Detection Likelihoods for Safer Occupancy Mapping

John Mullane
Martin Adams
Wijerupage Sardha Wijesoma

School of Electrical & Electronic Engineering
Nanyang Technological University
Singapore

Presentation Outline

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Motivation

- Accurate map estimation is critical for safe and reliable autonomous navigation

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- Exteroceptive sensors can be noisy, stochastic methods popular
- As sensing noise increases, performance of current occupancy grid approaches deteriorate

✔ Examine mathematical foundation of standard occupancy measurement likelihoods

✔ Improve accuracy of maps estimated, as sensor noise increases

Environmental Perception

- ✔ Provides real-time situational awareness
- ✔ Provides absolute correction data for real-time path estimation

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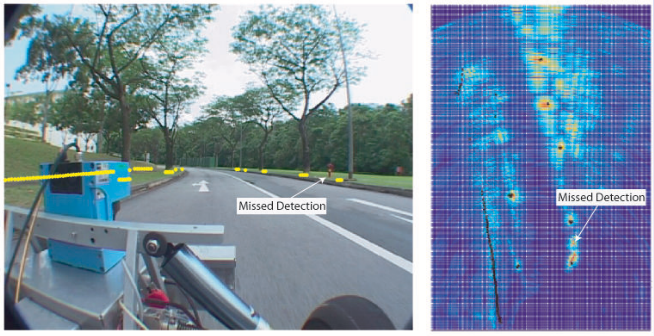

- Measurement Uncertainty:
  - ✘ Measurement noise
  - ✘ Spurious measurements
  - ✘ Detection uncertainty
  - ✘ Data association uncertainty

Presentation Outline

Sample Environment

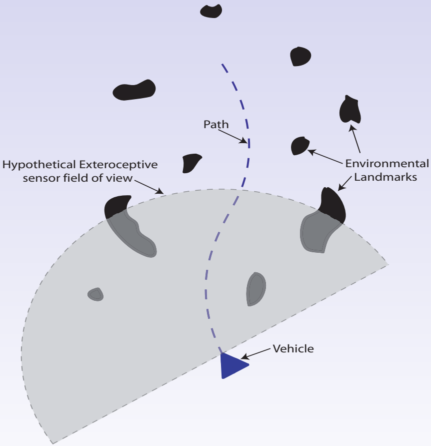

Figure: *A General 2D Autonomous Navigation Scenario.*

Map Representation

- ✔ Represent the map, M, as a mathematical object.

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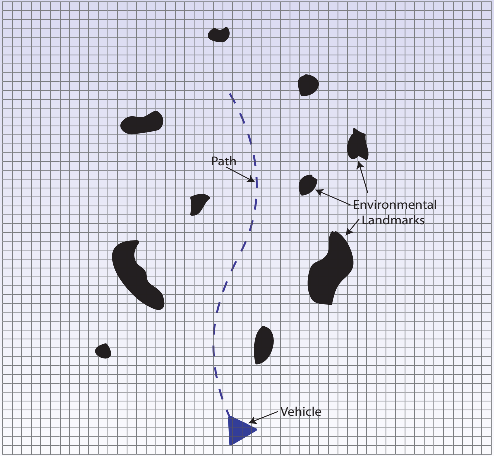


Grid-based Map

- [Moravec, ’85]
- [Elfes, ’89]
- [Martin, ’96]
- [Konolgie, ’97]
- [Pagac, ’98]
- [Gutmann, ’99]
- [Foessel, ’02]
- [Thrun, ’02]
- [Hahnel, ’03]
- [Grisetti, ’03]
- [Thrun, ’03]
- [Yang, ’06]

Presentation Outline

Dealing with Measurement Uncertainty

- ✔ Environmental Estimation: Robotic Mapping
- ✔ Bayesian approach, widely accepted in robotics

- Assuming vehicle path is known (RM):

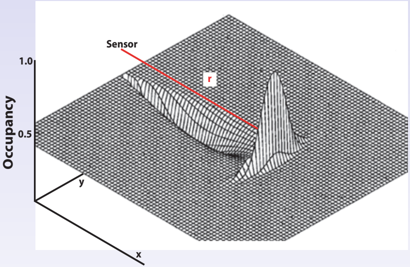
$$p_{k|k}(M_k|Z^k, X^k, u^{k-1}, X_0) = \frac{g_k(Z_k|M_k, X_k)p_{k|k-1}(M_k|Z^{k-1}, X^k, u^{k-1}, X_0)}{\int g_k(Z_k|M_k, X_k)p_{k|k-1}(M_k|Z^{k-1}, u^{k-1}, X_0)dM_k}$$

$p_{k|k}(M_k|Z^k, X^k, u^{k-1}, X_0)$: encapsulates all uncertainty about the map at time k.

The Measurement Likelihoods

- Widely adopted in the GBRM literature.
  - **Grid Maps**:

$$G_k(Z_k = r | M_k = \mathrm{GRID}, X_k) \qquad \text{[Elfes, '89]}$$



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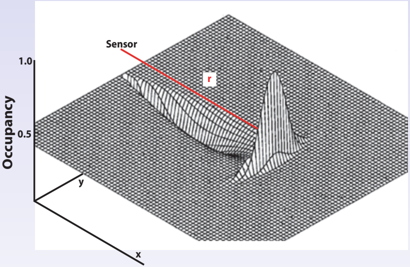

  - How are the measurement likelihoods calculated ?

$$G_k(Z_k = r|M_k = E, X_k) \text{ ?}$$

$$G_k(Z_k = r|M_k = O, X_k) \text{ ?}$$

Motivation Summary

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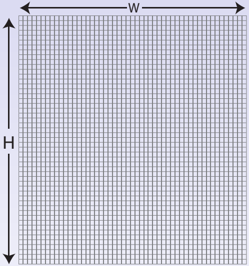
- ✘ Occupancy mapping likelihoods appear to have some inconsistencies
- ✔ Change the measurement space from range/bearing to detection/non-detection
- ✔ Improve robustness of occupancy grid framework to noisy environments and sensors

Presentation Outline

Grid Mapping Example

The GBRM Problem

$$p_{k|k}(M_k = [m_1, \ldots, m_{W \times H}] | Z^k, X^k, u^{k-1}, X_0) = \frac{g_k(Z_k|M_k, X_k) p_{k|k-1}(M_k|Z^{k-1}, X^k, u^{k-1}, X_0)}{\int g_k(Z_k|M_k, X_k) p_{k|k-1}(M_k|Z^{k-1}, u^{k-1}, X_0) dM_k}$$



- Bayesian recursive approach
- Measurement uncertainty
- Map occupancy uncertainty
- Decompose map into $W \times H$ independent estimation problems

$$p_{k|k}(M_k|Z^k, X_k) = \prod_{i=1}^{i=W\times H} p_{k|k}(m_k^i|Z^k, X^k)$$

GBRM requires the propagation of the map occupancy state.

Current Approach: The Range-based Recursion

The Range-based GBRM Filter

$$p_{k|k}(M_k=[m_1,\ldots,m_{W\times H}]|Z^k,X^k,u^{k-1},X_0)=\frac{g_k(Z_k|M_k,X_k)p_{k|k-1}(M_k|Z^{k-1},X^k,u^{k-1},X_0)}{\int g_k(Z_k|M_k,X_k)p_{k|k-1}(M_k|Z^{k-1},u^{k-1},X_0)dM_k}$$

- State is binary: $M=[O,E]$
- Prediction: $p_{k|k-1}(M_k|Z^{k-1},X^k,u^{k-1},X_0)$
- Measurement: Z_k =range/bearing
- Form likelihood: $g_k(Z_k|M_k,X_k)$
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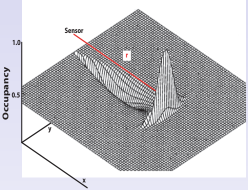
The Measurement Likelihood: State Dependency

$$G_k(z_k = r|m_{k,(x)} = O, X_k)$$



- The likelihood of a range measurement conditioned on the occupancy state and vehicle pose

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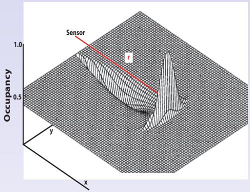

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Q. What is the function that relates $m_{k,(x)}$ and X_k to z_k, where z_k is range reading?

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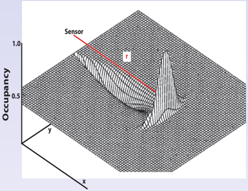

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A. Use $z_k = \{\text{Detection,No Detection}\}$ to get state dependant measurement equation.

The Measurement Likelihood: Uncertainty

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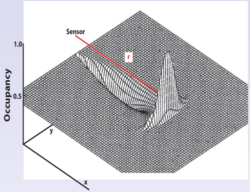

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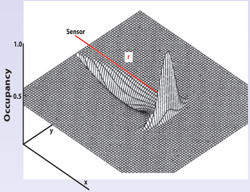

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Q. For no range reading, how is $G_k(z_k = r|m_{k,(x)} = [O, E], X_k)$ defined ?

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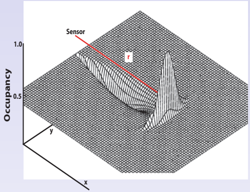

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Current approach: Drawbacks

- Grid-based Framework
  - ✔ Estimation state space: Occupancy
  - ✔ Map representation
  - Measurement Likelihood:
    - ✔ Measurement Noise
    - ✗ State dependent
    - ✗ Detection Uncertainty
    - ✗ Spurious Measurements

Presentation Outline

Proposed Approach: Advantages

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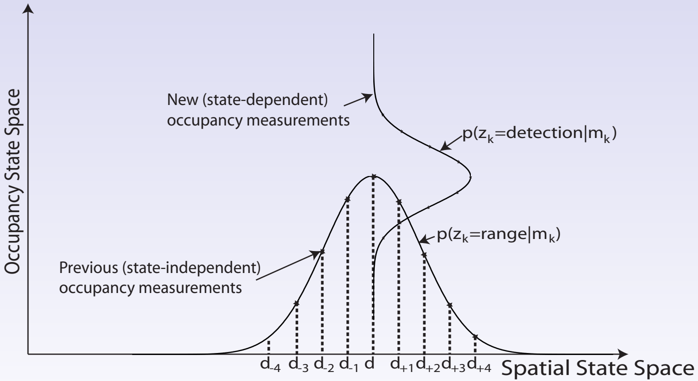
Proposed Approach: The Filtering State-Space



Proposed Approach: Key Observations

With z={Detection, Non-Detection}:

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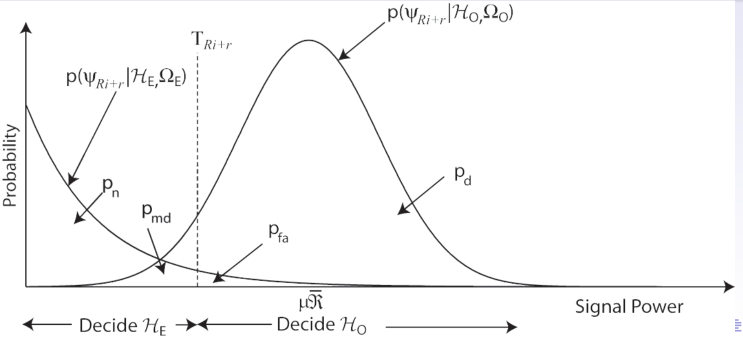
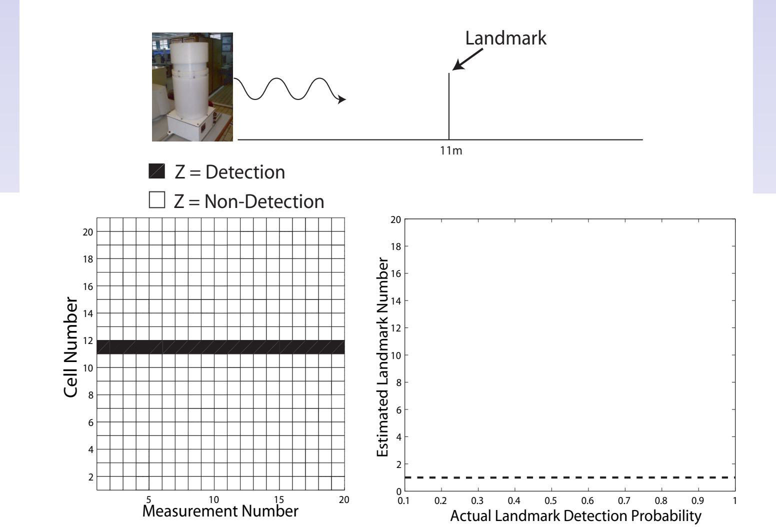
the measurement likelihood becomes the detection statistics



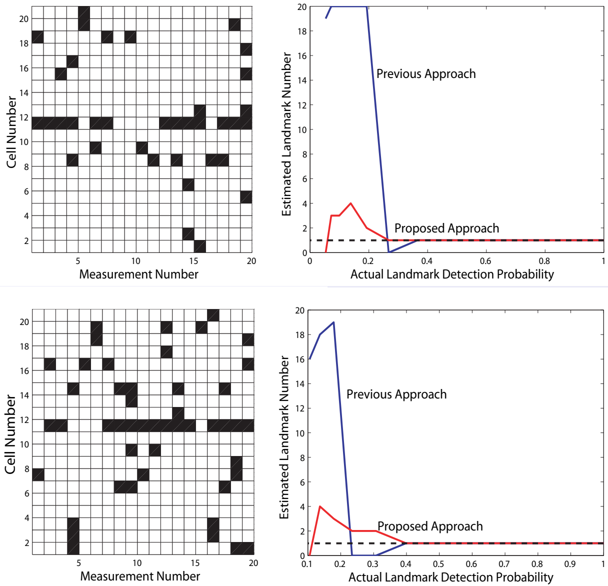
Presentation Outline

Simulation: Known Likelihoods



1D Scenario

Simulation: Known Likelihoods



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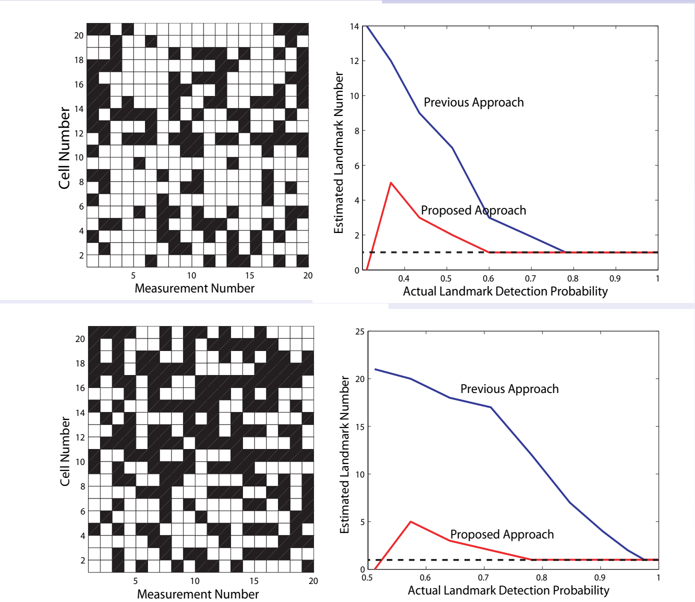

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- ✔ Likelihoods are sensor dependent
  - Detection theory may differ between sensor - MMWR, LMS, Camera, Sonar etc. etc.

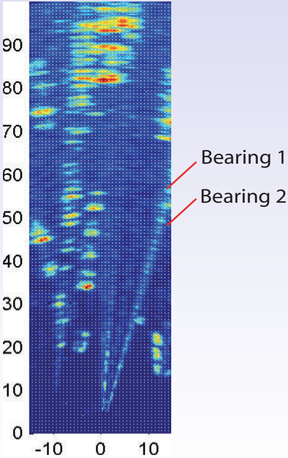
Presentation Outline

The MMW Radar

- Operates at 77GHz
- Returns unprocessed data allowing for custom detector design

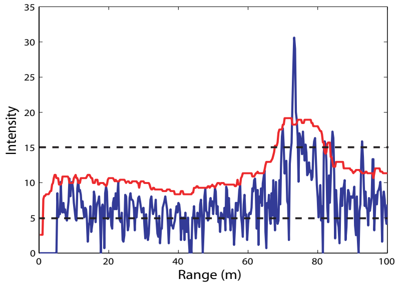
The Detection Problem: A Stochastic Approach

- ✘ Rarely considered in current navigation algorithms
- ✔ Stochastic detectors exploit statistics of underlying signals

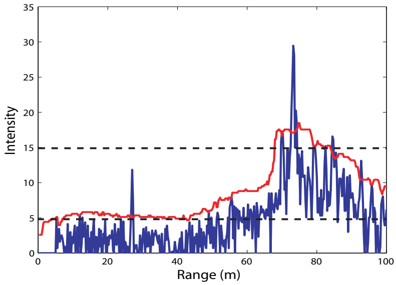


The Detection Problem: A Stochastic Approach

- ✔ Outperform classically adopted constant thresholds
- ✔ Detections (and non-detections) are statistically significant

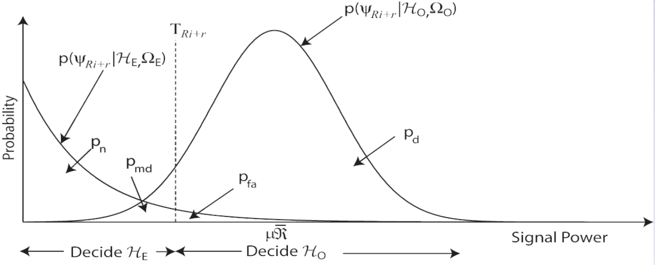


Spectrum at Bearing Angle 1



Spectrum at Bearing Angle 2

Detection Statistics



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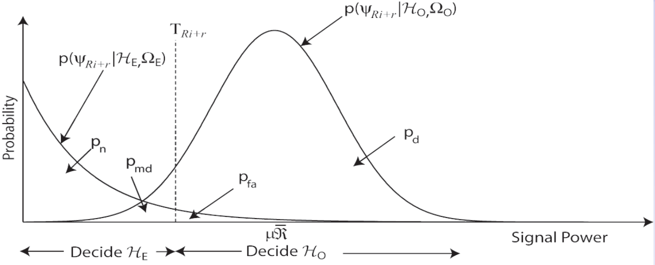


$$P_d = \int_0^\infty P[\psi_{Ri+r} \geq T_{Ri+r} | \mathcal{H}_O] f_\mu(\mu) d\mu$$

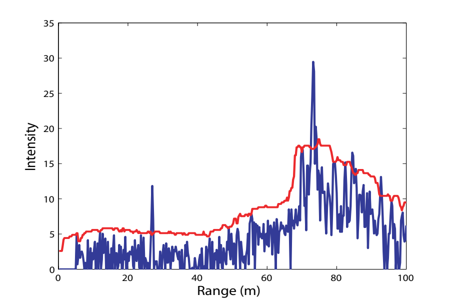
$$P_n = \int_0^\infty P[\psi_{Ri+r} < T_{Ri+r} | \mathcal{H}_E] f_\mu(\mu) d\mu$$

$$P_{md} = \int_0^\infty P[\psi_{Ri+r} < T_{Ri+r} | \mathcal{H}_O] f_\mu(\mu) d\mu$$

$$P_{fa} = \int_0^\infty P[\psi_{Ri+r} \geq T_{Ri+r} | \mathcal{H}_E] f_\mu(\mu) d\mu$$

MMWR: Stochastic Detection

- Form a threshold: $T = \tau\hat{\Omega}_E$
- Popular approaches: OS, CA, GO, SO, ...



- At a given range, r, assuming $\Omega_E = \mu$ then,

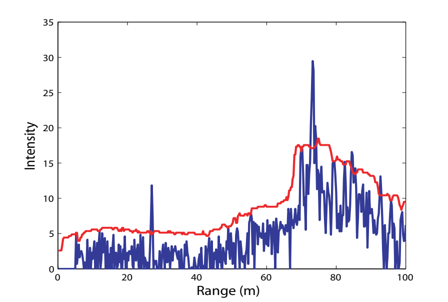
$$g_k(z_k = D | m_{k,(r)} = O, X_k) = \int_0^\infty P[\psi_r \geq T_r | \mathcal{H}_O] f_\mu(\hat{\mu}) d\hat{\mu}$$

$$g_k(z_k = D | m_{k,(r)} = E, X_k) = \int_0^\infty P[\psi_r \geq T_r | \mathcal{H}_E] f_\mu(\hat{\mu}) d\hat{\mu}$$

MMWR: OS-CFAR Likelihoods

- Assume $v(\Omega_E)$ is IID,

$$g_k(z_k = D | m_{k,(r)} = E, X_k) = K$$



- Analgous to data association threshold
  - ✔ χ^2 test accepts a *correct* assignment with a fixed probability
  - ✔ CFAR test accepts an *incorrect* assignment with a fixed probability
  - ✔ Both threshold give *no* information of the converse

MMWR: OS-CFAR Likelihoods

$$g_k(z_k = D | m_{k,(r)} = O, X_k) = \left(1 + \frac{T_r}{1+\bar{\mathfrak{R}}_r}\right)^{-2W}$$

where,

$$T_r = \tau \hat{\mu}_r$$

$$\tau = \arg\min_{\tau} \left(k_{os} \binom{2W}{k_{os}} \frac{(k_{os}-1)!(\tau + 2W - k_{os})!}{(\tau + 2W)!} - P_{fa} \right)$$

$$\hat{\mu}_r = \Psi_{os,k_{os}}$$

$$\Psi_{os} = \text{sort}([\psi_{r-G-W}, \ldots, \psi_{r-G}] \cup [\psi_{r+G+1}, \ldots, \psi_{r+G+W}])$$

$$\bar{\mathfrak{R}}_r = \frac{\psi_r - \hat{\mu}_r}{\hat{\mu}_r}$$

MMWR: Evaluating the Likelihoods

$$p_{k|k}(m_{k,(x)} = O|z^k, X^k) = \frac{g_k(z_k|m_{k,(x)} = O, X_k)p_{k|k-1}(m_{k,(x)} = O|z^{k-1}, X^k)}{p_{k|k}(z_k|m_{k,(x)}, X^k)}$$

Likelihood	**Filter**
Discrete probabilistic	Binary Bayes Filter
Discrete evidential	Dempster-Shafer Evidential Filter
Continuous probabilistic	if Gaussian - Kalman Filter if non-Gaussian - Particle Filter.

Presentation Outline

MMWR: Discrete Probabilistic GBRM Filter

$$P_{k|k}(M_k = O|z_{k,(r)} = D) = \frac{Z_{P_d} P_{k|k-1}(M_k = O|Z_{k-1})}{Z_{P_d} P_{k|k-1}(M_k = O|Z_{k-1}) + Z_{P_{fa}} P_{k|k-1}(M_k = E|Z_{k-1})}$$

$$P_{k|k}(M_k = O|z_{k,(r)} = \bar{D}) = \frac{Z_{P_{md}} P_{k|k-1}(M_k = O|Z_{k-1})}{Z_{P_{md}} P_{k|k-1}(M_k = O|Z_{k-1}) + Z_{P_n} P_{k|k-1}(M_k = E|Z_{k-1})}$$

MMWR: Discrete Evidential GBRM Filter

$$\mathfrak{m}_m(C) = \frac{\sum_{A \cap B = C} \mathfrak{m}_z(A)\mathfrak{m}_m(B)}{1 - \sum_{A \cap B = \emptyset} \mathfrak{m}_z(A)\mathfrak{m}_m(B)}$$

$$\mathfrak{m}_z(m_k^F | z_k = \bar{D}) = \frac{Z_{P_{md}}}{Z_{P_{md}} + Z_{P_n} + Z_{P_u}}$$

$$\mathfrak{m}_z(m_k^E | z_k = \bar{D}) = \frac{Z_{P_n}}{Z_{P_{md}} + Z_{P_n} + Z_{P_u}}$$

$$\mathfrak{m}_z(m_k^U | z_k = \bar{D}) = \frac{Z_{P_u}}{Z_{P_{md}} + Z_{P_n} + Z_{P_u}}$$

$$\mathfrak{m}_z(m_k^{\emptyset} | z_k) = 0$$

MMWR: Continuous Probabilistic GBRM Filter

$$p_{k-1|k-1}(o_{k-1,(r)}|z_{(r)}^{k-1}) \approx \sum_{i=1}^{N} w_{k-1,(r)}^{(i)} \delta_{o_{k-1,(r)}^{(i)}}(o_{k-1,(r)})$$

where,

$$o_{t-1,(r)} = \begin{bmatrix} m_{k-1,(r)} \\ \lambda_{k-1,(r)} \end{bmatrix}$$

$$o_{k,(r)}^{(i)} \sim q(o_{k,(r)}|o_{k-1,(r)}^{(i)}, z_{k,(r)})$$

$$w_{k,(r)}^{(i)} = w_{k-1,(r)}^{(i)} \frac{p(z_{k,(r)}|o_{k,(r)}^{(i)})p(o_{k,(r)}^{(i)}|o_{k-1,(r)}^{(i)})}{q(o_{k,(r)}^{(i)}|o_{k-1,(r)}^{(i)}, z_{k,(r)})}$$

MMWR: Continuous Probabilistic GBRM Filter

$$p(m_{k-1,(r)}|z_{(r)}^{k-1}) \sim p(m_{k-1,(r)} = 1|z_{(r)}^{k-1}, \Pi)$$
$$p_{k|k-1}(\lambda_{k,(r)}|z_{(r)}^{k-1}) = p_{k-1|k-1}(\lambda_{k-1,(r)}|z_{(r)}^{k-1})$$

$$q(o_{k,(r)}^{(i)}|o_{k-1,(r)}^{(i)}, z_{k,(r)}) = p(o_{k,(r)}^{(i)}|o_{k-1,(r)}^{(i)}).$$

$$p(z_{k,(r)}|o_{k,(r)}^{(i)}) = \frac{p(\psi_r|m_{(r)} = 1, \Omega_O)}{p(\psi_r|m_{(r)} = 0, \Omega_E)}$$

$$\hat{o}_{k,(r)} = \sum_{i=1}^{N} w_{k,(r)}^{(i)} o_{k,(r)}^{(i)}.$$

Presentation Outline

Testing Environment

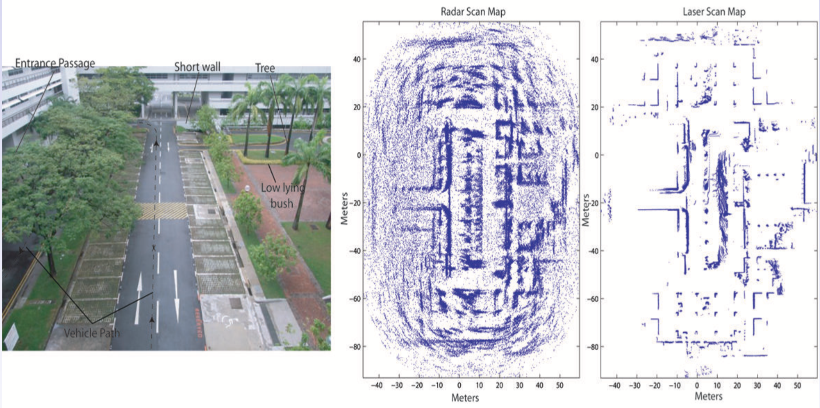


Figure: *Testing Ground overview with corresponding scan maps*

Testing Environment

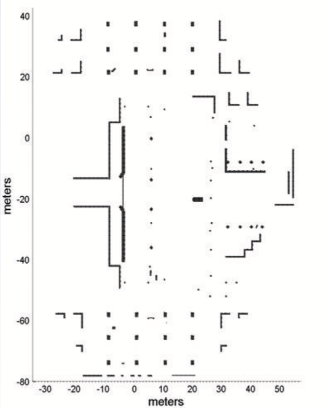


Figure: *Carpark binary ground-truth GB map.*

GBRM: Error Quantification

- Vector Map Comparison
- Sum of Squared Error (SSE) common [Martin, '96], [Collins '98], [Rachlin, '05]

$$\sum_{i}^{q}(m_i - \hat{m}_i)^2$$

✗ Not applicable to outdoor environments

$$NASSE = 0.5\left(\frac{1}{\mathfrak{q}_O}\sum_{i=0}^{\mathfrak{q}_O}\left(P(m_k^i|z^{i,k}, m^i = 1) - 1\right)^2 + \frac{1}{(\mathfrak{q} - \mathfrak{q}_O)}\sum_{i=\mathfrak{q}_O+1}^{\mathfrak{q}}\left(P(m_k^i|z^{i,k}, m^i = 0) - 0\right)^2\right)$$

GBRM: Error Quantification

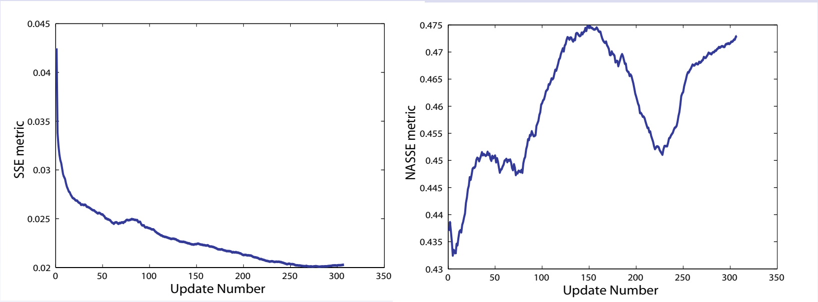


Figure: *Grid-based error metric comparison with localisation error.*

Discrete Probabilistic Implementation

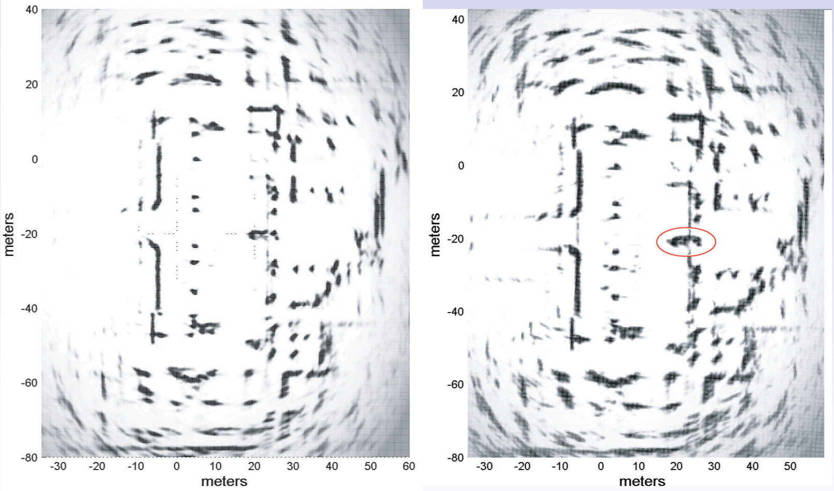


Figure: *Discrete probabilistic detection filter (left) and discrete range filter (right).*

Discrete Probabilistic Implementation

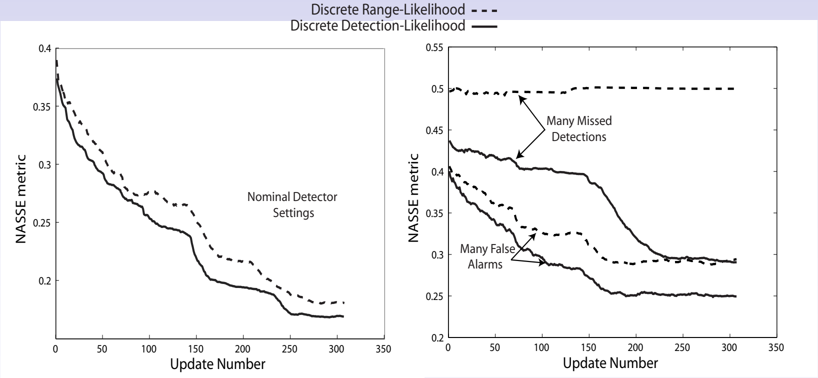
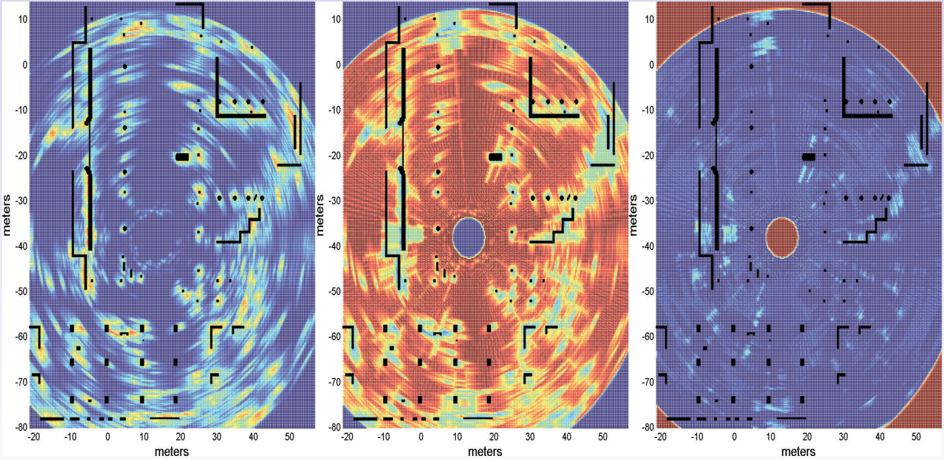


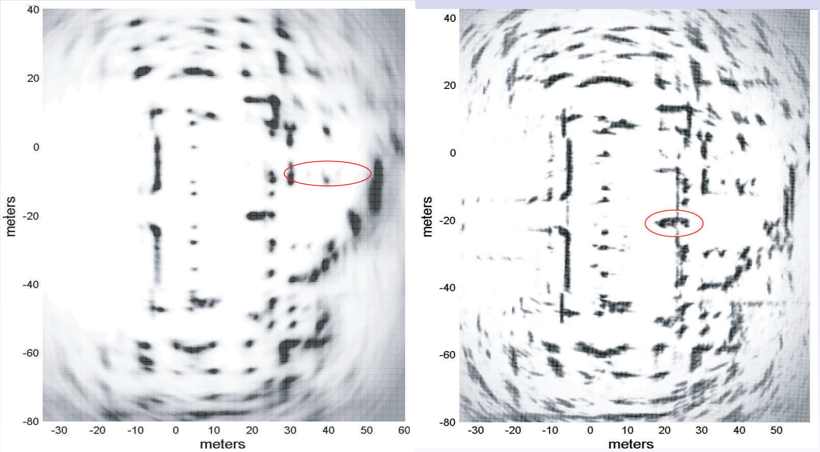
Figure: *Discrete probabilistic detection vs range likelihood NASSE comparison.*

Discrete Evidential Implementation



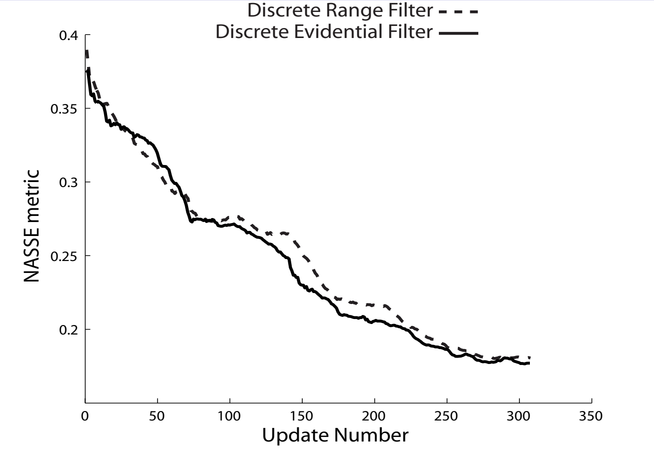
Instantaneous mass distributions on the map.

Discrete Evidential Implementation



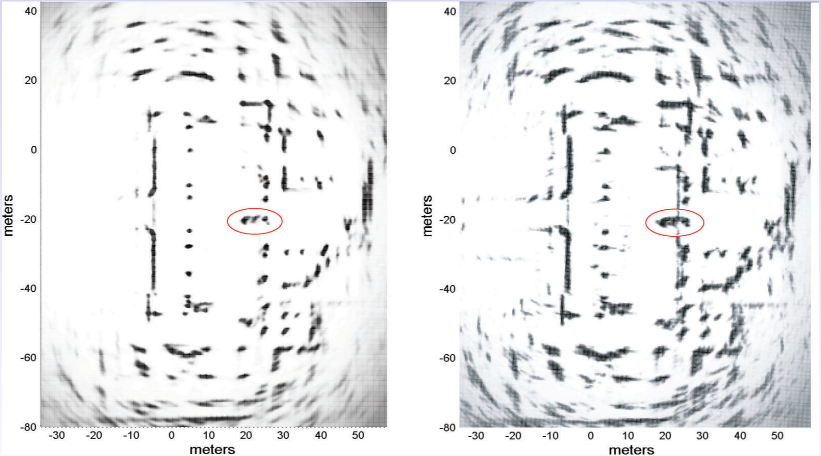
Discrete evidential detection filter (left) and discrete range filter (right).

Discrete Evidential Implementation



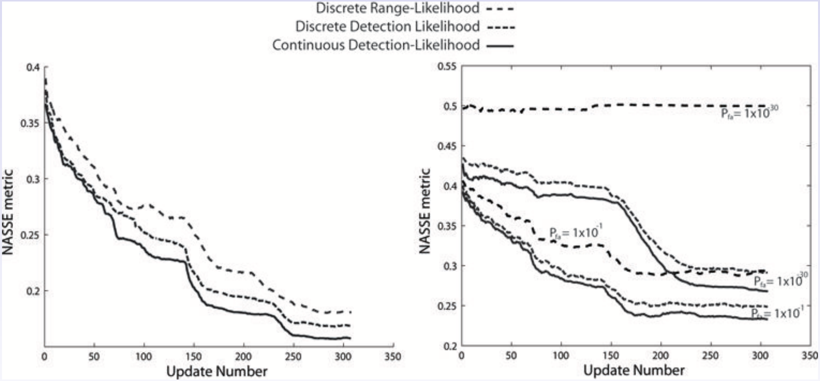
Discrete evidential filter NASSE.

Continuous Probabilistic Implementation



Continuous detection filter (left) and discrete range filter (right).

NASSE Comparisons



NASSE comparison.

NASSE Comparison vs. Detector Parameter

$$g_k(z_k = D | m_{k,(r)} = O, X_k) = \left(1 + \frac{T_r}{1 + \bar{\Re}_r}\right)^{-2W}$$

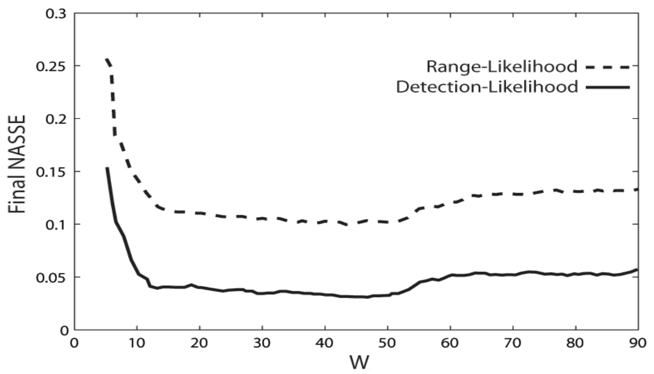


Figure: *Continuous detection vs range likelihood NASSE vs sliding window width.*

Campus Results

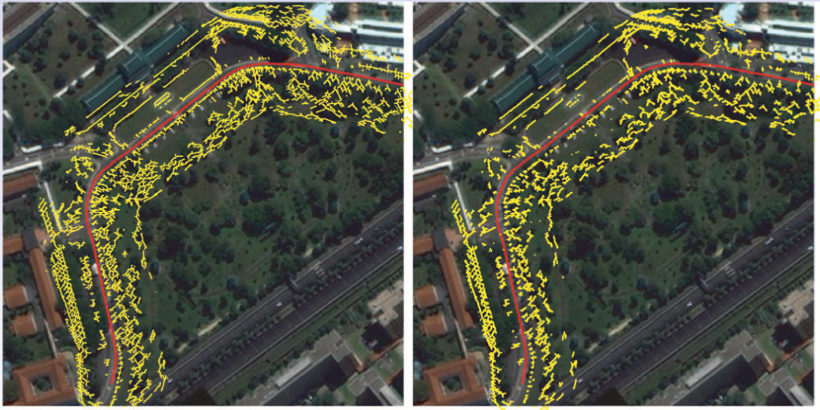


Figure: *Campus excerpt map estimate*

Campus Results

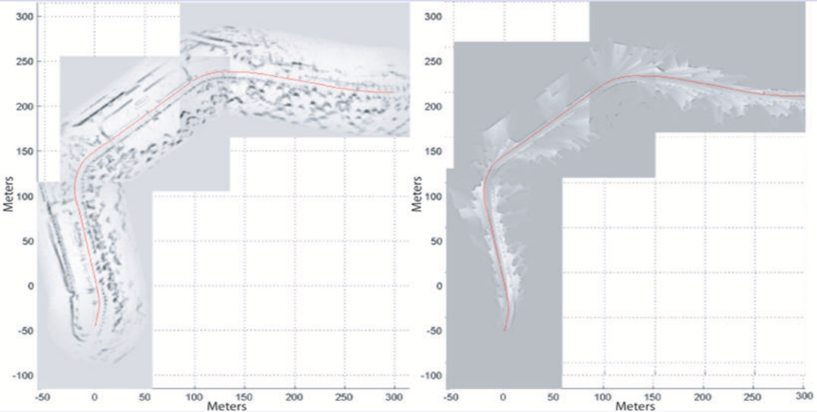


Figure: *Comparison of radar and laser posterior grid map estimates*

Campus Results

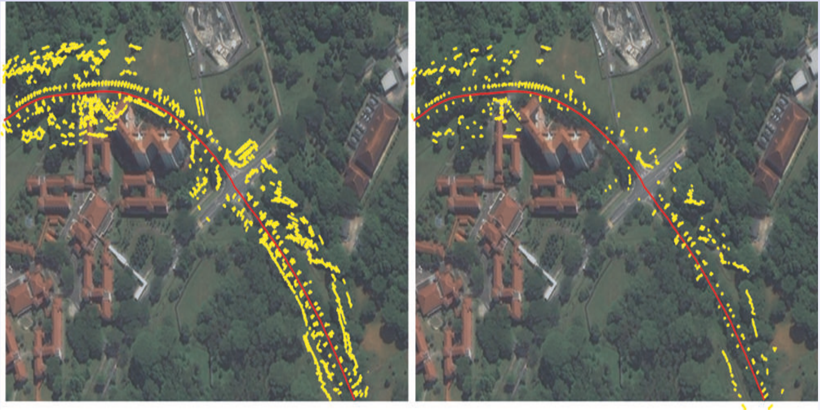


Figure: *Campus excerpt map estimate*

Campus Results

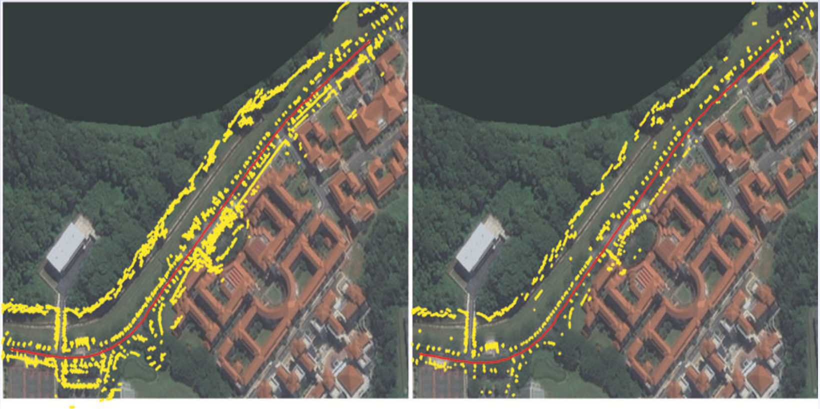


Figure: *Campus excerpt map estimate*

Presentation Outline

Conclusions

- Autonomous safety is highly dependant on accurate environmental representation
- ✔ Error of estimated grid maps can be reduced by incorporating the measurement uncertainty directly into the measurement likelihood
- ✔ Changing measurement space to detection/non-detection makes the likelihoods physically intuitive
- ✔ Likelihoods derived and mapping filters implemented using a MMWR sensor
- ✔ Improved mapping accuracy, particulary in situations of high false alarm and missed detection probability

Future directions

- Extend detection recursion to other sensing modalities
- For radar: Develop the EKF - Evidential Kalman Filter (Continuous evidential likelihoods)
- Extend to feature extraction algorithms - estimating the probability of feature existence

Acknowledgements

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