Pose Invariant, Robust Feature Extraction From Range Data With a Modified Scale Space Approach

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Abstract—**Feature-based Simultaneous Localization and Map building (SLAM) approaches require a robust method to extract position invariant landmarks from the surrounding environment. 2D laser range finders are currently one of the most common sensors used to obtain environmental information for mobile robot navigation due to their reliability, accuracy and low cost. However, the 2D laser scan data only give very limited information, making it difficult to extract meaningful features particularly in unstructured environments. The most important steps to extract features are segmentation and noise reduction. Scale space and adaptive smoothing are two common techniques within the vision community. They are used to remove high frequency noise and represent image data in multi-scale spaces. They allow for an easier segmentation of images and the extraction of features in the appropriate scale. In this paper, a modified adaptive smoothing algorithm is proposed and applied to laser range data within a modified scale space framework. This algorithm smoothes range data and segments it at the same time by translating a line model mask over the range data. Lines can be extracted from the segments by using a standard fitting algorithm.**

Index terms—**Feature extraction, scale space, adaptive smoothing, simultaneous localization and map building.**

I. INTRODUCTION

The localization of outdoor robotic vehicles in unknown environment is still a major problem to be solved for their successful deployment. Hence the current interest on the Simultaneous Localization And Map building (SLAM) approach as a solution was firstly introduced in [1] and demonstrated mathematically in [2]. Today efforts are concentrated on feature extraction, data association and the reduction of the computing complexities [3], [4] in the SLAM approach. There is much work on data association such as Joint Probabilistic Data Association (JPDA) and Multiple Hypothesis Tracking (MHT) [5], [6]. However, there are fewer efforts on feature extraction from sensor data. Due to its reliability, low cost and accuracy, the most common sensor used is the 2D laser range finder (also known as LADAR) despite its low resolution and limited field of view (a single line). The 2D LADAR range scan data available do not have sufficient information in terms of resolution, field of view and data rates, which together with data uncertainty make it difficult to extract features in the environment. Moreover, if moving in natural environments, feature recognition is very difficult.

In the feature-based SLAM, features need to be position invariant landmarks used to build a map and to update the robot location. Once pose invariant features are extracted from the environment, the robot location and map uncertainty will be reduced. In order to extract features from laser sensor data, the first step is to segment the whole laser scan into several groups of data, then features can be extracted from each segment. In previous work, segmentation was mainly achieved using the split-and-merge algorithm, which splits data into two segments according to the Mahalanobis [7], [8]. In [9], the multi-scale space representation used by the vision community [10], [11], [12], has been applied to laser range data called in an approach curvature scale space. However, because this approach is not able to determine the edge points and segment data, it can only be applied after the laser scan has been segmented into homogeneous groups. Furthermore, it will be shown that the Gaussian mask used in image processing does not render good results for range data.

This paper presents a pose invariant feature extraction approach applied to LADAR data in order to attain an efficient understanding of the data available. In this paper, a new adaptive smoothing algorithm is proposed to represent laser range data in multi-scale space and segment it at the same time. This algorithm employs *a line model mask* to smooth range data and preserves the discontinuities at different scales.

The paper is organized as follows. Section II gives a short introduction to scale space theory and the adaptive smoothing algorithm. Section III presents the proposed adaptive smoothing algorithm developed for LADAR data, which differs from those used for vision application. Section IV demonstrates how the LADAR data can be reliably segmented in a truly pose invariant manner. Section V shows how the modified scale space method can be applied to navigation problems, as lines are extracted from the segments. The results shown are taken from real semi-structured environment based LADAR data.

II. SCALE SPACE THEORY

Scale space theory is a framework for early visual operations, which has been developed and mainly used by the vision community to handle the multi-scale nature of real-world objects. An inherent property of real-world objects is that they only exist as meaningful entities over certain ranges of scale [13]. A simple example is the concept of a branch of a tree, which makes sense only at a view point scale form from a few centimeters to at most a few meters. Thus, the need for multi-scale representation arises when designing
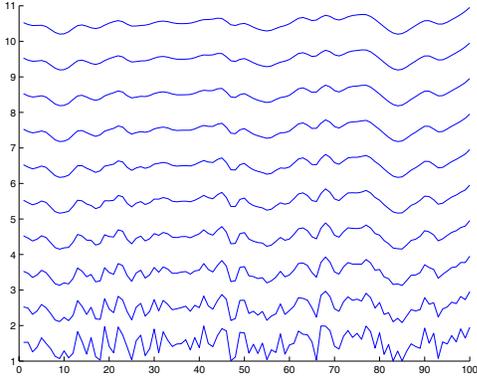
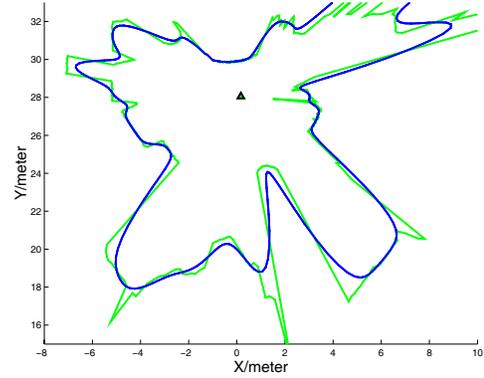Fig. 1. 1-D signal in multi-scale space representation.

Fig. 2. A 2D laser range scan was smoothed by a Gaussian mask with variance 1. The green line is the raw range data from the laser sensor and the blue line is the smoothed range data.

methods for automatically extracting information from real-world measurements should be. Since it is not obvious in advance what the proper scales, the only reasonable approach is to represent at multiple scales and then determine the best one.

Multi-scale signal representation may be achieved by imparting smoothing at different levels of scale. The crucial requirement for the multi-scale representation is that structures at coarse scales should constitute simplifications of corresponding structures at finer scales [10]. The finest scale corresponds to the finest level of detail and the coarsest scale corresponds to highest level of smoothing desired. At coarse scale, *the significant features of the signal must still remain and no new features should be created during the smoothing process*. Figure 1 shows the result of applying Gaussian smoothing to a one-dimensional signal to build up a multi-scale representation. Notice that no new local extrema were created in coarse scales and only significant local extrema will persist in coarse scales.

In the vision literature, the common way of constructing the scale space representation is to smooth the image data with a Gaussian kernel [10], [11]. The Gaussian kernel is a very well known low-pass filter, which will remove high frequency noise from the image data. By convolving the original image $I_0(x, y)$ with a Gaussian mask $G(x, y)$ with different variances t, the multi-scale representation will be constructed.

$$I(x, y, t) = I_0(x, y) * G(x, y, t) \qquad (1)$$

For a given 2D laser scan, the data is a range series with different bearing angles. If the bearing angle was used as the index, the 2D laser scan can be considered as a 1D signal. A corresponding 1D Gaussian mask has been used to smooth range data in [9], and a 1D averaging mask was used in [12].

$$S(x) = \frac{1}{N} \sum_{i=-1}^{1} S(x+i) w(x+i) \qquad (2)$$

with

$$N = \sum_{i=-1}^{1} w(x+i) \qquad (3)$$

Where S represents the 1D range signal, x is the index and i is the offset to the center pixel, and w is the weight of the Gaussian mask or averaging mask.

For 2D image data, each pixel value is assumed to follow a Gaussian distribution. It is therefore reasonable to convolve the pixel values with the Gaussian mask in order to 'blur' the image and remove detail and noise. However for 1D range signals, the range value of each scan point is related to its neighboring scan points according to the surface shape and the bearing angle. If the Gaussian mask or averaging mask is used to smooth 1D range data, the smoothed segment will tend to become circular as in figure 2. This can be particularly noticed at the corner region with coordinate(-5,30). This will cause the original shape of the environment to be changed, which will contradicting the Scale Space theory. The Gaussian mask and averaging mask are therefore not suitable for smoothing range data. A new line model smoothing mask that can smooth 1D range data and retain the original shape will be introduced in section III.

The smoothing operation can effectively remove detail and noise but it can not detect the discontinuities in the range data. The discontinuities are very important for segmentation and can be used to determine the physical boundaries of objects. Adaptive smoothing methods should be used to smooth range data in the same segment and determine the boundaries of each segment.

A. Adaptive Smoothing

The general idea behind adaptive smoothing is to apply a versatile operator which adapts itself to the local topology of the data to smooth. This filter can smooth the data everywhere, even across discontinuities. The weights of the convolution mask $w(x)$ can be computed by a decreasing function $f(d(x))$ such that $f(0) = 1$ and $f(d(x)) \to 0$ as $d(x)$ increases, where $d(x)$ represents the amount of discontinuity at point x. In [12], it has been suggested that $d(x)$ should depend on the magnitude of the derivative at that point.

$$w(x) = f(d(x)) = f(S'(x)) = e^{-\frac{S'(x)^2}{2k^2}} \qquad (4)$$

Where $S'(x)$ is the derivative of $S(x)$ and k is a parameter, which determines the magnitude of the derivative at that edge point to be preserved.

After choosing the line model smoothing mask for 1D laser range data, the discontinuity $d(x)$ can be calculated by the Mahalanobis distance of the line model fitting, which will be shown in section III.

B. Multi-Scale Space Representation

The purpose of the adaptive smoothing algorithm is not only to determine the discontinuities of the signal but also construct a representation with different degrees of detail. The multi-scale space representation of the original signal will be a one-parameter family of derived signals, where the one parameter will control the scale of the signal. By using an adaptive smoothing algorithm, there can be two scale space representations for the original signal. The first approach, which is similar to Gaussian scale-space, is to fix k in equation 4, and then use the number of iterations to serve as scale. The other approach is to use k as the scale while fixing the number of iterations.

Scale space theory has been applied successfully in image data. For SLAM, a 2D laser range finder is used here to obtain information from the surrounding environment. It would be useful to extract features from these range data. Until now, Scale space theory has attracted little attention when applied to laser range data [9], [12]. By examining the problem of Gaussian mask smoothing on laser range data, as discussed above, a new segmentation and feature extraction algorithm based on the multi-scale space and adaptive smoothing theory will be introduced in this paper.

III. ADAPTIVE LINE SMOOTHING

From figure 2, it has been shown that the 1D laser range data tends to become circular after smoothed by the Gaussian mask. Obviously the original shape of the laser scan has been changed. A new smoothing mask is therefore required to smooth the 1D laser range data in multi-scale space. Due to the complexity of the real environment, it's impossible to use one smoothing mask for every shape of the environment. In many semi-structured outdoor environments, a useful objective can be to segment laser range data and extract features such as line segments. It is noted here that a smoothing mask can easily be formulated for any chosen surface representation within an environment using the method which follows. For demonstration purposes here, a method of smoothing only line segments within a laser scan, while leaving all other parts of the scan in tact can successfully meet our requirements to segment laser data and extract lines. A modified scale space approach, based on a line model mask with weights calculated from the line fitting errors, is presented.

A. Modified Line Model

In [14], a recursive line model estimator is initialized using the first two range values to predict the third range value. Then the measurement will be used to update the state, forming an
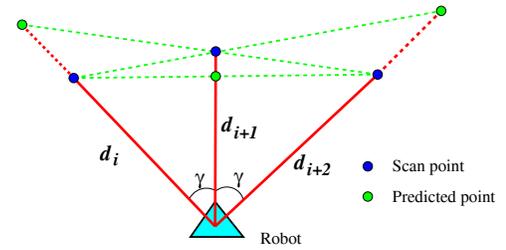

Fig. 3. Modified line models.

extracted line. In this model, it's assumed that the range data are obtained sequentially while the laser sensor is scanning. Commonly used SICK LMS laser sensors often produce a whole scan data batch at a time. Therefore any two range values can be used to predict the third range value in figure 3. d_i, d_{i+1} and d_{i+2} are the distance between the robot and three consecutive scan points within a planar surface. γ is the constant incrementing angle of the laser sensor.

Based on any 3 point combinations shown in figure 3, there will be three line models. Often, LADAR scan points are assumed to have only measurement noise in the range direction [7]. The estimated values of these three range points are required to minimize the sum of square errors $Min(\Delta d_1{}^2 + \Delta d_2{}^2 + \Delta d_3{}^2)$ in the range direction, where Δd is the distance between the scan point and predicted point. This can be achieved by using Kalman Filter based on the modified line model. The state vector to be estimated are the range values of these three points.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} d_i \\ d_{i+1} \\ d_{i+2} \end{bmatrix} \tag{5}$$

The prediction model can be shown to be [14]:

$$\widehat{d}_i = \frac{d_{i+2}d_{i+1}}{2d_{i+2}cos\gamma - d_{i+1}} \tag{6}$$

$$\widehat{d}_{i+1} = \frac{2d_i d_{i+2} cos\gamma}{d_i d_{i+2}} \tag{7}$$

$$\widehat{d}_{i+2} = \frac{d_i d_{i+1}}{2d_i cos\gamma - d_{i+1}} \tag{8}$$

where $\widehat{d}_i$, $\widehat{d}_{i+1}$ and $\widehat{d}_{i+2}$ are the distances between the robot and the predicted points.

Obviously these prediction equations are nonlinear, so that the Extended Kalman Filter (EKF) or Unscented Kalman Filter (UKF) [15] are required to calculate the predicted state vector. The observation model is:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{9}$$

And the three range values are the measurements, and each point has the same measurement noise, which is assumed to be Gaussian and independent of the target reflectivity. This has

been shown to be a reasonable assumption in TOF LADARs [16].

By applying a Kalman filter to update the state in equation 5, the innovation v and innovation variance S can be used to calculate the Mahalanobis distance D.

$$D = v S^{-1} v^T \qquad (10)$$

The innovation $v(k+1)$ is the difference between predicted observation and measurement, which is a random variable with Gaussian distribution. The Mahalanobis distance D will be a random variable following the χ^2 distribution, which will reflect whether the three range points should lie in one straight line with a certain probability. In the adaptive smoothing step, this property will be used to calculate the weight of the smoothing mask.

B. Smoothing Mask Weight

In figure 3, a modified line model is used to smooth three range points by assuming they may lie in the same straight line. The estimated range values can be calculated from a Kalman filter to minimize the total square errors in the range direction. This modified line model can be considered as a smoothing mask similar in nature to the Gaussian mask for image data, but does not make range scan become circular.

In order to construct the smoothing mask, the weight for each estimated range value must be determined. In the line model, the Mahalanobis distance has been calculated from the Kalman filter. This distance actually indicates how these three range points can be fitted by a straight line. If this distance becomes large, it is less plausible to fit a straight line through these three range points. It can be concluded that these three range points may not lie in one line and the estimated range values are less trust worthy.

To apply this to the adaptive smoothing algorithm, the weights of the smoothing mask should be calculated by a decreasing function depending on the discontinuity in equation 4. In the line model, the innovation v is the difference between the predicted range value and the measured value, which reflects the discontinuity in the laser segment. So according to equation 4 and 10, the weight of the line mask can be calculated as:

$$w = e^{-\frac{v v^T}{2s}} = e^{-\frac{D}{2}} \qquad (11)$$

Where D is the Mahalanobis distance.

According to the adaptive smoothing algorithm in multi-scale space, the modified line model mask is shifting from the first scan point to the last scan point. Figure 4 shows how the modified line model mask smooths through the line segment. For each scan point such as C in figure 4, it will have three estimates C_1, C_2 and C_3 by its neighboring scan points A, B, D and E. Each estimate has a weight calculated from equation 11. The observation will has a weight to make the total weight to be unity. The final range value will be the sum of the estimated values and observation multiplying with corresponding weights.
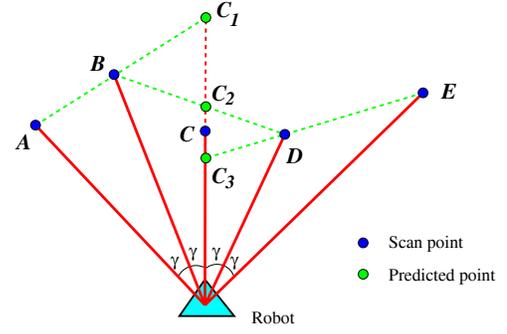

Fig. 4. For each scan point, it has three estimates by its neighbors when the modified line model mask is shifting through the whole scan.

$$\hat{C} = \frac{w_C * C + \sum\limits_{i=1}^{3} (w_{Ci} * C_i)}{w_C + \sum\limits_{i=1}^{3} w_{Ci}} \qquad (12)$$

Since the weight is a decreasing function depending on the Mahalanobis distance, it will adapt itself to the local topology of the input range data. The weight for the line fitting across the boundary of different segments will be small, the estimated range value will have little effect on the final estimated value. Therefore this smoothing operation can cross different segments and preserve the discontinuities. This will be demonstrated with real LADAR range data in the next section.

C. Multi-Scale Space Representation

Two forms of "scale" are used to represent data in the vision literature. The first representation is to fix the variance of the mask and use the number of iterations as a scale variable. This means that better output data can be obtained by applying the adaptive smoothing mask several times. In [12], it has been shown that the input data will have little change after 20 iterations of smoothing. In figure 5, a laser range scan with a line segment and non-line segment scan points has been smoothed once by the adaptive line smoothing algorithm. In figure 6, the same laser range scan has been smoothed recursively 20 times. It can be seen that the line segment becomes smoother when increasing the number of iterations but the non-line segment scan points has little change, which is the required result.

The second representation is to fix the number of iterations and use the variance of the mask as the scale variable. With a larger variance, the smoothed data will be at coarse scale. Most of the noise will have been removed and only the main structures will remain. In this paper, the modified line model was used as smoothing mask. The variance of this mask can be tuned by choosing different process noise values for the line model. By choosing a large line model noise, which means large variance of smoothing mask, the result signal will be in coarse scale.
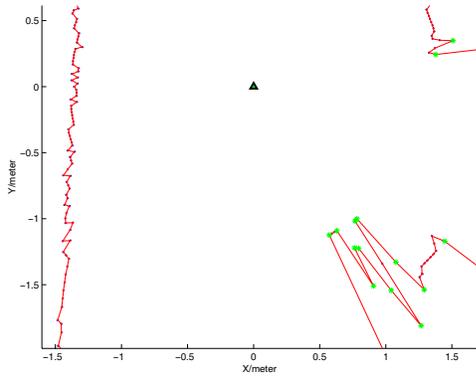
Fig. 5. A laser range scan was smoothed by the adaptive smoothing mask. The number of smoothing iteration is 1.
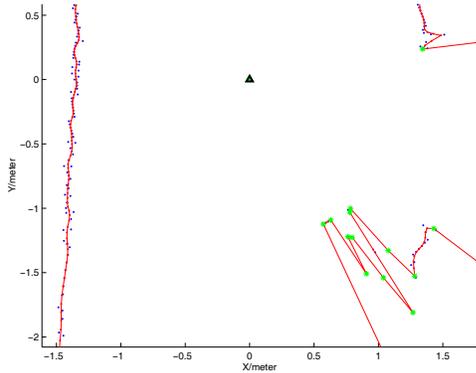

Fig. 6. A laser range scan was smoothed by the adaptive smoothing mask. The number of smoothing iteration is 20.
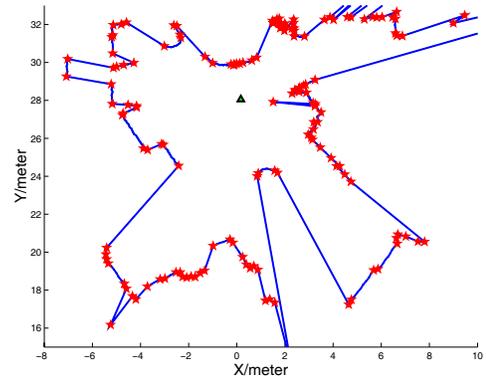

Fig. 7. Segmentation after one iteration with process noise equal to zero.
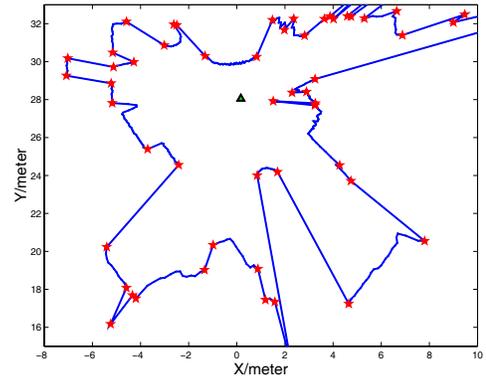
Fig. 8. Segmentation after one iterations with process noise equal to one.

IV. SEGMENTATION

From the Kalman filter with the modified line model, the innovation V is the difference between the predicted range value and the measured value from the laser sensor, which is assumed to follow a Gaussian distribution [16]. Therefore, the innovation is a Gaussian random variable, and according to equation 10, the Mahalanobis D will be a random variable following the χ^2 distribution [5]. As we know, the Mahalanobis distance indicates the modified line model fidelity. So it's a good measure for segmentation. A threshold is normally set to distinguish line segments and non-line segments. If the Mahalanobis distance is less than 9, it has a probability of 99.7% that these three range points may lie on a straight line; otherwise these three scan points are not in one line segment and a new segment will start from that point.

The segmentation for a real indoor laser scan has been done by using different line model variance and iteration number. In figure 7, an indoor laser scan was smoothed by the modified line model once and the process model noise for the Kalman filter was set to zero. That means the modified line model is assumed to be perfect. After smoothing, the laser scan was segmented based on the Mahalanobis distance for each scan point calculated from the Kalman filter. The triangle in the figure is the robot and the blue lines are the smoothed laser

scan from the surrounding environment. The red star points indicate the edge points, which will be used to segment the whole scan. From figure 7, it can be seen that the filter is very "strict" to divide some desired line segments into more smaller line segments. This is due to the zero process model noise. When the process model noise is smaller, the filter will be more "strict". So the laser scan with higher measurement noise will be easily segments into more small line segments.

In figure 8, a very high process model noise of unity was chosen. The obvious difference with figure 7 is that the edge points reduce a lot. The whole laser scan was roughly segmented into several parts and many dominant edge points disappeared. But for each part, it can still be segmented into several smaller line segments. Due to the high process model noise, the filter is more flexible to stand for small changes in line segments.

After that, the iterative smoothing will show the effects on segmentation. In figure 9, the laser scan was smoothed by the Kalman filter with the same process model noise equal to zero as the figure 7. Except that, it has been smoothed with 30 iterations and then segmented according the Mahalanobis distance calculated in the last iteration. The same effect as in figure 8 has occurred, in that the number of edge points was reduced. However, the dominant edge points are all preserved and the laser scan becomes smoother. The missing edge points
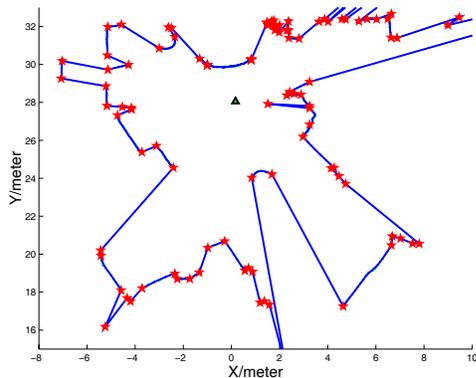
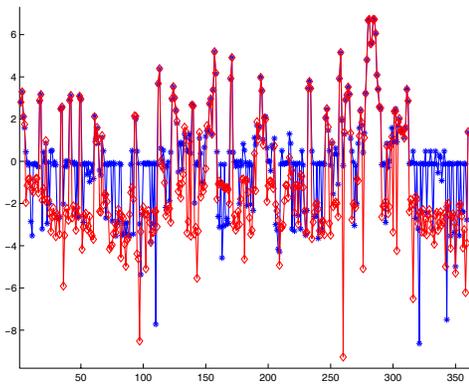Fig. 9. Segmentation after 30 iterations with a zero process noise.

Fig. 10. The Mahalanobis distances of the modified line smoothing when the smoothing mask is shifting from the first three scan points to the last three scan points. The blue stars denote the Mahalanobis distances when the number of iteration is one. The red diamonds denote the Mahalanobis distances when the number of iteration is 30.
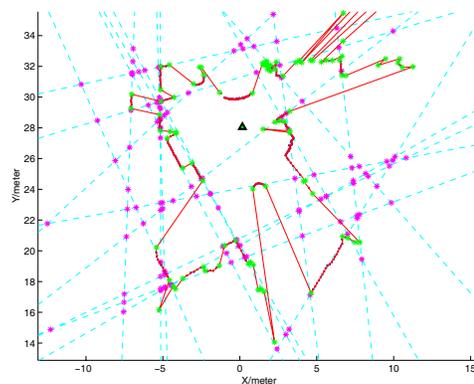

Fig. 11. Lines extracted from an indoor laser range scan after adaptive smoothing and segmentation. The lines or the intersection points of lines can be used as features for SLAM.

are mostly caused by the measurement noise. After smoothed for 30 iterations, the noise for laser scan has been largely reduced. Therefore, the filter can easily determine the true edge points from the laser scan.

From figure 9, it is clear to see that the segmentation becomes better. As discussed above, the measurement noise in laser scans will be remove as the number of iterations increases. So the Mahalanobis distance of the Kalman filter for scan points in the line segments will be largely reduced and the distance for non-line segment points should still be large as before. In figure 10, the Mahalanobis distance for each scan point was shown. The blue stars denote the distance calculated when the laser scan was smoothed once. And the red diamonds denote the distance calculated when the laser scan was smoothed for 30 times. The X axis is the index of scan points and the Y axis is the Mahalanobis distance, which has been converted to the logarithm of the true distance. It's clear to see that most of the distances has been reduced a lot after smoothed for 30 times, which correspond to the scan points in the line segments. There are still several scan points, whose Mahalanobis distance is approximately the same as before. In effect, these points correspond to the edge points in the laser scan. Therefore, it's easier now to distinguish the edge points according to the Mahalanobis distance without too much noise.

V. LINE SEGMENT FITTING

After segmentation, extraction of geometric features from segments such as lines is needed. A standard regression method is mostly used to find the best line in polar coordinates as features. For laser range data,"best" is with respect to squared measurement errors in *range direction*. If the fitting error is beyond a predefined threshold, the extracted line will be ignored. The line fitting method has been specifically discussed in [7], [17]. The extracted features and their covariance matrix can also be determined, which is essential in feature-based SLAM approaches. In figure 11, an indoor laser range scan was segmented by using adaptive line model smoothing and extended line segments and their intersections (shown as crosses) have been extracted by using a line fitting algorithm. In figure 12, this algorithm has been applied to an outdoor laser

scan and line segments and their intersections were extracted.

VI. CONCLUSIONS

A robust feature extraction algorithm is required for feature-based SLAM approach. The most important steps for reliable feature extraction from 2D laser range data are to segment laser scans and reduce noise. The proposed modified adaptive smoothing algorithm based on scale space theory shows several attractive properties. Based on scale space theory, laser data is represented in multi-scale space. This representation is sufficiently robust to noise inherently present in the laser scans. Instead of the Gaussian mask used in image processing, the modified line model mask is used to smooth only line segments that will be finally extracted as features. This mask will avoid the problem of making laser range segments become circular caused by the Gaussian mask. Because the weights of the line model mask are calculated adaptively, this adaptive smoothing algorithm can preserve the discontinuities and segment laser scans during smoothing operations.

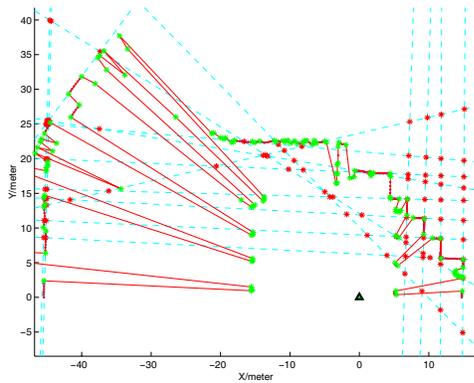This adaptive line-model smoothing algorithm requires the

Fig. 12. An outdoor laser range scan has been smoothed and segmented by the adaptive smoothing algorithm. Lines were extracted and the intersection points of lines are used as features.

presence of line segments in the environment. Some segments such as the circular segment with large radius may be considered to be a line segment. The extracted lines from these non-line segments turn out not to be position invariant. A solution is to calculate the fitting error of each segment. If the fitting error is large, it will not be considered as a line segment. Another problem is that the intersection of two nearly parallel lines is very sensitive to the robot position - the classical *ill-conditioning* problem in numerical analysis of line intersection. These estimated intersection points would have a large error and affect the whole SLAM performance. Extracted line intersections need to have such information attached to them as attributes to avoid the algorithmic intersection calculations of nearly parallel lines. This remains a focus of future research.

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