}, X_0) =$

$$\int f_k(X^k, M_k|X_{k-1}, M_{k-1}, u_{k-1})\times p_{k-1}(X^{k-1}, M_{k-1}|Z^{k-1}, u^{k-2}, X_0)dX_{k-1}\mu(dM_{k-1})$$

  - Similar to the classical FB-SLAM recursion, except that the dimensionality of Z_k and M_k are not fixed with time.

Presentation Outline

The First Order Approach

- Append each map element, $m \in M_k$, with the vehicle trajectory,

$$\zeta_k^i = \begin{bmatrix} m_i \\ X^k \end{bmatrix}$$

and form the joint SLAM RFS at time k,

$$\zeta_k = \left\{ \zeta_k^1, \zeta_k^2, \cdots, \zeta_k^{q(k)} \right\}$$

- Invoke a conditional vehicle-map state independence by reducing the problem to multiple estimations of a *single* trajectory and *single* map feature.

The First Order Approach

- $|M_k|$ is monotonically increasing with time however,
  - $|M_{k|k-1} \cap FOV(X_k)|$ can be assumed Poisson with probability density $v(m)/\hat{N}$.
  - However, PHD-SLAM requires $\zeta^i_{k|k-1}$ to be Poisson with probability density $v(X^k, m)/\hat{N}$

$$v(X^k, m) = p(X^k|m)\tilde{v}(m)$$

The First Order Approach

It can then be shown that,

- PHD FB-SLAM Time update:

$$v_{k|k-1}(\zeta_k) = \int f(\zeta_k|\zeta_{k-1}, u_{k-1}) v_{k-1}(\zeta_{k-1}) d\zeta_{k-1} + b_k$$
$$= \int f(\zeta_k|x_{k-1}, m_k, u_{k-1}) v_{k-1}(x_{k-1}, m_k) dx_{k-1} + b_k$$

- PHD FB-SLAM Measurement update:

$$v_k(\zeta_k) = \Bigg[1 - p_D(\zeta_k) + \sum_{z \in Z_k} \frac{p_D(\zeta_k) g_k(z|\zeta_k)}{c_k(z) + \int p_D(\xi) g(z|\xi) v_{k|k-1}(\xi) d\xi}\Bigg] v_{k|k-1}(\zeta_k)$$

Presentation Outline

The GMM Implementation

- More efficient than SMC approaches for mildly non-linear systems
  - If the prior and birth intensities are GMM,

$$v_{k-1}(\zeta_{k-1}) = \sum_{i=1}^{N \times J_{k-1}} w_{k-1}^{(i)} \mathcal{N}(\zeta; \mu_{k-1}^{(i)}, P_{k-1}^{(i)})$$

$$b_k = \sum_{i=1}^{N \times J_{b,k}} w_{b,k}^{(i)} \mathcal{N}(\zeta; \mu_{b,k}^{(i)}, P_{b,k}^{(i)})$$

then the predicted intensity is also GMM,

$$v_{k|k-1}(\zeta_k) = \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}(\zeta; \mu_{k|k-1}^{(i)}, P_{k|k-1}^{(i)})$$

The GMM Implementation

- Assuming a Gaussian measurement likelihood, $g(z|\zeta_k)$, the joint posterior intensity, $v_k(\zeta_k)$, is consequently a Gaussian mixture,

$$v_k(\zeta_k) = v_{k|k-1}(\zeta_k)\left[1 - P_D(\zeta_k) + \sum_{z \in Z_k} \sum_{i=1}^{J_{k|k-1}} v_{G,k}^{(i)}(z|\zeta_k)\right]$$

  - where,

$$v_{G,k}^{(i)}(z|\zeta_k) = w_k^{(i)} \mathcal{N}(\zeta; \mu_{k|k}^{(i)}, P_{k|k}^{(i)})$$

$$w_k^{(i)} = \frac{P_D(\zeta_k) w_{k|k-1}^{(i)} q^{(i)}(z, \zeta_k)}{c_k(z) + \sum_{j=1}^{J_{k|k-1}} P_D(\zeta_k) w_{k|k-1}^{(j)} q^{(j)}(z, \zeta_k)}$$

with, $q^{(i)}(z, \zeta_k) = \mathcal{N}\left(z; H_k \mu_{k|k-1}^{(i)}, S_k^{(i)}\right)$.

The GMM Implementation

- The individual components are obtained from the standard EKF update equations,

$$\mu_{k|k}^{(i)} = \mu_{k|k-1}^{(i)} + K_k^{(i)}(z - H_k \mu_{k|k-1}^{(i)})$$

$$P_{k|k}^{(i)} = [I - K_k^{(i)} \nabla H_k] P_{k|k-1}^{(i)}$$

$$K_k^{(i)} = P_{k|k-1}^{(i)} \nabla H_k^T [S_k^{(i)}]^{-1}$$

$$S_k^{(i)} = R_k + \nabla H_k P_{k|k-1}^{(i)} \nabla H_k^T$$

  - ∇H_k being the Jacobian of the measurement equation with respect to the features estimated location.
  - $c_k(z) = \lambda_c V \mathcal{U}(z)$ with λ_c being the clutter density.

Presentation Outline

FB-SLAM: Simulated Environment

- Comparison of proposed ‘PHD-SLAM’ with ‘FastSLAM + ML DA + Map Management’ in feature map environments
- Vehicle speed and steering input noise of $1ms^{-1}$ and 5^o
- Two scenarios tested, with $R_{max} = 10m$:

Scenario	P_D	λ_c	σ_R	σ_α
Simple	0.95	$0m^{-2}$	$0.25m$	0.5^o
Challenging	0.95	$0.03m^{-2}$	$12.5m$	25^o

- $N = 10$ for both filters

FB-SLAM: Simulated Environment

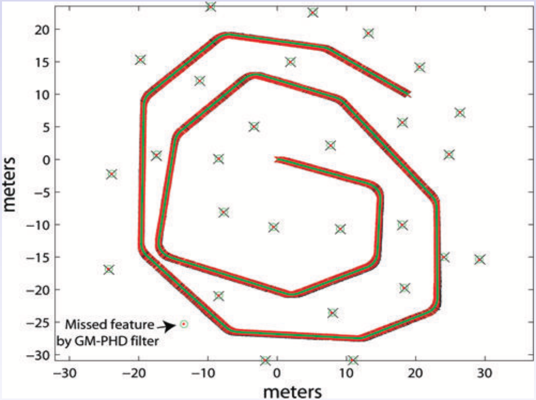


Figure: *Comparative results for the proposed GM-PHD SLAM filter (black) and that of FastSLAM (red), compared to ground truth (green).*

FB-SLAM: Simulated Environment

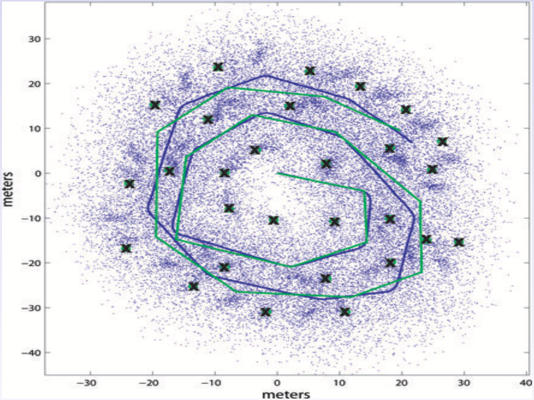


Figure: *The raw dataset at a clutter density of* $0.03m^{-2}$.

FB-SLAM: Simulated Environment

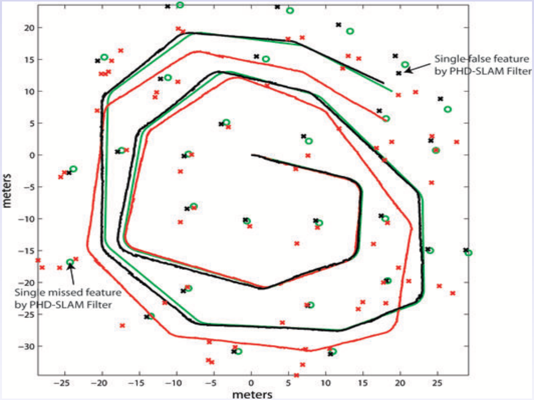


Figure: *The estimated trajectories of the GM-PHD SLAM filter (black) and that of FastSLAM (red). Estimated feature locations (crosses) are also shown with the true features (green circles)*

FB-SLAM: Simulated Environment

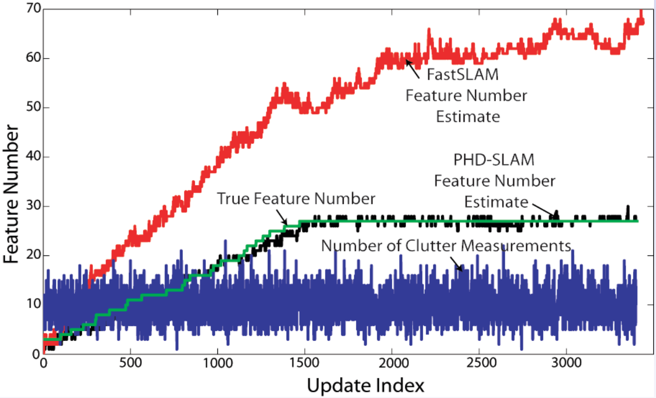


Figure: *Feature number estimates.*

FB-SLAM: Simulated Environment

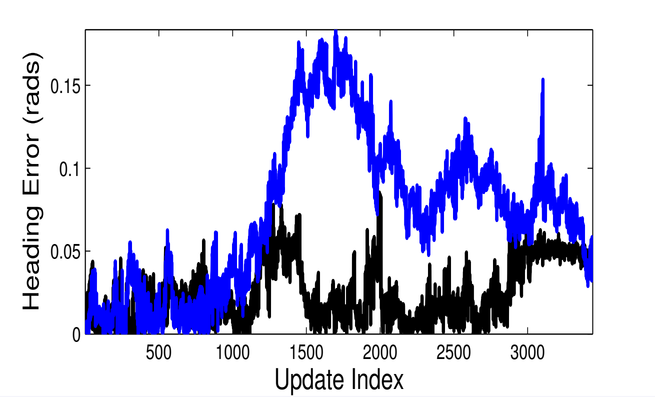


Figure: *The error in vehicle heading estimate for the PHD-SLAM (black) and FastSLAM (red) filters.*

FB-SLAM: Real Environment

- Outdoor carpark environment

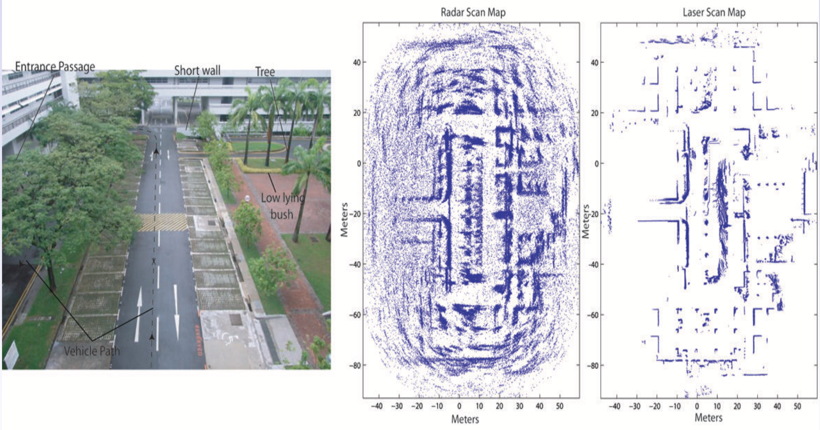


Figure: *Sample data registered from radar and laser sensors.*

FB-SLAM: Real Environment

- Odometry with extract feature measurements

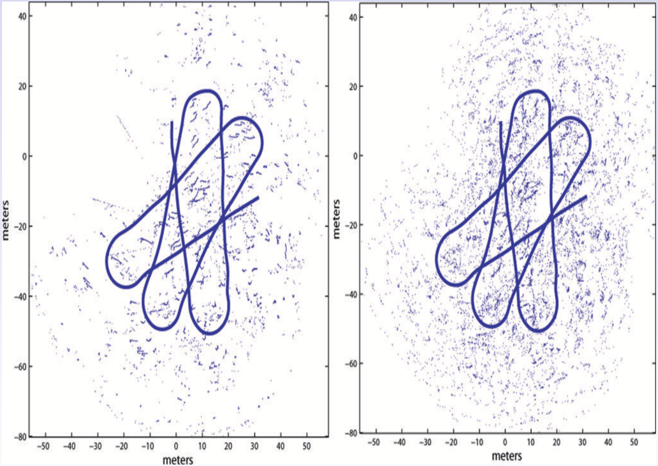


Figure: *Extracted point feature measurements with odometry.*

FB-SLAM: Real Environment

- EKF, FastSLAM and PHD-SLAM with laser data

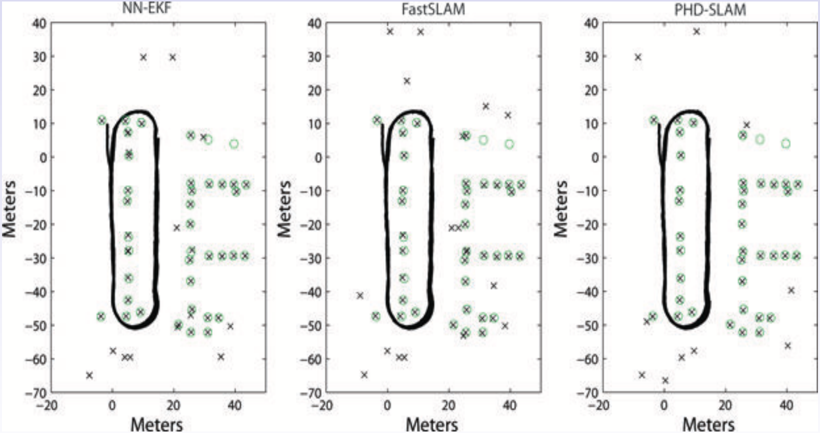


Figure: *Posterior SLAM estimate using laser data.*

FB-SLAM: Real Environment

- EKF, FastSLAM and PHD-SLAM with Radar data

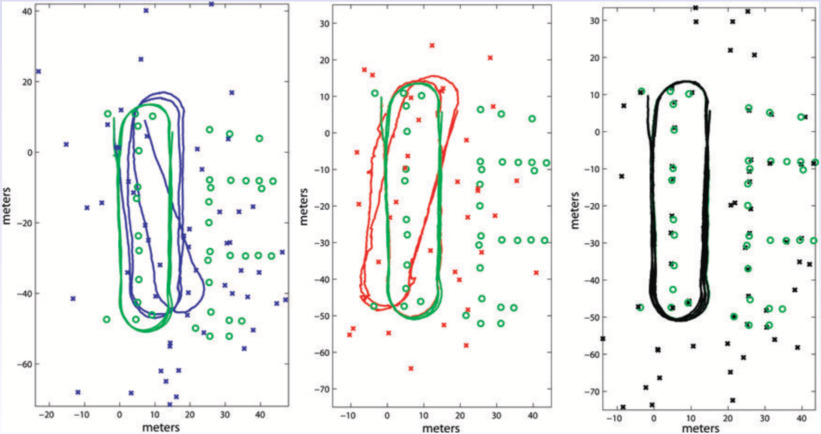


Figure: *Posterior SLAM estimate using radar data.*

Conclusions

- Previous vector based SLAM sub-optimally deal with the dimensionality and data association problems
- The Set-SLAM recursion presents an alternative feature-based Bayesian SLAM recursion which jointly considers the entire system uncertainty
- The proposed PHD-SLAM and GM implementation demonstrate the validity of the framework.
- Promising results shown, especially in scenarios of high-clutter and large data association ambiguity.

Future directions

- Extend FBRM/FB-SLAM solution to higher order recursions - propagate density on feature number
- Exploit robot motion in a Jump Markov approach to switch between lower and higher moment filters
- Extend RFS-SLAM to other formulations (FastSLAM ?, Full covariance EKF ?)

RFS approach highlights that the majority of work on FB-SLAM to date is conditioned on a known number of features.

Acknowledgements

- The research described in this project was funded in part by the Singapore National Research Foundation (NRF) through the Singapore-MIT Alliance for Research and Technology (SMART) Center for Environmental Sensing and Monitoring (CENSAM).