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Brief paper

Robust adaptive control of uncertain force/motion constrained nonholonomic mobile manipulators[☆]

Z. Li[a,b], S.S. Ge[a,*], M. Adams[b], W.S. Wijesoma[b]

[a]*Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576, Singapore*
[b]*School of Electrical and Electronics Engineering, Nanyang Technological University, Singapore 639798, Singapore*

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Abstract

In this paper, force/motion tracking control is investigated for nonholonomic mobile manipulators with unknown parameters and disturbances under uncertain holonomic constraints. The nonholonomic mobile manipulator is transformed into a reduced chained form, and then, robust adaptive force/motion control with hybrid variable signals is proposed to compensate for parametric uncertainties and suppress bounded disturbances. The control scheme guarantees that the outputs of the dynamic system track some bounded auxiliary signals, which subsequently drive the kinematic system to the desired trajectory/force. Simulation studies on the control of a wheeled mobile manipulator are used to show the effectiveness of the proposed scheme.

Keywords: Mobile manipulator; Trajectory/force control; Nonholonomic/holonomic constraint

1. Introduction

Mobile manipulators have received increasing attention over the last 15 years, because of their industrial relevance and academic interest (Dong, 2002; Liu & Lewis, 1990; Tan, Xi, & Wang, 2003; Watanabe, Sato, Izumi, & Kunitake, 2000). However, the motion planning and control of mobile manipulators cannot be addressed by traditional methods due to the nonholonomic nature of the systems which are imposed by the wheeled platforms. Due to Brockett's theorem (1983), it is well known that nonholonomic systems with restricted mobility cannot be stabilized to a desired configuration nor posture-via differentiable, or even continuous, pure state feedback. Therefore, the control design for these systems is a challenging problem, and has attracted much attention in the robotics and control community (Ge, Wang, Lee, & Zhou, 2001, 2003; Oya & Su, 2003; Wang, Ge, & Lee, 2004).

With the assumption of known dynamics, much research has been carried out to control mobile manipulators including input–output feedback linearization (Tan et al., 2003), nonlinear feedback control (Yamamoto & Yun, 1996). Because of dynamic uncertainty, adaptive neural network controls (Lin & Goldenberg, 2001) have been proposed for motion control of mobile manipulators. However, for practical applications, not only the motion but also the force of the end-effector of the arm should be considered. In Dong (2002), adaptive control was proposed for trajectory/force control of mobile manipulators subjected to holonomic and nonholonomic constraints with unknown inertia parameters. Most control approaches for mobile manipulator deal with uncertainty in system dynamics and assume that the exact holonomic constraints of the interconnected system are known (Ge & Lewis, 2006). However, in practical applications, environmental uncertainties arise in mobile manipulator applications which can affect the system stability and performance. In this paper, under holonomic uncertainty, we consider the trajectory and force tracking for general dynamic nonholonomic mobile manipulator systems in which the system is transformed into a chained form. Since the general motion/force decomposition control is not valid with constraint uncertainties, we develop a low-pass force filter to assure the

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* Corresponding author. Tel.: +65 6516 6821; fax: +65 67791103.
E-mail address: elegesz@nus.edu.sg (S.S. Ge).

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boundedness of force error and simultaneously estimate the uncertain constraint Jacobian matrix. A main concern in our design is the magnitude of the Jacobian error as it influences the residual force error. By investigating the dynamics error, we apply a reduction procedure and define a mixed tracking error of trajectory and force. Then, robust adaptive force/motion control is presented in which adaptive controls are used to compensate for parametric uncertainties, suppress constraint uncertainties and bounded disturbances. The control guarantees the outputs of the dynamic system to track some bounded hybrid signals which subsequently drive the kinematic system to the desired trajectory/force.

2. System description

2.1. Dynamics of mobile manipulators

Consider an n DOF manipulator mounted on a nonholonomic mobile platform (Li, Ge, & Ming, 2007)

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d(t) = B(q)\tau + f, \tag{1}$$

where $q = [q_1, \ldots, q_n]^{\mathrm{T}} \in R^n$ denote the generalized coordinates; $M(q) \in R^{n\times n}$ is the symmetric bounded positive definite inertia matrix; $C(q,\dot{q})\dot{q} \in R^n$ denotes the Centripetal and Coriolis torques; $G(q) \in R^n$ is the gravitational torque vector; $d(t) \in R^n$ denotes the external disturbances; $\tau \in R^p$ are the control inputs; $B(q) \in R^{n\times p}$ is a known input transformation matrix; and $f = J^{\mathrm{T}}(q)\lambda \in R^n$ denotes the generalized constraint forces. The generalized coordinates may be separated into two sets as $q = [q_v, q_a]^{\mathrm{T}}$ with $q_v \in R^{n_v}$ denoting the generalized coordinates for the vehicle and $q_a \in R^{n_a}$ denoting the coordinates of the arm.

2.2. Nonholonomic constraints

When the system is subjected to nonholonomic constraints, assume that the $n_v - m$ nonintegrable and independent velocity constraints can be expressed as $A(q_v)\dot{q}_v = 0$. Since $A(q_v) \in R^{(n_v-m)\times n_v}$, it is always possible to find an m rank matrix $H_v(q_v) \in R^{n_v\times m}$ formed by a set of smooth and linearly independent vector fields spanning the null space of $A(q_v)$, i.e., $H_v^{\mathrm{T}}(q_v)A^{\mathrm{T}}(q_v) = 0$. Since $H_v(q_v) = [H_{v1}(q_v), \ldots, H_{vm}(q_v)]$ is formed by a set of smooth and linearly independent vectors spanning the null space of $A(q_v)$, define an auxiliary function $v_b = [v_{b1}, \ldots, v_{bm}]^{\mathrm{T}} \in R^m$ such that

$$\dot{q}_v = H_v(q_v)v_b = H_{v1}(q_v)v_{b1} + \cdots + H_{vm}(q_v)v_{bm} \tag{2}$$

which is the so-called kinematic model of nonholonomic system. Differentiating (2) yields

$$\ddot{q}_v = \dot{H}_v(q_v)v_b + H_v(q_v)\dot{v}_b. \tag{3}$$

Therefore, the dynamics (1), after eliminating the nonholonomic constraint $A^{\mathrm{T}}\lambda_n$, can be reformulated as

$$\dot{q} = H(q)v, \tag{4}$$

$$M_1(q)\dot{v} + C_1(q,\dot{q})v + G_1(q) + d_1 = B_1(q)\tau + f_1, \tag{5}$$

where $H = \mathrm{diag}[H_v(q_v), I_a]$, $M_1(q) = H^{\mathrm{T}}(q)M(q)H(q)$, $C_1(q,\dot{q}) = H^{\mathrm{T}}(q)[M(q)\dot{H}(q) + C(q,\dot{q})H(q)]$, $G_1(q) = H^{\mathrm{T}}(q)G(q)$, $d_1 = H^{\mathrm{T}}(q)d(t)$, $B_1(q) = H^{\mathrm{T}}(q)B(q)$, $v = [v_b, \dot{q}_a]^{\mathrm{T}}$, and $f_1 = J_1(q)^{\mathrm{T}}\lambda_h$.

2.3. Reduced model and state transformation

There exist a coordinate transformation $T_1(q)$ and a state feedback $T_2(q)$, such that

$$\zeta = [\xi, q_a]^{\mathrm{T}} = T_1(q) = [T_{11}(q_v), q_a]^{\mathrm{T}}, \tag{6}$$

$$v = [v_b, \dot{q}_a]^{\mathrm{T}} = T_2(q)u = [T_{21}(q_v)u_b, u_a]^{\mathrm{T}} \tag{7}$$

with $T_2(q) = \mathrm{diag}[T_{21}(q_v), I]$ and $u = [u_b, u_a]^{\mathrm{T}}$, then the kinematic system (4) could be locally or globally converted to the chained form (Walsh & Bushnell, 1995)

$$\dot{\xi}_1 = u_1, \quad \dot{\xi}_i = u_1\xi_{i+1} \; (2 \leqslant i \leqslant n_v - 1), \quad \dot{\xi}_{n_v} = u_2, \quad \dot{q}_a = u_a, \tag{8}$$

where $u_a = [u_{a1}, \ldots, u_{an_a}]^{\mathrm{T}}$.

Remark 2.1. In Murray and Sastry (1993), a necessary and sufficient condition for the transformation of the kinematic system (4) of a differential drive wheeled mobile platform into single chain was given. In Walsh and Bushnell (1995), the existence condition of the transformation (multi-chain case) was discussed for the other types of mobile platform.

Consider the above transformations, the dynamics (5) is converted as

$$M_2(\zeta)\dot{u} + C_2(\zeta,\dot{\zeta})u + G_2(\zeta) + d_2 = B_2\tau + f_2, \tag{9}$$

where $M_2(\zeta) = T_2^{\mathrm{T}}(q)M_1(q)T_2(q)|_{q=T_1^{-1}(\zeta)}$, $C_2(\zeta,\dot{\zeta}) = T_2^{\mathrm{T}}(q)[M_1(q)\dot{T}_2(q) + C_1(q,\dot{q})T_2(q)]|_{q=T_1^{-1}(\zeta)}$, $G_2(\zeta) = T_2^{\mathrm{T}}(q)G_1(q)|_{q=T_1^{-1}(\zeta)}$, $B_2 = T_2^{\mathrm{T}}(q)B_1(q)|_{q=T_1^{-1}(\zeta)}$, $d_2 = T_2^{\mathrm{T}}(q)d_1|_{q=T_1^{-1}(\zeta)}$, $f_2 = T_2^{\mathrm{T}}(q)f_1 = J_2^{\mathrm{T}}(q)\lambda_h|_{q=T_1^{-1}(\zeta)}$.

Property 2.1. *The matrix M_2 is symmetric and positive definite, and the matrix $\dot{M}_2 - 2C_2$ is skew-symmetric* (Dong, 2002).

Property 2.2. *There exist some finite non-negative constants $c_i \geqslant 0$ $(i=1,\ldots,5)$ such that $\forall\zeta \in R^n$, $\forall\dot{\zeta} \in R^n$, $\|M_2(\zeta)\| \leqslant c_1$, $\|C_2(\zeta,\dot{\zeta})\| \leqslant c_2 + c_3\|\dot{\zeta}\|$, $\|G_2(\zeta)\| \leqslant c_4$, and $\sup_{t\geqslant 0}\|d_2(t)\| \leqslant c_5$* (Wang et al., 2004).

2.4. Uncertain holonomic constraints

Assume that the r-rigid-link manipulator is in contact with a certain constrained surface $\Phi(q)$ can be represented as: $\Phi(\chi(q)) = 0$, where $\Phi(\chi(q))$ is a given scalar function, $\chi(q) \in R^l$ denotes the position vector of the end-effector in contact with the environment. If the constraint surface is rigid and has a continuous gradient, the contact force in (9) is then given by $f_2 = J_2^{\mathrm{T}}(q)\lambda_h$, where λ_h is the magnitude of the contact force. However, when the robot end-effector is rigidly in contact with the uncertain surface, the environmental constraint could be expressed as an algebraic equation of the coordinate χ in the task space. Without loss of generality, the uncertain surface constraint $\bar{\Phi}(\chi(q))$ can be decomposed into a nominal part $\Phi(\chi(q))$ and an unknown constraint error part $\Delta\Phi(\chi(q))$ in an additive manner as follows:

$$\bar{\Phi}(\chi(q)) = \Phi(\chi(q)) + \Delta\Phi(\chi(q)), \tag{10}$$

where $\bar{\Phi}(\chi(q))$ is the constrained surface.

Denoting $\bar{J}_2$ and J_2 as the Jacobian matrix of $\bar{\Phi}(\chi(q))$ and $\Phi(\chi(q))$ with respect to q, and since $\dot{q}_v = H_v(q_v)T_{21}(q_v)u_b$, $\bar{J}_2(q) = \bar{J}_\chi[J_{2v}H_v(q_v)T_{21}(q_v), J_{2a}]$, $J_2(q) = J_\chi[J_{2v}H_v(q_v)T_{21}(q_v), J_{2a}]$, $\bar{J}_\chi = \bar{\Phi}(\chi(q))/\partial\chi$, $J_\chi = \partial\Phi(\chi(q))/\partial\chi$, $J_{2v} = \partial\chi/\partial q_v$ and $J_{2a} = \partial\chi/\partial q_a$.

Integrating (4) and (7), we have

$$J_2(q)u + \frac{\partial\Delta\Phi(\chi(q))}{\partial t} = 0. \tag{11}$$

Assume that

$$\bar{J}_2(q) = J_2(q) + \Delta J_2(q) \tag{12}$$

with $\Delta J_2(q)$ defined later. Since the uncertain constraint error (10) is introduced, integrating (12) into (9) yields

$$M_2(\zeta)\dot{u} + C_2(\zeta,\dot{\zeta})u + G_2(\zeta) + d_2 = B_2\tau + (J_2(T_1^{-1}(\zeta)) + \Delta J_2(T_1^{-1}(\zeta)))^{\mathrm{T}}\lambda_h. \tag{13}$$

Assumption 2.1. The Jacobian matrices $\bar{J}_2(q)$ is uniformly bounded and uniformly continuous if q is uniformly bounded and continuous, respectively.

Assumption 2.2. The manipulator is operating away from any singularity.

Remark 2.2. Under Assumption 2.2, the Jacobian $J_{2a}=\partial\chi/\partial q_a$ is of full row rank l, such that the joint coordinate q_a can be partitioned into $q_a = [q_{a1}^{\mathrm{T}}, q_{a2}^{\mathrm{T}}]^{\mathrm{T}}$ where $q_{a1} \in R^{n_a - l}$ and $q_{a2} \in R^l$, with $q_{a2} = \Omega(q_{a1})$ with a nonlinear mapping function $\Omega(\cdot)$ from an open set $R^{n_a-l} \times R \to R^l$. The terms $\partial\Omega/\partial q_{a1}$, $\partial^2\Omega/\partial q_{a1}^2$, $\partial\Omega/\partial t$, $\partial^2\Omega/\partial t^2$ exist and are bounded in the workspace.

Since the dimension of the constraint is l, the configuration space of the manipulator is left with $n_a - l$ degrees of freedom. Based on the full row rank for J_a, the existence of $\Omega(q_{a1})$ (You & Chen, 1993; Yuan, 1997), it is easy to obtain

$$J_2(q) = J_\chi[J_{2v}H_v(q_v)T_{21}(q_v), J_{2a1}, J_{2a2}]. \tag{14}$$

Integrating (14) into (11) and considering (14) and letting $\delta_h = \partial\Delta\Phi(\chi(q))/\partial t$, we have

$$u = \begin{bmatrix} u_b \\ \dot{q}_{a1} \\ -J_{2a2}^{-1}[J_{1v}H_v(q_v)T_{21}(q_v)u_b + J_{1a1}\dot{q}_{a1}] \\ -J_{1a2}^{-1}J_\chi^{-1}\delta_h \end{bmatrix} = Lu^1 + \varepsilon, \tag{15}$$

where

$$L = [L_v \;\; L_a]^{\mathrm{T}} = \begin{bmatrix} I_v & 0 \\ 0 & I_{a1} \\ -J_{2a2}^{-1}J_{2v}H_v(q_v)T_{21}(q_v) & -J_{2a2}^{-1}J_{2a1} \end{bmatrix}, \tag{16}$$

$$u^1 = [u_b \;\; \dot{q}_{a1}]^{\mathrm{T}}, \tag{17}$$

$$\varepsilon = \mathscr{J}\delta_h, \tag{18}$$

$$\mathscr{J} = [0 \;\; 0 \;\; -J_{2a2}^{-1}J_\chi^{-1}]^{\mathrm{T}}. \tag{19}$$

It is easy to have

$$L^{\mathrm{T}}J_2^{\mathrm{T}}(q) = 0. \tag{20}$$

Differentiating (15) and substituting it into (13), we have

$$M_3(\zeta)\dot{u}^1 + C_3u^1 + G_3(\zeta) + d_3 = B_3\tau + (J_2(T_1^{-1}(\zeta)) + \Delta J_2(T_1^{-1}(\zeta)))^{\mathrm{T}}\lambda_h, \tag{21}$$

where $M_3(\zeta) = M_2(\zeta)L$, $C_3(\zeta,\dot{\zeta}) = M_2(\zeta)\dot{L} + C_2(\zeta,\dot{\zeta})L$, $G_3(\zeta) = G_2(\zeta)$, $B_3 = B_2$, $d_3 = M_2(\zeta)\dot{\varepsilon} + C_2(\zeta,\dot{\zeta})\varepsilon + d_2$.

Assumption 2.3. The set of the constrained surface reachable by the end-effector of mobile manipulator, defined by

$$\mathscr{S} := \{\chi : \bar{\Phi}(\chi, \alpha) = 0, \alpha \in R^{l_1}\} \tag{22}$$

is bounded and belong to a class of continuously differentiable manifolds $\bar{\Phi}(\chi,\alpha) = f(\chi_1, \chi_2, \ldots, \chi_{l_1})\alpha + g(\chi_{l_1+1}, \chi_{l_1+2}, \ldots, \chi_l)\varepsilon + \varpi$ with $l_1 \leqslant l \leqslant n$ and $\chi \in R^n$, where $\alpha = [\alpha_1, \ldots, \alpha_{l_1}, 0, \ldots, 0]^{\mathrm{T}} \in R^l$ and $\varepsilon = [0, \ldots, 0, 1, \ldots, 1]^{\mathrm{T}} \in R^l$ are constant vectors, $f(*) = [f_1, \ldots, f_{l_1}, 0, \ldots, 0] \in R^{1\times l}$ and $g(*) = [0, \ldots, 0, g_1, \ldots, g_l] \in R^{1\times l}$ are bounded and uniformly continuous differentiable vectors, and ϖ is a constant scalar.

Considering Assumption 2.3, the uncertainty δ_h could be expressed with

$$\delta_h = \Delta J_2(T_1^{-1}(\zeta))\frac{\mathrm{d}}{\mathrm{d}t}(T_1^{-1}(\zeta)), \tag{23}$$

$$\Delta J_2^{\mathrm{T}} = J_\zeta^{\mathrm{T}}\rho(S_\Phi(\chi)W + C_\Phi\upsilon), \tag{24}$$

where $S_\Phi = [\partial f/\partial\chi_1, \partial f/\partial\chi_2, \ldots, \partial f/\partial\chi_{l_1}, 0, \ldots, 0]^{\mathrm{T}} \in R^{l\times l}$, $C_\Phi = [0, \ldots, 0, \partial g/\partial\chi_{l_1+1}, \partial g/\partial\chi_{l_1+2}, \ldots, \partial g/\partial\chi_l]^{\mathrm{T}} \in R^{l\times l}$, $\rho = 1/\|S_\Phi(\chi)W + C_\Phi\upsilon\|$, and $J_\zeta = \partial\chi/\partial\zeta$. From Assumption 2.3, the weight vector $W = [W_1, \ldots, W_{l_1}, 0, \ldots, 0]^{\mathrm{T}} \in R^l$ is unknown positive, and $\upsilon = [0, \ldots, 0, 1, \ldots, 1]^{\mathrm{T}} \in R^l$.

Property 2.3. *$S_\Phi C_\Phi = \mathbf{0}$ and $W^{\mathrm{T}}\upsilon = 0$.*

Define the estimated Jacobian $\Delta\hat{J}_2$ by

$$\Delta\hat{J}_2^{\mathrm{T}} = J_\zeta^{\mathrm{T}}\hat{\rho}(S_\Phi\hat{W} + C_\Phi\upsilon) \tag{25}$$

with $\hat{\rho} = 1/\|S_\Phi(\chi)\hat{W} + C_\Phi\upsilon\|$. Consider Property 2.3, the error in Jacobian matrix $\Delta\tilde{J}_2 = \Delta\hat{J}_2 - \Delta J_2$ can be expressed by

$$\Delta\tilde{J}_2^{\mathrm{T}} = J_\zeta^{\mathrm{T}}(S_\Phi(\hat{\rho}\hat{W} - \rho W) + \tilde{\rho}C_\Phi\upsilon), \tag{26}$$

where $\tilde{\rho} = \hat{\rho} - \rho$. Consider (26), the force error can be expressed as

$$e_f = J_\zeta^{+\mathrm{T}} J_2^{\mathrm{T}}\lambda_h - F = J_\zeta^{+\mathrm{T}}\Delta\tilde{J}_2^{\mathrm{T}}\lambda_h, \tag{27}$$

where $J_\zeta^{+\mathrm{T}} = J_\zeta(J_\zeta^{\mathrm{T}}J_\zeta)^{-1}$ and the force error e_f can be calculated from $J_\zeta^{+\mathrm{T}}J_2^{\mathrm{T}}\lambda_h$ and F from force sensor.

Assumption 2.4. There exist some finite non-negative known constants $b_{\delta1}$ and $b_{\delta2}$, such that, $\forall\chi \in \Omega_\chi$, $\|\delta_h\| \leqslant b_{\delta1}\|(\mathrm{d}/\mathrm{d}t)T_1^{-1}(\zeta)\|$, $\|\dot{\delta}_h\| \leqslant b_{\delta1}\|(\mathrm{d}^2/\mathrm{d}t^2)T_1^{-1}(\zeta)\| + b_{\delta2}\|(\mathrm{d}/\mathrm{d}t)T_1^{-1}(\zeta)\|$.

3. Adaptive control

Given a desired motion trajectory q_d and a desired constraint force, or, equivalently, a desired multiplier $\lambda_h^d(t)$ should satisfy the constrained equations. Since the desired trajectory q_d should satisfy Eq. (6), we can have the desired ζ_d. After the transformation for the chained form through $\zeta_d = T_1(q_d)$ and $\upsilon_d = T_2(q_d)u_d$, we can obtain ζ_d and u_d and, equivalently, u_d^1. The trajectory and force tracking control can be restated as seeking a strategy for specifying a control law subjected to the uncertain holonomic constraint, such that $\{\lambda_h, \zeta, \dot{\zeta}\} \to \{\lambda_h^d, \zeta_d, \dot{\zeta}_d\}$.

Assumption 3.1. The desired reference trajectory $\zeta_d(t)$ is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the $(n-1)$th order. The desired Lagrangian multiplier λ_h^d is also bounded and uniformly continuous.

Assumption 3.2. Time varying positive functions ϱ_i and h_i converge to zero as $t \to \infty$ and satisfy $\lim_{t\to\infty}\int_0^t \varrho_i(\omega)\,\mathrm{d}\omega = a_i < \infty$, $\lim_{t\to\infty}\int_0^t h_i(\omega)\,\mathrm{d}\omega = b_i < \infty$ with finite constants a_i and b_i, $i = 1, \ldots, 5$.

There are many choices for ϱ_i and h_i that satisfy the Assumption 3.2, for example, $\varrho_i = h_i = 1/(1+t)^2$. Denote the tracking errors as $e = \zeta - \zeta_d$ and $e_\lambda = \lambda_h - \lambda_h^d$, and define the reference signals

$$u_d^1 = [u_{bd}^{1\mathrm{T}}, u_{ad}^{1\mathrm{T}}]^{\mathrm{T}},$$

$$u_{bd}^1 = \begin{bmatrix} u_{d1} + \eta \\ u_{d2} - s_{n_v-1}u_{d1} - k_{n_v}s_{n_v} \\ +\sum_{i=0}^{n_v-3}\dfrac{\partial(e_{n_v} - s_{n_v})}{\partial u_{d1}^{(i)}}u_{d1}^{(i+1)} \\ +\sum_{i=2}^{n_v-1}\dfrac{\partial(e_{n_v} - s_{n_v})}{\partial e_i}e_{i+1} \end{bmatrix},$$

$$u_{ad}^1 = \dot{q}_{a1d} - K_a(q_{a1} - q_{a1d}), \tag{28}$$

where

$$s = \begin{bmatrix} e_1 \\ e_2 \\ e_3 + k_2e_2u_{d1}^{2p-1} \\ e_4 + s_2 + k_3s_3u_{d1}^{2p-1} \\ -\dfrac{1}{u_{d1}}\sum_{i=0}^{0}\dfrac{\partial(e_3 - s_3)}{\partial u_{d1}^{(i)}}u_{d1}^{(i+1)} - \sum_{i=2}^{2}\dfrac{\partial(e_3 - s_3)}{\partial e_i}e_{i+1} \\ \vdots \\ e_{n_v} + s_{n_v-2} + k_{n_v-1}s_{n_v-1}u_{d1}^{2p-1} \\ -\frac{1}{u_{d1}}\sum_{i=0}^{n_v-4}\frac{\partial(e_{n_v-1} - s_{n_v-1})}{\partial u_{d1}^{i}}u_{d1}^{(i+1)} \\ -\sum_{i=2}^{n_v-2}\frac{\partial(e_{n_v-1} - s_{n_v-1})}{\partial e_i}e_{i+1} \end{bmatrix},$$

$$\dot{\eta} = -k_0\eta - k_1s_1 - \sum_{i=2}^{n_v-1}s_i\zeta_{i+1} + \sum_{j=3}^{n_v}s_j\sum_{i=2}^{j-1}\frac{\partial(e_j - s_j)}{\partial e_i}\zeta_{i+1},$$

$p = n_v - 2$, $u_{d1}^{(i)}$ is the ith derivative of u_{d1} with respect to t, and k_i is positive constant, $K_a \in R^{(n_a-l)\times(n_a-l)}$ is diagonal positive.

Define new variables to handle the force control

$$\dot{\vartheta} = -K_\vartheta\vartheta - K_\vartheta J_2^{\mathrm{T}}e_\lambda, \tag{29}$$

where $\vartheta = [0, \vartheta_1]$ with $\vartheta_1 \in R^{n_a}$, $e_\lambda = \lambda_h - \lambda_h^d$, and $K_\vartheta = \mathrm{diag}[0, k_{\vartheta i}] > 0$. Defining the following auxiliary signals as $\tilde{u}^1 = u^1 - u_d^1 = [\tilde{u}_1, \tilde{u}_2, \tilde{u}_{a1}]^{\mathrm{T}}$, $\dot{\mu} = \tilde{u}^1$ and $u_r = u_d^1 - K_u\mu$, we have

$$r = \dot{\mu} + K_u\mu, \tag{30}$$

$$\sigma = Lr + \vartheta, \tag{31}$$

$$v = Lu_r - \vartheta, \tag{32}$$

where $K_u = \mathrm{diag}[0, K_{u1}] > 0$ and $K_{u1} \in R^{(n_a-l)\times(n_a-l)}$. From (30) and the definitions of $\tilde{u}^1$, u_r and $\dot{\mu}$ above, we have

$$u^1 = u_r + r. \tag{33}$$

The time derivatives of σ and v are given by

$$\dot{\sigma} = \dot{L}u^1 + L\dot{u}^1 - \dot{v}, \tag{34}$$

$$\dot{v} = \dot{L}u_r + L\dot{u}_r - \dot{\vartheta}. \tag{35}$$

From (31) to (33) we have

$$\sigma + v = Lu^1. \tag{36}$$

From the dynamics (21) together with (34)–(36), we have

$$M_2(\zeta)\dot{\sigma} + C_2(\zeta,\dot{\zeta})\sigma + M_2(\zeta)\dot{v} + C_2(\zeta,\dot{\zeta})v + G_2(\zeta) + d_3(t) = B_2(\zeta)\tau + (J_2^{\mathrm{T}} + \Delta J_2^{\mathrm{T}})\lambda_h. \tag{37}$$

Consider the control law given by

$$B_2\tau = -\sum_{i=1}^{5}\frac{\hat{c}_i\beta_i^2\|\sigma\|^2Y_i^2(v,\dot{v})}{\beta_iY_i(v,\dot{v})\|\sigma\| + \varrho_i} - K_\sigma\sigma - \Lambda - (J_2^{\mathrm{T}} + \Delta\hat{J}_2^{\mathrm{T}})\lambda_h^d + (J_2^{\mathrm{T}} + \Delta\hat{J}_2^{\mathrm{T}})K_\lambda(\lambda_h - \lambda_h^d) - \tau_h, \tag{38}$$

$$\Lambda = [\Lambda_1 \;\; 0]^{\mathrm{T}}, \tag{39}$$

$$\Lambda_1 = \begin{bmatrix} k_1s_1 + \sum_{i=2}^{n_v-1} s_i\zeta_{i+1} \\ -\sum_{j=3}^{n_v} s_j\sum_{i=2}^{j-1}\frac{\partial(e_j - s_j)}{\partial e_i}\zeta_{i+1} \\ s_{n_v}\end{bmatrix},$$

$$\tau_h = \frac{\|J_\zeta\|}{2\|C_\Phi\upsilon\|}\mathrm{sgn}(\sigma)(\|S_\Phi\|^2\hat{\omega} + \|C_\Phi\upsilon\|^2) + \frac{\|J_\zeta\|}{2\|C_\Phi\upsilon\|}\mathrm{sgn}(\sigma)\|e_f\|^2 + (K_\lambda + I)\Delta\hat{J}_2^{\mathrm{T}}(\lambda_h - \lambda_h^d), \tag{40}$$

$$\mathrm{sgn}(\sigma) = \begin{cases} 1 & \text{if } \sigma \geqslant 0,\\ -1 & \text{if } \sigma < 0\end{cases}$$

and the adaptive laws

$$\dot{\hat{W}} = \hat{\rho}\lambda_h S_\Phi^{\mathrm{T}}J_\zeta\sigma, \tag{41}$$

$$\dot{\hat{\omega}} = \frac{1}{2\|C_\Phi\upsilon\|}\|\sigma\|\|J_\zeta\|\|S_\Phi\|^2, \tag{42}$$

$$\dot{\hat{c}}_i = -h_i\hat{c}_i + \frac{\gamma_i\beta_i^2Y_i^2\|\sigma\|^2}{\beta_iY_i(v,\dot{v})\|\sigma\| + \varrho_i} \quad (i = 1,\ldots,5), \tag{43}$$

where

$$Y_1(v,\dot{v}) = \|\dot{v}\| + \left(b_{\delta1}\left\|\frac{\mathrm{d}}{\mathrm{d}t}\mathscr{J}\right\| + b_{\delta2}\|\mathscr{J}\|\right)\left\|\frac{\mathrm{d}}{\mathrm{d}t}T_1^{-1}(\zeta)\right\| + b_{\delta1}\|\mathscr{J}\|\left\|\frac{\mathrm{d}^2}{\mathrm{d}t^2}T_1^{-1}(\zeta)\right\|,$$

$$Y_2(v,\dot{v}) = \|v\| + b_{\delta1}\|\mathscr{J}\|\left\|\frac{\mathrm{d}}{\mathrm{d}t}T_1^{-1}(\zeta)\right\|,$$

$$Y_3(v,\dot{v}) = \|\dot{\zeta}\|\left(\|v\| + b_{\delta1}\|\mathscr{J}\|\left\|\frac{\mathrm{d}}{\mathrm{d}t}T_1^{-1}(\zeta)\right\|\right),$$

$$Y_4(v,\dot{v}) = Y_5(v,\dot{v}) = 1.$$

K_σ and K_λ are positive definite matrices, $\beta_i > 0$ and $\gamma_i > 0$ are constant. From the dynamic equation (37) together with (38), the closed-loop system dynamics can be written as

$$\dot{s}_1 = \eta + \tilde{u}_1, \tag{44}$$

$$\dot{s}_2 = (\eta + \tilde{u}_1)\zeta_3 + s_3u_{d1} - k_2s_2u_{d1}^{2p}, \tag{45}$$

$$\dot{s}_3 = (\eta + \tilde{u}_1)\left(\zeta_4 - \frac{\partial(e_3 - s_3)}{\partial e_2}\zeta_3\right) + s_4u_{d1} - s_2u_{d1} - k_3s_3u_{d1}^{2p} \quad \vdots \tag{46}$$

$$\dot{s}_{n_v-1} = (\eta + \tilde{u}_1)\left(\zeta_{n_v} - \sum_{i=2}^{n_v-2}\frac{\partial(e_{n_v-1} - s_{n_v-1})}{\partial e_i}\zeta_{i+1}\right) + s_{n_v}u_{d1} - s_{n_v-2}u_{d1} - k_{n_v-1}s_{n_v-1}u_{d1}^{2p}, \tag{47}$$

$$\dot{s}_{n_v} = (\eta + \tilde{u}_1)\sum_{i=2}^{n_v-2}\frac{\partial(e_{n_v} - s_{n_v})}{\partial e_i}\zeta_{i+1} - k_{n_v}s_{n_v} - s_{n_v-1}u_{d1} + \tilde{u}_2, \tag{48}$$

$$\dot{\eta} = -k_0\eta - \Lambda_1, \tag{49}$$

$$M_2\dot{\sigma} = -C_2\sigma - \xi - K_\sigma\sigma - \sum_{i=1}^{5}\frac{\hat{c}_i\beta_i^2\|\sigma\|^2Y_i^2(v,\dot{v})}{\beta_iY_i(v,\dot{v})\|\sigma\| + \varrho_i} - J_2^{\mathrm{T}}(\lambda_h^d - K_\lambda e_\lambda) + J_2^{\mathrm{T}}\lambda_h - \Delta\tilde{J}_2^{\mathrm{T}}\lambda_h + \Delta\hat{J}_2^{\mathrm{T}}(K_\lambda + I)e_\lambda - \tau_h - \Lambda, \tag{50}$$

where $\Upsilon = M_2\dot{v} + C_2v + G_2 + d_3$.

Theorem 3.1. *Consider the mechanical system described by* (1), (2), *and* (10). *Under Assumption* 3.1, *the control law* (38) *leads to*: (i) $\zeta, \dot{\zeta}, \lambda_h$ *converge to* $\zeta_d, \dot{\zeta}_d, \lambda_h^d$ *at* $t \to \infty$; *and* (ii) *all the signals in the closed-loop are bounded for all* $t \geqslant 0$.

Proof. Consider the Lyapunov function candidate:

$$V(t) = \frac{1}{2}\sum_{i=2}^{n_v}s_i^2 + \frac{1}{2}k_1s_1^2 + \frac{1}{2}\eta^2 + \frac{1}{2}\sigma^{\mathrm{T}}M_2\sigma + \sum_{i=1}^{5}\frac{1}{2\gamma_i}\tilde{c}_i^2 + \frac{1}{2}\tilde{W}^{\mathrm{T}}\tilde{W} + \frac{1}{2}\vartheta^{\mathrm{T}}(I + K_\lambda)K_\vartheta^{-1}\vartheta + \frac{1}{2}(\|W\|^2 - \hat{\omega})^2, \tag{51}$$

where $\tilde{W} = \hat{W} - W$, $\tilde{c}_i = \hat{c}_i - c_i$, we have $\dot{\tilde{W}} = \dot{\hat{W}}$, $\dot{\tilde{c}}_i = \dot{\hat{c}}_i$. Considering Property 2.1 and integrating (43) and (50) into the derivative of V yield

$$\dot V \leqslant -\sum_{i=2}^{n_v-1} k_i s_i^2 u_{d1}^{2l} - k_{n_v} s_{n_v}^2 - k_0\eta^2 + \tilde u^{1\mathrm T}\Lambda - \sigma^{\mathrm T}K_\sigma\sigma$$
$$-\sum_{i=1}^{5}\frac{\hat c_i\beta_i^2\|\sigma\|^2Y_i^2(v,\dot v)}{\beta_iY_i(v,\dot v)\|\sigma\|+\varrho_i}+\|\sigma\|\|\Upsilon\|$$
$$-\sigma^{\mathrm T}J_2^{\mathrm T}(\lambda_h^d-K_\lambda e_\lambda)+\sigma^{\mathrm T}J_2^{\mathrm T}\lambda_h$$
$$-\sigma^{\mathrm T}\Delta\tilde J_2^{\mathrm T}\lambda_h+\sigma^{\mathrm T}\Delta\hat J_2^{\mathrm T}(K_\lambda+I)e_\lambda-\sigma^{\mathrm T}\tau_h-\sigma^{\mathrm T}\Lambda$$
$$-\sum_{i=1}^{5}\frac{h_i\hat c_i\tilde c_i}{\gamma_i}+\sum_{i=1}^{5}\frac{\tilde c_i\beta_i^2\|\sigma\|^2Y_i^2(v,\dot v)}{\beta_iY_i(v,\dot v)\|\sigma\|+\varrho_i}+\dot{\hat W}^{\mathrm T}\tilde W$$
$$+\vartheta^{\mathrm T}(I+K_\lambda)K_\vartheta^{-1}\dot\vartheta-(\|W\|^2-\hat\omega)\dot{\hat\omega}. \quad (52)$$

From (26) and (27), we can obtain

$$e_f = J_\zeta^{+\mathrm T}\Delta\tilde J_2^{\mathrm T}\lambda_h$$
$$= J_\zeta^{+\mathrm T}J_\zeta^{\mathrm T}S_\Phi(\hat\rho\hat W-\rho W)\lambda_h + J_\zeta^{+}J_\zeta^{\mathrm T}\tilde\rho C_\Phi\upsilon\lambda_h. \quad (53)$$

Considering Property 2.3, we have

$$\|e_f\|^2=\|J_\zeta^{+\mathrm T}J_\zeta^{\mathrm T}\|^2\|S_\Phi(\hat\rho\hat W-\rho W)\|^2\|\lambda_h\|^2$$
$$+\|J_\zeta^{+\mathrm T}J_\zeta^{\mathrm T}\|^2\|C_\Phi\upsilon\|^2\|\tilde\rho\lambda_h\|. \quad (54)$$

Therefore, we obtain

$$\|\tilde\rho\lambda_h\|\leqslant\|e_f\|/\|C_\Phi\upsilon\|. \quad (55)$$

Considering (26), (53) and using the adaptive parameter law (41), rewriting the 10th, the 11th and the 16th right-hand terms in (52), we have

$$-\sigma^{\mathrm T}\Delta\tilde J_2^{\mathrm T}\lambda_h+\dot{\hat W}^{\mathrm T}\tilde W+\sigma^{\mathrm T}\Delta\hat J^{\mathrm T}(k_\lambda+I)e_\lambda$$
$$=-\sigma^{\mathrm T}J_\zeta^{\mathrm T}\tilde\rho(S_\Phi W+C_\Phi\upsilon)\lambda_h+\sigma^{\mathrm T}\psi, \quad (56)$$

where $\psi=\Delta\hat J_2^{\mathrm T}(K_\lambda+I)e_\lambda$. From (55) and (56), we have

$$-\sigma^{\mathrm T}J_\zeta^{\mathrm T}\lambda_h\tilde\rho(S_\Phi W+C_\Phi\upsilon)$$
$$\leqslant\frac{\|\sigma\|\|J_\zeta\|}{2\|C_\Phi\upsilon\|}(\|e_f\|^2+\|S_\Phi\|^2\|W\|^2+\|C_\Phi\upsilon\|^2) \quad (57)$$

by noting Property 2.3. Moreover

$$\|\sigma\|\|\Upsilon\|-\sum_{i=1}^{5}\frac{\hat c_i\beta_i^2\|\sigma\|^2Y_i^2(v,\dot v)}{\beta_iY_i(v,\dot v)\|\sigma\|+\varrho_i}-\sum_{i=1}^{5}\frac{h_i\hat c_i\tilde c_i}{\gamma_i}$$
$$+\sum_{i=1}^{5}\frac{\tilde c_i\beta_i^2\|\sigma\|^2Y_i^2(v,\dot v)}{\beta_iY_i(v,\dot v)\|\sigma\|+\varrho_i}\leqslant\sum_{i=1}^{5}\left(\frac{h_i}{4\gamma_i}c_i^2+c_i\varrho_i\right). \quad (58)$$

In addition, from (29) and (31), $\sigma^{\mathrm T}=r^{\mathrm T}L^{\mathrm T}+\vartheta^{\mathrm T}$. Thus, we have

$$-\sigma^{\mathrm T}J_2^{\mathrm T}(\lambda_h^d-K_\lambda e_\lambda)+\sigma^{\mathrm T}J_2^{\mathrm T}\lambda_h+\vartheta^{\mathrm T}(I+K_\lambda)K_\vartheta^{-1}\dot\vartheta$$
$$=-\vartheta^{\mathrm T}(I+K_\lambda)\vartheta+r^{\mathrm T}L^{\mathrm T}J_2^{\mathrm T}(I+K_\lambda)e_\lambda \quad (59)$$

by noting $L^{\mathrm T}J_2^{\mathrm T}=0$ from (20).

From (41), (57)–(59), it can be shown that

$$\dot V\leqslant -k_0\eta^2-k_{n_v}s_{n_v}-\sum_{i=2}^{n_v-1}k_is_i^2u_{d1}^{2l}+\tilde u^{1\mathrm T}\Lambda-\sigma^{\mathrm T}K_\sigma\sigma$$
$$+\sum_{i=1}^{5}\left(\frac{h_i}{4\gamma_i}c_i^2+c_i\varrho_i\right)-\vartheta^{\mathrm T}(K_\lambda+I)\vartheta+\frac{\|\sigma\|\|J_\zeta\|}{2\|C_\Phi\upsilon\|}\|e_f\|^2$$
$$+\frac{\|\sigma\|\|J_\zeta\|}{2\|C_\Phi\upsilon\|}(\|S_\Phi\|^2\|W\|^2+\|C_\Phi\upsilon\|^2)+\sigma^{\mathrm T}\psi-\sigma^{\mathrm T}\tau_h$$
$$-\sigma^{\mathrm T}\Lambda-(\|W\|^2-\hat\omega)\dot{\hat\omega}-\vartheta^{\mathrm T}(I+K_\lambda)\vartheta. \quad (60)$$

From (16), (29), and (39), considering the fourth and the 12th right-hand terms in (60), we have

$$\tilde u^{1\mathrm T}\Lambda-\sigma^{\mathrm T}\Lambda=\tilde u^{1\mathrm T}\Lambda-(r^{\mathrm T}L^{\mathrm T}\Lambda+\vartheta^{\mathrm T}\Lambda)$$
$$=\tilde u^{1\mathrm T}\begin{bmatrix}\Lambda_1\\0\end{bmatrix}-r^{\mathrm T}[L_v^{\mathrm T}\ \ L_a^{\mathrm T}]\begin{bmatrix}\Lambda_1\\0\end{bmatrix}-[0\ \ \vartheta_1]\begin{bmatrix}\Lambda_1\\0\end{bmatrix}.$$

From (16) and (30), we have $L_v=[I_v,0]$, and $K_u=\mathrm{diag}[0,K_{u1}]$, $r^{\mathrm T}[L_v^{\mathrm T}\ \ L_a^{\mathrm T}][\Lambda_1\ \ 0]^{\mathrm T}=\tilde u^{1\mathrm T}\Lambda_1$, subsequently we obtain

$$\tilde u^{1T}\Lambda-\sigma^{\mathrm T}\Lambda=0. \quad (61)$$

Integrating (61), one obtains

$$\dot V\leqslant -k_0\eta^2-k_{n_v}s_{n_v}-\sum_{i=2}^{n_v-1}k_is_i^2u_{d1}^{2l}-\sigma^{\mathrm T}K_\sigma\sigma$$
$$+\sum_{i=1}^{5}\left(\frac{h_i}{4\gamma_i}c_i^2+c_i\varrho_i\right)-\vartheta^{\mathrm T}(I+K_\lambda)\vartheta$$
$$+\frac{\|\sigma\|\|J_\zeta\|\|S_\Phi\|^2}{2\|C_\Phi\upsilon\|}(\|W\|^2-\hat\omega)-(\|W\|^2-\hat\omega)\dot{\hat\omega}. \quad (62)$$

Considering the parameter $\hat\omega$ update law (42), it results that $\dot V\leqslant -k_0\eta^2-k_{n_v}s_{n_v}-\sum_{i=2}^{n_v-1}k_is_i^2u_{d1}^{2l}-\sigma^{\mathrm T}K_\sigma\sigma+\sum_{i=1}^{5}((h_i/4\gamma_i)c_i^2+c_i\varrho_i)-\vartheta^{\mathrm T}(I+K_\lambda)\vartheta$. Noting Assumption 3.2, we have $\sum_{i=1}^{5}((h_i/4\gamma_i)c_i^2+c_i\varrho_i)\to 0$ as $t\to\infty$. Integrating both sides of the above equation gives $V(t)-V(0)<-\int_0^t(k_0\eta^2+k_{n_v}s_{n_v}+\sum_{i=2}^{n_v-1}k_is_i^2u_{d1}^{2l}+\sigma^{\mathrm T}K_\sigma\sigma+\vartheta^{\mathrm T}(I+K_\lambda)\vartheta)\,\mathrm ds+\sum_{i=1}^{5}((a_i/4\gamma_i)c_i^2+c_ib_i)<\infty$. Thus, $V(t)<V(0)+\sum_{i=1}^{5}((a_i/4\gamma_i)c_i^2+c_ib_i)$, therefore $V(t)$ is bounded, which implies that $\eta, s_i, \sigma, \hat c_i, \hat W, \vartheta$ and $\hat\omega$ are bounded. From the definition of s_i in (29), it can be concluded that $[e_1,e_2,\ldots,e_{n_v}]^{\mathrm T}$ is bounded, which follows that η is bounded. Since σ is bounded, we can obtain $r,\tilde u^1\in L_2^{n-l}$ from (31), therefore, $q_{a1}-q_{a1d}$ and $\dot q_{a1}-\dot q_{a1d}$ are bounded, which follows that q_{a1} is bounded. Since ϑ is bounded, from (29), we have e_λ is bounded. Therefore, it is concluded that $s_iu_{d1}, s_{n_v}, \eta\in L_2$, it can be shown that $s_iu_{d1}\to 0, s_{n_v}\to 0, \eta\to 0$ as $t\to 0$, respectively. It is further concluded that $s_i\to 0$ as $t\to 0$. Differentiating $u_{d1}^p\eta$ yields $(\mathrm d/\mathrm dt)u_{d1}^p\eta=-k_1u_{d1}^ps_1+lu_{d1}^{p-1}\dot u_{d1}^p\eta-k_0u_{d1}^p\eta-u_{d1}^l\{\sum_{i=2}^{n_v-1}s_i\zeta_{i+1}-\sum_{j=3}^{n_v}s_j\sum_{i=2}^{j-1}(\partial(e_j-s_j)/\partial e_i)\zeta_{i+1}\}$, where the first term is uniformly continuous and the other terms converges to zero. Since $(\mathrm d/\mathrm dt)u_{d1}^p\eta$ converge to zero, s and $\dot s$

also tend to zero. It is obvious that $s_i = 0$, yields that $\xi_i \to \xi_{di}$ and $\dot{\xi}_i \to \dot{\xi}_{di}$ as $t \to \infty$. Since σ, $\dot{\sigma}$, d_3, $\Delta\tilde{J}_2$, e_λ and τ_h are all bounded, it can be concluded that τ is bounded from (38). $\square$

4. Simulation studies

Consider a 3-DOF robotic manipulator with two revolute joints and one prismatic joint mounted on two wheeled mobile platform shown in Fig. 1, which is subjected to the following constraints: $\dot{x}\cos\theta + \dot{y}\sin\theta = 0$. Using the Lagrangian approach, we can obtain the standard form (1) with $q_v = [x, y, \theta]^{\mathrm{T}}$, $q_a = [\theta_1, \theta_2, \theta_3]^{\mathrm{T}}$, $\theta_2 = \pi/2$, $q = [q_v, q_a]^{\mathrm{T}}$, and $A_v = [\cos\theta, \sin\theta, 0]^{\mathrm{T}}$, and

$$M_v = \begin{bmatrix} M_{v11} & M_{v12} \\ M_{v21} & M_{v22} \end{bmatrix}, \quad C_v = \begin{bmatrix} C_{v11} & C_{v12} \\ C_{v21} & C_{v22} \end{bmatrix},$$

$$B_v = \begin{bmatrix} \sin\theta/r & -\cos\theta/r & -l/r \\ -\sin\theta/r & \cos\theta/r & l/r \end{bmatrix}^{\mathrm{T}},$$

$M_{v12} = [m_{123}d\cos\theta + m_{23}\cos(\theta+\theta_1), m_{123}d\sin\theta + m_{23}\sin(\theta+\theta_1)]^{\mathrm{T}}$, $M_{v11} = \mathrm{diag}[m_{p123}]$, $m_{23} = m_2 l_2 + m_3 L_3$, $L_3 = 2l_2 + l_3 + \theta_3$, $M_{v22} = I_p + I_{123} + m_{123}d^2 + m_2(l_2^2 + 2dl_2\cos\theta_1) + m_3(L_3^2 + 2dL_3\cos\theta_1)$, $M_{va} = [M_{va1}, M_{va2}, M_{va3}]$, $M_{va1} = [m_{23}\cos(\theta+\theta_1), m_{23}\sin(\theta+\theta_1), I_{123} + m_2(l_2^2 + 2dl_2\cos\theta_1) + m_3(L_3^2 + 2dL_3\cos\theta_1)]^{\mathrm{T}}$, $M_{va2} = 0$, $M_{va3} = [\sin(\theta+\theta_1), -\cos(\theta+\theta_1), 0]^{\mathrm{T}}$, $B_a = \mathrm{diag}[1.0]$, $M_a = \mathrm{diag}[I_{123}, I_{23}, m_3]$, $\tau = [\tau_l, \tau_r, \tau_1, \tau_2, \tau_3]^{\mathrm{T}}$, $G_v = [0.0, 0.0, 0.0]^{\mathrm{T}}$, $m_{p123} = m_p + m_{123}$, $m_{123} = m_1 + m_2 + m_3$, $I_{123} = I_1 + I_2 + I_3 + m_3 L_3^2$, $I_{23} = I_2 + I_3 + m_3 L_3^2$, $C_{v11} = 0$, $C_{v12} = C_{v21}^{\mathrm{T}}$, $C_{v22} = -2m_{23}d\sin\theta_1\dot{\theta}_1$, $C_a = \mathrm{diag}[-m_{23}d\sin\theta_1\dot{\theta}, -m_{23}d\sin\theta_1\dot{\theta}, 0]$, $C_{v12} = [-m_{123}d\dot{\theta}\sin\theta - m_{23}\sin(\theta+\theta_1)(\dot{\theta}+\dot{\theta}_1), m_{123}d\dot{\theta}\cos\theta + m_{23}\cos(\theta+\theta_1)(\dot{\theta}+\dot{\theta}_1)]^{\mathrm{T}}$, $G_a = [0.0, m_2 g l_2, m_3 g L_3]^{\mathrm{T}}$, $C_{va} = [C_{va1}, C_{va2}, C_{va3}]$, $C_{va1} = C_{va2} = [-m_{23}\sin(\theta+\theta_1)(\dot{\theta}+\dot{\theta}_1), -m_{23}\sin\cos(\theta+\theta_1)(\dot{\theta}+\dot{\theta}_1), 0]^{\mathrm{T}}$, $C_{va3} = [-m_3\cos(\theta+\theta_1)(\dot{\theta}+\dot{\theta}_1), -m_3\sin\cos(\theta+\theta_1)(\dot{\theta}+\dot{\theta}_1), 0]^{\mathrm{T}}$, $C_{av1} = C_{va1}^{\mathrm{T}}$, $C_{av2} = C_{va2}^{\mathrm{T}}$, $C_{av3} = [m_3\cos(\theta+\theta_1)(\dot{\theta}+\dot{\theta}_1), m_3\sin(\theta+\theta_1)(\dot{\theta}+\dot{\theta}_1), m_3 d\sin\theta\dot{\theta}_1]$.

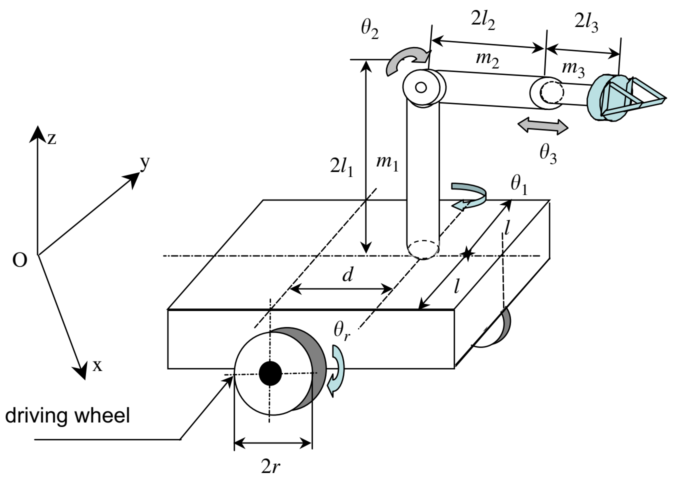

Fig. 1. 3-DOF robotic manipulator mounted on a mobile platform.

Remark 4.1. In such case that 3-DOF mobile manipulator consists of two revolute joints and one prismatic joint, $\forall\zeta \in R^n$, $\forall\dot{\zeta} \in R^n$, $\|M_2(\zeta)\| \leqslant k_{m1} + k_{m2}\|\zeta_6\|^2$ with k_{m1} and $k_{m2} > 0$, $\|G_2(\zeta)\| \leqslant k_{g1} + k_{g2}\|\zeta_6\|$ with k_{g1} and $k_{g2} > 0$, where $\zeta_6 = \theta_3$, if the boundedness of ζ_6 is known, there still exist some finite non-negative constants $c_i \geqslant 0$ $(i = 1, \ldots, 4)$, therefore Property 2.2 holds.

Remark 4.2. The existence of sign-function in the controller (46) may inevitably lead to chattering in control torques. To avoid chattering, a sat-function can be used to replace the sign-function (Slotine & Li, 1991).

Given the desired trajectory $q_d = [x_d, y_d, \theta_d, \theta_{1d}, \theta_{2d}]^{\mathrm{T}}$ with $x_d = 2.0\cos(t)$, $y_d = 2.0\sin(t)$, $\theta_d = t$, $\theta_{1d} = \pi/2$ rad, $\theta_{2d} = \pi/2$ rad and the geometric constraint which the end-effector subjected to: $\Phi = \alpha(x^2 + y^2) + z - c = 0$ with $c = 2.25$ m, and $\lambda_d = 10.0$ N, the desired value of the parameter α is 1.0, and the joint 3 is redundant prismatic joint used to compensate the position errors caused by uncertain holonomic constraints. Assume that $\theta_3 \in [0.0\,\mathrm{m}, 0.3\,\mathrm{m}]$. The transformation $T_1(q)$ is defined as $\zeta_1 = \theta$, $\zeta_2 = x\cos\theta + y\sin\theta$, $\zeta_3 = -x\sin\theta + y\cos\theta$, $\zeta_4 = \theta_1$, $\zeta_5 = \theta_2$, $\zeta_6 = \theta_3$, $u_1 = v_{b2}$, $u_2 = v_{b1} - (x\cos\theta + y\sin\theta)v_{b2}$, $u_3 = \dot{\zeta}_3$, $u_4 = \dot{\zeta}_4$, $u_5 = \dot{\zeta}_5$, one can obtain the kinematic system in the chained form as $\dot{\zeta}_1 = u_1$, $\dot{\zeta}_2 = \zeta_3 u_1$, $\dot{\zeta}_3 = u_2$, $\dot{\zeta}_4 = u_3$, $\dot{\zeta}_5 = u_4$, $\dot{\zeta}_6 = u_5$. Using the above diffeomorphism

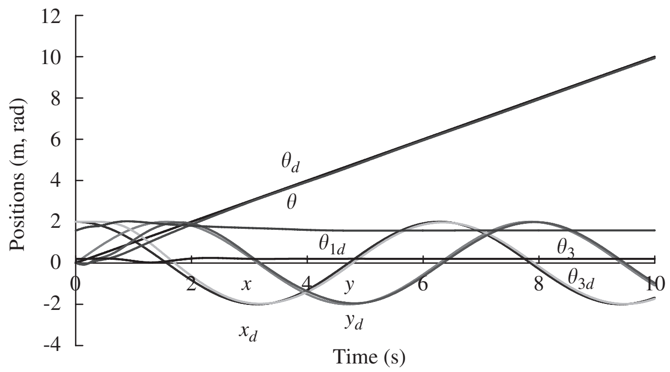

Fig. 2. Positions tracking.

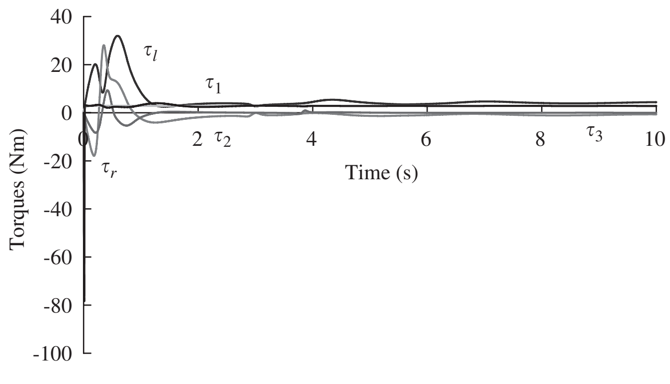

Fig. 3. Torques of joints.

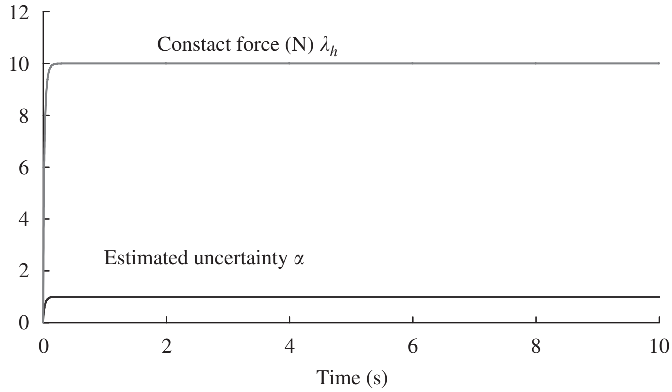

Fig. 4. The contact force (N) and estimated α.

transformation, we can obtain $\zeta_{d1} = t$, $\zeta_{d2} = 2.0$, $\zeta_{d3} = 0.0$, $\zeta_{d4} = \pi/2$, $\zeta_{d5} = \pi/2$, $\lambda_h^d = 10.0\,\mathrm{N}$ with $u_{d1} = 1.0$, $u_{d2} = 0.0$, $u_{d3} = 0.0$, $u_{d4} = 0.0$. In the simulation, $m_p = 5.0\,\mathrm{kg}$, $m_1 = 1.0\,\mathrm{kg}$, $m_2 = m_3 = 0.5\,\mathrm{kg}$, $I_p = 2.5\,\mathrm{kg\,m^2}$, $I_1 = 1.0\,\mathrm{kg\,m^2}$, $I_2 = 0.5\,\mathrm{kg\,m^2}$, $I_3 = 0.5 + m_3\theta_3^2\,\mathrm{kg\,m^2}$, $d = l = r = 0.5\,\mathrm{m}$, $2l_1 = 1.0\,\mathrm{m}$, $2l_2 = 0.3\,\mathrm{m}$. The initial condition select $x(0) = 2.0\,\mathrm{m}$, $y(0) = 0.0\,\mathrm{m}$, $\theta(0) = \pi/2\,\mathrm{rad}$, $\theta_1(0) = \pi/2\,\mathrm{rad}$, $\theta_2(0) = \pi/2\,\mathrm{rad}$, $\theta_3(0) = 0.1\,\mathrm{m}$, $\lambda(0) = 0.0\,\mathrm{N}$ and $\dot{x}(0) = 0.5\,\mathrm{m/s}$, $\dot{y}(0) = \dot{\theta}(0) = \dot{\theta}_1(0) = \dot{\theta}_2(0) = \dot{\theta}_3(0) = 0.0$, $\alpha(0) = 0.1$, $\hat{c}_i(0) = 1.0$, $i = 1, \ldots, 5$. In the simulation, the design parameters are set as $b_{\delta 1} = b_{\delta 2} = 1.0$, $k_0 = 30.0$, $k_1 = 200.0$, $k_2 = 1.0$, $k_3 = 1.0$, $K_\vartheta = \mathrm{diag}[0, 0.01]$, $K_\lambda = 0.5$, $\eta(0) = 0$, $K_\sigma = \mathrm{diag}[1.0]$, $K_a = \mathrm{diag}[1.0]$, the adaptive gains in the adaptive laws are chosen as $\gamma_i = 0.1$, $\beta_i = 1.0$, $h_i = \varrho_i = 1/(1+t)^2$. The disturbances on the mobile base are set to a time varying form as $0.5\sin(t)$ and $0.5\cos(t)$. The control results are shown in Figs. 2–4. Fig. 2 show the trajectory tracking $(q - q_d)$ with the disturbances, and the corresponding torques are shown in Fig. 3. Fig. 4 shows the contact force tracking $\lambda_h - \lambda_h^d$ and the evolution of α. The simulation results show that the position tracking error converges to zero, the estimated uncertainty converges and the contact force error converges to the desired contact force in Fig. 4.

5. Conclusion

In this paper, the trajectory and force tracking controls of nonholonomic mobile manipulators have been investigated with unknown inertia parameters, constraints and disturbances. The controls ensure that the output of the dynamic system tracks hybrid variable signals and makes the whole system stable with respect to the desired force/motion. Throughout this paper, feedback control design and stability analysis are performed via explicit Lyapunov techniques. Simulation studies on the control of a two wheels driven mobile manipulator illustrate the effectiveness of the proposed control.

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Zhijun Li received the B.E. and M.E. degrees in mechanical engineering, and the Dr. Eng. degree in mechatronics from Wuhan Institute of Technology (currently, Wuhan University of Technology), PR China and Shanghai Jiao Tong University, PR China, in 1996, 1999 and 2002, respectively. From 2003 to 2005, he was a postdoctoral fellow in Department of Mechanical Engineering and Intelligent systems, The University of Electro-Communications, Tokyo, Japan. From 2005 to 2006, he was a research fellow in the Department of Electrical and Computer Engineering, National University of Singapore and Nanyang Technological University, Singapore. Currently, he is with Department of Automation, Shanghai Jiao Tong University as a Lecturer. Dr. Li's current research interests are in the adaptive/robust control, friendly manipulator, nonholonomic system, etc.

Shuzhi Sam Ge, IEEE Fellow, P. Eng, is the Director of Social Robotics Lab, Interactive Digital Media Institute, and Professor at Department of Electrical and Computer Engineering, the National University of Singapore. He received his B.Sc. degree from Beijing University of Aeronautics and Astronautics (BUAA), and the Ph.D. degree and the Diploma of Imperial College (DIC) from Imperial College of Science, Technology and Medicine, University of London.

He has (co)-authored three books: Adaptive Neural Network Control of Robotic Manipulators (World Scientific, 1998), Stable Adaptive Neural Network Control (Kluwer, 2001) and Switched Linear Systems: Control and Design (Springer, 2005), and over 300 international journal and conference papers. He has been serving as Associate Editors for IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, Automatica, and Editor for International Journal of Control, Automation & Systems, and Taylor & Francis Automation and Control Engineering Series. Dr. Ge is the recipient of the 1999 National Technology Award, 2001 University Young Research Award, 2002 Temasek Young Investigator Award, Singapore, and 2004 Outstanding Overseas Young Researcher Award from National Science Foundation, China. His current research interests include social robotics, intelligent control, hybrid systems, and intelligent multimedia fusion.

Martin Adams is an Associate Professor at the School of Electrical and Electronic Engineering, Nanyang Technological University (NTU), Singapore. He obtained his first degree in Engineering Science at the University of Oxford, UK, in 1988 and continued to study for a D.Phil. at the Robotics Research Group, University of Oxford, which he received in 1992. He continued his research in autonomous robot navigation as a project leader and part time lecturer at the Institute of Robotics, Swiss Federal Institute of Technology (ETH), Zurich, Switzerland. He was employed as a guest professor and taught control theory in St. Gallen (Switzerland) from 1994 to 1995. From 1996 to 2000, he served as a senior research scientist in robotics and control, in the field of semiconductor assembly automation, at the European Semiconductor Equipment Centre (ESEC), Switzerland. His research work focuses on autonomous robot navigation, sensing, sensor data interpretation and control, and he has published many technical papers in these fields. He has been the principle investigator and leader of many robotics projects, coordinating researchers from local industries and local and overseas universities and is an associate editor of the International Journal of Systems Science.

Wijerupage Sardha Wijesoma received the B.Sc. Engineering Hons. degree in electronics and telecommunication engineering from the University of Moratuwa, Sri Lanka, in 1983, and the Ph.D. degree in robotics from Cambridge University, Cambridge, UK, in 1990. He is an Associate Professor of the School of Electrical and Electronic Engineering, Nanyang Technological University (NTU), Singapore. He is also the Program Director for Mobile Robotics of the Center for Intelligent Machines, NTU. His research interests are in autonomous land and underwater vehicles, with emphasis on problems related to navigation and perception.