Incorporating Estimated Feature Descriptor Information into Rao Blackwellized-PHD-SLAM

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***Abstract*—Recently, various techniques which adopt Random Finite Set (RFS) based techniques for the solution of the fundamental, autonomous robotic, feature based, Simultaneous Localization and Mapping (SLAM) problem, have been proposed. In contrast to their vector based counterparts, these techniques offer the advantage that feature detection, as well as the usually considered spatial, statistics can be incorporated into the Bayesian recursion in a joint manner. Most of the proposed solutions are based on the Probability Hypothesis Density (PHD) filter approximation of an RFS estimator. With the aim of further improving such solutions, this article demonstrates the importance of modelling feature detection uncertainty, based on the commonly used range/bearing sensors such as laser range finders used in robotics. In particular, a feature descriptor is defined, based on the number of unoccluded range/bearing values which can be estimated via ray-tracing techniques from estimated SLAM robot pose/feature coordinates. A modified version of the PHD corrector equation is introduced, which incorporates this extra information. An example of such a descriptor, based on the center location and radii of trees in a park, is demonstrated, and statistical information obtained from such an environment is used in a SLAM simulation. This demonstrates the potential of achieving superior SLAM performance, when feature descriptor statistics are incorporated directly into the PHD filter update stage.**

I. Introduction

Within the autonomous robotic feature based navigation and Simultaneous Localization and Mapping (SLAM) literature, feature detection statistics are often ignored, and feature uncertainty is considered to lie solely in the spatial domain, and typically modelled with range and bearing uncertainties [1, 2]. It is considered the task of external map management and association algorithms to minimise the problems of false alarms and missed detections, before map estimation takes place so that mathematically, the probabilities of detection of features that have been associated are assumed to be unity, and the probabilities of false alarm of the associated measurements are assumed to be zero. Also, the probabilities of detection of unassociated features are considered to be zero, while the probabilities of false alarms of unassociated measurements are assumed to be unity. In some state of the art SLAM solutions, detection statistics are considered via the use of a binary Bayes filter to update each feature's confidence measure [1]. However, this takes place independently of the SLAM Bayesian recursion, therefore ignoring the correlation between feature existence and the SLAM state spatial uncertainties. In contrast, the Random Finite Set (RFS) based filters include probability of detection and false alarm statistics directly into the filter's update step, making the feature detector's detection statistics an intrinsic part of the Bayesian state estimation process and its solution. Multiple RFS-based filters have been applied to the SLAM problem. Mullane et. al. introduced the RFS methods to the SLAM problem by using PHD filter [3]. A Single Cluster (SC)-PHD filter has also been applied to SLAM [4]. More recent research has applied newer RFS filters from the tracking literature, such as the Labeled Multi Bernoulli filter [5, 6], to the SLAM problem [7].

In the related field of target tracking, detection statistics are considered to be of prime importance. For example, it has long been recognized that a sensor's received signal amplitudes related to true targets should be higher than those corresponding to false alarms, and that this information should be utilized. However, this requires the Signal-to-Noise Ratio (SNR) corresponding to targets to be known when detected from any sensor to target view point. Since such information is typically not available, object detection probabilities are usually naively considered to be constant[1], despite the fact that the varying relative positions of objects and the sensor, and any occlusions, typically have a large effect on that object's detection probability [8]. Little attention is given to the shape of a sensor's FoV and the possibility of partial or total object occlusion, and their quantified effects on the expected detection statistics.

The use of the received power from a radar sensor directly in the PHD update equation has been proposed by Clark et. al. [9], in which it was demonstrated that the additional power information improves the performance of the PHD filter, even when the SNR is unknown. This article applies a similar principle, based on the expected number of range point in-liers of environmental features which can be totally visible, partially occluded or completely occluded to range/bearing sensors. Such sensors, for example Laser Range Finders (LRFs), are typically used in autonomous robotics research to provide feature measurements for SLAM.

The article proceeds as follows. In Section II, vector and RFS based SLAM solutions are explained, high-lighting the importance of incorporating knowledge of detection statistics into both solutions. Section III then reviews the modified PHD filter proposed in [9], which allows for the incorporation of

[1] but not necessarily zero or unity.

radar amplitude information into its estimation equations. The contribution of this article is presented in Section IV, in which the concepts are applied to the expected number of sensed inliers corresponding to features, from any range/bearing type sensor, and integrated into a RFS SLAM solution. Finally, Section V presents the simulation results obtained, demonstrating the importance of including detection based statistical information into the SLAM problem.

II. Detection Statistics in SLAM

The importance, and incorporation of, detection statistics into both feature vector and RFS based SLAM solutions is now summarized, in order to provide the basis for a modified PHD SLAM formulation in Section IV.

A. Vector Based SLAM Techniques & Detection Statistics

Most feature vector based solutions to SLAM require a map management routine to manage the addition and removal of features from the map estimate. Such methods vary in mathematical rigor. While some are heuristic in nature, others use a binary Bayes filter as used in the occupancy grid based mapping [1, 10].

In the binary Bayes filter the probability of existence $P_{\mathrm{E}}(\boldsymbol{m}^i|\boldsymbol{x}_{0:k},\mathcal{Z}_{0:k})$ of the ith map feature vector $\boldsymbol{m}^i$, given the history of robot poses $\boldsymbol{x}_{0:k}$ from discrete time 0 to k and all feature measurement sets $\mathcal{Z}_{0:k}$, is updated at each step using the probabilistic evidence provided by the current measurement $\mathcal{Z}_k$, and assumed or known data association - i.e.

$$P_{\mathrm{E}}(\boldsymbol{m}^i|\boldsymbol{x}_{0:k},\mathcal{Z}_{0:k}) = \frac{P_{\mathrm{E}}(\boldsymbol{m}^i|\boldsymbol{x}_k,\mathcal{Z}_k)P_{\mathrm{E}}(\boldsymbol{m}^i|\boldsymbol{x}_{0:k-1},\mathcal{Z}_{0:k-1})}{p(\mathcal{Z}_k|\mathcal{Z}_{0:k-1})}, \quad (1)$$

where $\boldsymbol{x}_k$ represents the robot pose (spatial coordinates and orientation) at time-step k. (1) can be expressed in log-odds form as follows

$$\begin{aligned} l_k(\boldsymbol{m}^i) &= l_{k-1}(\boldsymbol{m}^i) \\ &+ log\frac{P_{\mathrm{E}}(\boldsymbol{m}^i|\boldsymbol{x}_k,\mathcal{Z}_k)}{1-P_{\mathrm{E}}(\boldsymbol{m}^i|\boldsymbol{x}_k,\mathcal{Z}_k)} - log\frac{P_{\mathrm{E}}(\boldsymbol{m}^i)}{1-P_{\mathrm{E}}(\boldsymbol{m}^i)}. \end{aligned} \quad (2)$$

By setting the prior probability of existence $P_{\mathrm{E}}(\boldsymbol{m}^i)$ to an uninformative prior (0.5) the last term in equation (2) is eliminated. Then incorporating modelled or known probabilities of detection and false alarm, the probabilistic evidence provided by the measurements can be calculated as

$$\begin{aligned} &P_{\mathrm{E}}(\boldsymbol{m}^i|\boldsymbol{x}_k,\mathcal{Z}_k) \\ &\quad = \frac{(1-P_{\mathrm{D}}(\boldsymbol{m}^i))P_{\mathrm{FA}}P_{\mathrm{E}}(\boldsymbol{m}^i)+P_{\mathrm{D}}(\boldsymbol{m}^i)P_{\mathrm{E}}(\boldsymbol{m}^i)}{P_{\mathrm{FA}}(1-P_{\mathrm{FA}})P_{\mathrm{D}}(\boldsymbol{m}^i)P_{\mathrm{E}}(\boldsymbol{m}^i)} \end{aligned} \quad (3)$$

when $\boldsymbol{m}^i$ is associated to a measurement in $\mathcal{Z}_k$, and as

$$P_{\mathrm{E}}(\boldsymbol{m}^i|\boldsymbol{x}_k,\mathcal{Z}_k) = \frac{(1-P_{\mathrm{D}}(\boldsymbol{m}^i))P_{\mathrm{E}}(\boldsymbol{m}^i)}{(1-P_{\mathrm{E}}(\boldsymbol{m}^i))+(1-P_{\mathrm{D}}(\boldsymbol{m}^i))P_{\mathrm{E}}(\boldsymbol{m}^i)} \quad (4)$$

when $\boldsymbol{m}^i$ is unassociated. $P_{\mathrm{D}}(\boldsymbol{m}^i)$ is the probability of detection of feature $\boldsymbol{m}^i$, P_{FA} is the probability of a measurement being a false alarm. From equation (2) it can be seen that a simple measurement counting heuristic can be interpreted as a log odds binary Bayes filter with an implicitly assumed probability of detection and false alarm.

B. Random Finite Set SLAM and the Importance of Detection Statistics

Mullane et. al. [3] introduced the concept of Random Finite Sets into the SLAM problem. By recognizing that the SLAM state is more naturally represented by a random set instead of a random vector, they where able to include the data association and map management problems into the Bayesian estimation paradigm. Previous solutions to the SLAM problem, resolved these problems using external algorithms, which are executed outside of the Bayesian update, making them sub-optimal. Since an RFS implementation of SLAM will be used as the primary demonstration of the importance of determining detection statistics, a brief overview of RB-PHD-SLAM now follows.

1) SLAM Definitions with RFSs: SLAM is a state estimation problem in which the best estimate of the robot trajectory and map feature positions is sought over time, using all sensor measurements. Common to both random vector and RFS SLAM approaches, the underlying stochastic system representing the robot's pose component is modelled by the non-linear discrete-time equation

$$\boldsymbol{x}_k = \boldsymbol{g}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1},\boldsymbol{\delta}_{k-1}) \quad (5)$$

where:

- $\boldsymbol{g}$ is the robot motion model,
- $\boldsymbol{u}_k$ is the the odometry measurement at time-step k,
- $\boldsymbol{\delta}_k$ is the process noise at time-step k.

In the RFS SLAM approach, the observed landmarks up to and including time-step k, are defined as an RFS

$$\mathcal{M}_k \equiv \{\boldsymbol{m}_k^1,\boldsymbol{m}_k^2,...,\boldsymbol{m}_k^m\} \quad (6)$$

where

- $\boldsymbol{m}_k^j$ is a random vector containing the Cartesian position of landmark j, and
- the number of landmarks, $m = |\mathcal{M}_k|$, is also a random variable.

In general, the landmark from which a measurement is generated is unknown. Furthermore, there is a probability of detection, $P_{\mathrm{D}}(\boldsymbol{m}_k^j|\boldsymbol{x}_k)$, associated with every landmark, implying that it may be misdetected with probability $1-P_{\mathrm{D}}(\boldsymbol{m}_k^j|\boldsymbol{x}_k)$. Measurements may also be generated from sensor noise or objects of non-interest (clutter), with assumed known distributions. The set of all n measurement vectors at time-step k is defined as

$$\mathcal{Z}_k \equiv \{\boldsymbol{z}_k^1,\boldsymbol{z}_k^2,...,\boldsymbol{z}_k^n\}. \quad (7)$$

where

- $\boldsymbol{z}_k^i$ is the i-th measurement vector at time-step k.

With these definitions a set-based, measurement model can be defined as

$$\mathcal{Z}_k \equiv \mathcal{H}(\boldsymbol{x}_k, \mathcal{M}_k, \boldsymbol{\epsilon}_k) \cup \mathcal{E}_k \tag{8}$$

where

- $\mathcal{H}(\boldsymbol{x}_k, \mathcal{M}_k, \boldsymbol{\epsilon}_k)$ models all expected measurements based on $\boldsymbol{x}_k$ and the map set $\mathcal{M}_k$,
- $\boldsymbol{\epsilon}_k$ models the spatial noise associated with the measurements at time k, and
- $\mathcal{E}_k$ models the clutter or unexpected measurements (false alarms) at time k.

Using a Bayesian framework and a filtering approach, the Probability Density Function (PDF)

$$p\left(\boldsymbol{x}_{0:k}, \mathcal{M}_k | \mathcal{Z}_k, \boldsymbol{u}_{0:k}\right) \tag{9}$$

is sought through RFS approaches, requiring Finite Set Statistics (FISST) [11]. The estimates at each timestep are made relative to the reference frame defined by the robot's initial pose.

Using the Rao-Blackwellized Particle Filter as in FastSLAM, the SLAM posterior PDF (Equation (9)) can be factored into the form [12, 13]

$$p\left(\boldsymbol{x}_{0:k} | \mathcal{Z}_k, \boldsymbol{u}_{0:k}\right) p\left(\mathcal{M}_k | \boldsymbol{x}_{0:k}, \mathcal{Z}_k, \boldsymbol{u}_{0:k}\right) \tag{10}$$

such that the first term in (10) is a PDF on the robot's trajectory and can be sampled using particles. The second term in (10) is the PDF of the map conditioned on the robot's trajectory. In the RFS-based approach, the map RFS is assumed to follow a multi-object Poisson distribution such that features are independent and identically distributed (IID) as

$$p\left(\mathcal{M}_k = \{\boldsymbol{m}_k^1, \boldsymbol{m}_k^2, ..., \boldsymbol{m}_k^m\} | \ |\mathcal{M}_k| = m\right) = m! \prod_{i=1}^{m} p_{\boldsymbol{m}}(\boldsymbol{m}^i) \tag{11}$$

where $p_{\boldsymbol{m}}(\cdot)$ is the spatial distribution for the features in the map. Note that the $m!$ term is necessary since a set includes all possible permutations of its elements. The number of features is assumed Poisson distributed with parameter λ according to

$$|\mathcal{M}_k| = p(m) \sim \frac{\lambda^m e^{-\lambda}}{m!} \tag{12}$$

These assumptions allow the PDF of the map RFS to be approximated by a time varying PHD, which is also referred to as an intensity function, v_k:

$$v_k = v_k(\boldsymbol{m}) \equiv \lambda p_{\boldsymbol{m}}(\boldsymbol{m}). \tag{13}$$

The map PDF is then approximated as

$$p(\mathcal{M}_k = \{\boldsymbol{m}_k^1, \boldsymbol{m}_k^2 \ ... \ \boldsymbol{m}_k^m\}) = \frac{\prod_{i=1}^{m} v_k(\boldsymbol{m}_k^i)}{\exp(\int v_k(\boldsymbol{m}) d\boldsymbol{m})} \tag{14}$$

In contrast to vector-based RB-PF approaches, which typically use the EKF to update the Gaussians for individual landmarks, a PHD filter is used to update the map intensity function in RB-PHD-SLAM [13]. A brief overview of the main steps in the RB-PHD-SLAM filter now follows, highlighting the importance of detection statistics.

2) Particle Propagation: At time-step k, the particles representing the prior distribution,

$$\boldsymbol{x}_{k-1}^{[i]} \sim p\left(\boldsymbol{x}_{0:k-1} | \mathcal{Z}_{1:k-1}, \boldsymbol{u}_{0:k-1}\right) \tag{15}$$

are propagated forward in time by sampling the motion noise, $\boldsymbol{\delta}_k^{[i]}$, and using the robot motion model (5)

$$\boldsymbol{x}_k^{[i]} = \boldsymbol{g}(\boldsymbol{x}_{k-1}^{[i]}, \boldsymbol{u}_{k-1}, \boldsymbol{\delta}_{k-1}^{[i]}) \longrightarrow p\left(\boldsymbol{x}_{0:k} | \mathcal{Z}_{1:k-1}, \boldsymbol{u}_{0:k-1}\right) \tag{16}$$

This step is common to vector-based Rao-Blackwellized solutions to SLAM, such as FastSLAM [12].

3) Prediction: For each particle, its map intensity from the previous update, $v_{k-1}^{+}(\boldsymbol{m})$, is augmented with an arbitrarily small "birth" intensity v_k^b, according to the PHD filter predictor equation:

$$v_k^{-}(\boldsymbol{m}) = v_{k-1}^{+}(\boldsymbol{m}) + v_k^b(\boldsymbol{m}) \tag{17}$$

This "birth" intensity $v_k^b(\boldsymbol{m})$ represents the number of new features that might appear at $\boldsymbol{m}$ and is usually heuristically determined. This intensity is required to model the appearance of new features and is similar to the proposal distribution concept in FastSLAM.

4) Map Update: The map intensity for each particle is updated with the latest measurements according to the PHD filter corrector equation

$$\begin{aligned} v_k^{+}(\boldsymbol{m}) &= v_k^{-}(\boldsymbol{m})(1 - P_{\mathrm{D}}(\boldsymbol{m})) \\ &+ v_k^{-}(\boldsymbol{m}) \sum_{i}^{|\mathcal{Z}_k|} \frac{P_{\mathrm{D}}(\boldsymbol{m}) \boldsymbol{h}(\boldsymbol{z}_k^i | \boldsymbol{m}, \boldsymbol{x}_k)}{\kappa(\boldsymbol{z}_k^i | \boldsymbol{x}_k) + \int P_{\mathrm{D}}(\boldsymbol{m}) \boldsymbol{h}(\boldsymbol{z}_k^i | \boldsymbol{m}, \boldsymbol{x}_k) v_k^{-}(\boldsymbol{m}) d\boldsymbol{m}} \end{aligned} \tag{18}$$

where $\boldsymbol{h}(\boldsymbol{z}_k^i | \boldsymbol{m}, \boldsymbol{x}_k)$ is the ith measurement's/detected feature's spatial likelihood and $\kappa(\boldsymbol{z}_k^i | \boldsymbol{x}_k)$ is the intensity of the clutter RFS at time k. The first term in (18) is a copy of $v_k^{-}(\boldsymbol{m})$ scaled down by the factor $(1 - P_{\mathrm{D}}(\boldsymbol{m}))$ to account for the possibility that the predicted features are undetected. In the second term, note that instead of determining data association based on heuristics, the PHD filter determines how much a measurement should influence each and every landmark estimate.

5) Importance Weighting and Re-sampling: The weighting and re-sampling of particles is the method used to update the robot trajectory PDF after propagation (also known as the proposal distribution). This is given by

$$p\left(\boldsymbol{x}_{0:k} | \mathcal{Z}_{1:k-1}, \boldsymbol{u}_{0:k-1}\right). \tag{19}$$

This has to to be updated to become a new PDF representing the robot trajectory after measurement updates (or the target distribution),

$$p\left(\boldsymbol{x}_{0:k} | \mathcal{Z}_{1:k}, \boldsymbol{u}_{0:k-1}\right). \tag{20}$$

Bayes rule allows the weighting distribution in terms of (19) and (20) to be expressed as

$$\frac{p\left(\boldsymbol{x}_{0:k} | \mathcal{Z}_{1:k-1}, \boldsymbol{u}_{0:k-1}\right)}{p\left(\boldsymbol{x}_{0:k} | \mathcal{Z}_{1:k}, \boldsymbol{u}_{0:k-1}\right)} = \eta p\left(\mathcal{Z}_k | \boldsymbol{x}_{0:k}, \mathcal{Z}_{1:k-1}\right), \tag{21}$$

in which η is a normalizing constant. Since (19) and (20) are sampled using particles, the weighting distribution, defined as w_k, is also sampled such that a weight is calculated for each particle. To solve (21), Bayes theorem gives

$$\begin{aligned} w_k &\equiv p\left(\mathcal{Z}_k|\boldsymbol{x}_{0:k}, \mathcal{Z}_{1:k-1}\right) \\ &= p\left(\mathcal{Z}_k|\mathcal{M}_k, \boldsymbol{x}_{0:k}\right) \frac{p\left(\mathcal{M}_k|\mathcal{Z}_{1:k-1}, \boldsymbol{x}_{0:k}\right)}{p\left(\mathcal{M}_k|\mathcal{Z}_{1:k}, \boldsymbol{x}_{0:k}\right)} \end{aligned} \tag{22}$$

Equation (22) can be solved because the map RFS is assumed to be multi-object, Poisson distributed. Note from (22) that the choice of the map, $\mathcal{M}_k$, for which the expression is evaluated in its general form is theoretically arbitrary since the right hand side of the first line of (22) is independent of the map. This has led to multiple solutions that adopt the empty-set strategy, the single-feature strategy and multi-feature strategy in determining the particle weight w_k in (22). Although w_k is theoretically independent of the map, because of the approximations involved in the PHD Filter, it has been shown that the choice of the map can have a significant effect on the performance of the filter and that the multi-feature strategy is superior to the others [14]. This is achieved at the cost of an increased computational complexity. The multi-feature strategy is adopted in this work. In [13, 15] the implementation of RB-PHD-SLAM equations (16), (17) and (18) using Gaussian mixtures is shown.

Importantly, within the above four steps, the map update and particle weighting steps require the knowledge of both the probability of detection of the feature detector and the intensity (PHD) of its false alarms.

III. PHD SLAM USING TARGET AMPLITUDE FEATURE

In the SLAM literature features generated by a detector are often accompanied by a descriptor, which can be used to help in data association. In the related field of target tracking, Clark et. al. [9] proposed a modification to the PHD filter that uses RADAR measurement *amplitude information* together with its accompanying range value. For this, the measurement vectors $\boldsymbol{z}_k^i$ are redefined to be

$$\boldsymbol{z}_k^i \equiv [\mathring{\boldsymbol{z}}_k^i \ a_i] \ , \tag{23}$$

where $\mathring{\boldsymbol{z}}_k^i$ corresponds to the spatial part of the measurement (i.e., what used to be the entire measurement), and a_i is the *amplitude information*. Hence, $\boldsymbol{h}(\boldsymbol{z}_k^i|\boldsymbol{m}_k^j, \boldsymbol{x}_k)$ and $\kappa(\boldsymbol{z}_k^i|\boldsymbol{x}_k)$ account for the joint likelihood of target state and amplitude. The distributions of this amplitude, under false alarm and detection hypotheses, were modelled as Swerling type I and II models [16], which provide probabilistic (Rayleigh) models of received power fluctuations when the RADAR-to-target viewing aspect changes. The dependency on the environment is modelled by a single parameter d, where the expected (mean) SNR from a target is $1+d$ and the model is described by the equations

$$p_{\mathrm{FA}}(a) = a \exp\left(\frac{-a^2}{2}\right), \quad a \geq 0 \tag{24}$$

$$p_{\mathrm{D}}(a) = \frac{a}{2(1+d)} \exp\left(\frac{-a^2}{2(1+d)}\right), \quad a \geq 0 \tag{25}$$

where a is the received amplitude of the radar signal, $p_{\mathrm{FA}}(a)$ and $p_{\mathrm{D}}(a)$ are the distributions of a for false alarms and targets of interest respectively. The modified PHD update equation of [9] is then

$$\begin{aligned} {v_k}^{+}(\boldsymbol{m}) &= v_k^{-}(\boldsymbol{m})(1-P_{\mathrm{D}}(d)) + v_k^{-}(\boldsymbol{m}) \\ &\times \sum_i^{|\mathcal{Z}_k|} \frac{U_d(\boldsymbol{m}, \boldsymbol{z}_i)}{\kappa(\mathring{\boldsymbol{z}}_k^i|\boldsymbol{x}_k) g_{\mathrm{FA}}^{\tau}(a_i) + \int U_d(\boldsymbol{m}, \boldsymbol{z}_i) v_k^{-}(\boldsymbol{m}) d\boldsymbol{m}} \end{aligned} \tag{26}$$

where

$$U_d(\boldsymbol{m}, \boldsymbol{z}_i) = P_{\mathrm{D}}(d) g_a^{\tau}(a_i|d) \boldsymbol{h}(\mathring{\boldsymbol{z}}_k^i|\boldsymbol{m}, \boldsymbol{x}_k) \tag{27}$$

representing the product of the feature detection probability, the amplitude measurement likelihood and the spatial measurement likelihood. $g_a^{\tau}(a_i|d)$ and $g_{\mathrm{FA}}^{\tau}(a_i)$ are the measurement and false alarm likelihoods of the amplitude a_i of measurement z_i occurring, given that a detection threshold τ was exceeded - i.e.:

$$g_{\mathrm{FA}}^{\tau}(a_i) = \begin{cases} \frac{p_{\mathrm{FA}}(a_i)}{\int_{a_i > \tau} p_{\mathrm{FA}}(a_i) da_i}, & a_i \geq \tau \\ 0, & a_i < \tau \end{cases} \tag{28}$$

$$g_a^{\tau}(a_i|d) = \begin{cases} \frac{p_{\mathrm{D}}(a_i)}{\int_{a_i > \tau} p_{\mathrm{D}}(a_i) da_i}, & a_i \geq \tau \\ 0, & a_i < \tau \end{cases} \tag{29}$$

The difference between (26) and the standard PHD update (18) is the inclusion of the measurement and false alarm likelihoods. This will augment the weight of measurements which are more likely to be true detections rather than false alarms.

Clark et. al. [9] presented a reasonable prior distribution for d and showed that by marginalizing d using this distribution, the filter performance could be improved compared to experiments, which ignored the amplitude information. Its prior distribution was also used to determine the probability of detection used in the PHD filter.

Note that Equation (26) requires estimates of both $P_{\mathrm{D}}(d)$ and $g_a^{\tau}(a|d)$ as well as the clutter term $\kappa(\boldsymbol{z}_k^i|\boldsymbol{x}_k) g_{\mathrm{FA}}^{\tau}(a)$. The estimation of these statistical quantities for range/bearing sensors is addressed in the following sections.

IV. ESTIMATING FEATURE AMPLITUDE LIKELIHOOD BASED ON RANGE DATA

A. *Estimating a Likelihood Equivalent to $g_a^{\tau}(a|d)$*

Instead of the Swerling based Rayleigh distributions of the received signal amplitude, adopted for radar measurement likelihoods in [9], the measurement vectors $\boldsymbol{z}_k^i$ are redefined to be

$$\boldsymbol{z}_k^i \equiv [\mathring{\boldsymbol{z}}_k^i \ \boldsymbol{\theta}_i] \ , \tag{30}$$

where $\boldsymbol{\theta}_i$ is a descriptor vector associated to measurement $\mathring{\boldsymbol{z}}_k^i$. A general probability distribution $p(\boldsymbol{\theta})$ for true detections can be used to define the measurement/feature likelihood i.e.:

$$g_{\boldsymbol{\theta}}^{\gamma}(\boldsymbol{\theta}) = \begin{cases} \dfrac{p(\boldsymbol{\theta})}{\int_{\boldsymbol{\theta}_{\text{vol}}} p(\boldsymbol{\theta})d\boldsymbol{\theta}} & \text{if } \boldsymbol{\theta} \in \boldsymbol{\theta}_{\text{vol}} \\ 0 & \text{if } \boldsymbol{\theta} \notin \boldsymbol{\theta}_{\text{vol}} \end{cases} \tag{31}$$

where $\boldsymbol{\theta}$ is a feature parameter vector resulting from any general feature detection algorithm. An example of such a parameter vector will be given in Section V. In (31), $p(\boldsymbol{\theta})$ is a distribution on $\boldsymbol{\theta}$, with known parameters. $\boldsymbol{\theta}_{\text{vol}}$ is a volume in the $\boldsymbol{\theta}$ space such that if $\boldsymbol{\theta}$ falls within $\boldsymbol{\theta}_{\text{vol}}$, the detector makes a detection. This volume is completely determined by the detector. Further, a different distribution $p_{\text{FA}}(\boldsymbol{\theta})$ has to be used to model false alarms in the detectable volume.

$$g_{\text{FA}}^{\gamma}(\boldsymbol{\theta}) = \begin{cases} \dfrac{p_{\text{FA}}(\boldsymbol{\theta})}{\int_{\boldsymbol{\theta}_{\text{vol}}} p_{\text{FA}}(\boldsymbol{\theta})d\boldsymbol{\theta}} & \text{if } \boldsymbol{\theta} \in \boldsymbol{\theta}_{\text{vol}} \\ 0 & \text{if } \boldsymbol{\theta} \notin \boldsymbol{\theta}_{\text{vol}} \end{cases} \tag{32}$$

These likelihoods can then be used, along with the detection statistics $P_{\text{D}}(\boldsymbol{m}|n_p(\boldsymbol{m},\boldsymbol{x}_k))$ and P_{FA} introduced in [17], in a modified PHD filter update equation equivalent to (26), given by

$$v_k^+(\boldsymbol{m}) = v_k^-(\boldsymbol{m})(1 - P_{\text{D}}(\boldsymbol{m}|n_p(\boldsymbol{m},\boldsymbol{x}_k))) + v_k^-(\boldsymbol{m})\times \sum_{i}^{|\mathcal{Z}_k|} \frac{U_{n_p}(\boldsymbol{m},\boldsymbol{z}_i)}{\kappa(\mathring{\boldsymbol{z}}_k^i|\boldsymbol{x}_k)g_{\text{FA}}^{\gamma}(\boldsymbol{\theta}) + \int U_{n_p}(\boldsymbol{m},\boldsymbol{z}_i)v_k^-(\boldsymbol{m})d\boldsymbol{m}} \tag{33}$$

where

$$U_{n_p}(\boldsymbol{m},\boldsymbol{z}_i) = P_{\text{D}}(\boldsymbol{m}|n_p(\boldsymbol{m},\boldsymbol{x}_k))g_{\boldsymbol{\theta}}^{\gamma}(\boldsymbol{\theta}_i)\boldsymbol{h}(\mathring{\boldsymbol{z}}_k^i|\boldsymbol{m},\boldsymbol{x}_k) \tag{34}$$

In principle, $\boldsymbol{\theta}$ can be any descriptor based on the measurement. Clearly it is desirable that its distribution for measurements $p(\boldsymbol{\theta})$ and for false alarms $p_{\text{FA}}(\boldsymbol{\theta})$ are as separated in the $\boldsymbol{\theta}$ space as possible, so that the measurements that are more likely to be false alarms will have lower weights in the modified PHD update equation (34). Otherwise the likelihoods $g_{\text{FA}}^{\gamma}(\boldsymbol{\theta})$ and $g_{\boldsymbol{\theta}}^{\gamma}(\boldsymbol{\theta})$ will be approximately equal and the modified PHD filter from Equation (26) will return to its traditional form, given by (18).

B. Application to General Range Data

Since many autonomous robotic and SLAM solutions rely on features detected by range/bearing sensors, this section shows how the feature likelihoods can be estimated using range data. In particular the application used as an example is of a robot traversing a park environment and using a circular cross section object (such as a tree trunk) detector as in [17]. The likelihoods are obtained by comparing the estimated number of feature in-lier points, calculated using ray tracing as in [17], with the actual number of points that the feature detector extracts from the hypothesized feature.

1) Estimating Feature Likelihoods $g_{\boldsymbol{\theta}}^{\gamma}(\boldsymbol{\theta})$ and $g_{FA}^{\gamma}(\boldsymbol{\theta})$: To include the feature likelihood, a feature descriptor $\boldsymbol{\theta}$ which behaves in a similar manner as the amplitude a used in [9] has to be determined. This can be the same as the parameter vector that the feature detector uses to make its decision or some other value obtained from the measurement.

As explained in section IV, $\boldsymbol{\theta}$ should have well spaced distributions for false alarms and detections, wrt $\boldsymbol{\theta}$. For example in line detectors such as the Split and Merge and Random Sampling and Consensus (RANSAC) algorithms [18], the largest error of the line fit divided by the length of the line could be used (one would expect this value to be smaller for real lines than for false alarms).

Corner detectors could fit a line to the data and use the same descriptor, but expect the opposite result (i.e., a line should fit poorly). However, regardless of the detector used, the number of range values n used by the detector to extract the feature (e.g., in RANSAC this would be the number of inliers) is proposed as a descriptor while the predicted number of unoccluded range points $n_p(\boldsymbol{m},\boldsymbol{x}_k)$, is proposed as a sufficient statistic, equivalent to the signal to noise ratio $d+1$. As shown in Figure 1, $n_p(\boldsymbol{m},\boldsymbol{x}_k)$ is predicted by using ray tracing to simulate the range values that would be obtained from the object (hollow points). These range values are then compared with the actual range values from the sensor (solid color points). If the actual range values are considerably lower than predicted then the predicted range points are labelled as occluded (by the red points), the number of remaining points (green points) is $n_p(\boldsymbol{m},\boldsymbol{x}_k)$.

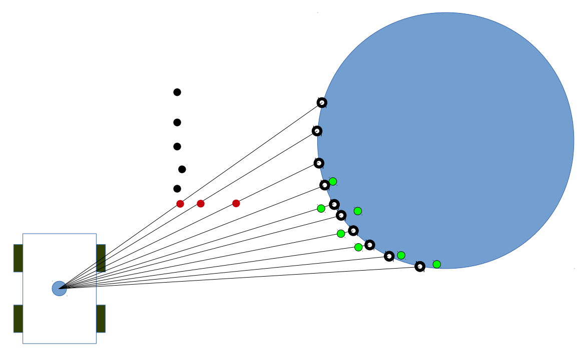

Fig. 1: Analysis of range data from a circular shaped feature. Based on the SLAM state estimate, the range sensor beams that would hit the feature (black lines) and their predicted range values (hollow points), can be determined. Beams with range values several times the range standard deviation shorter than expected (red points) are discarded from the detection probability analysis. The number of remaining (green) points is used to estimate the feature's probability of detection, and the difference between this number and the actual number of points used by the detector $n - n_p(\boldsymbol{m},\boldsymbol{x}_k)$ is used as a descriptor $(\boldsymbol{\theta})$. Black points are range values with angles that do not correspond to the feature being analysed.

After choosing the descriptor real data needs to be obtained in an environment with features identified by independent means in order to model the probability distributions $p(\boldsymbol{\theta})$ and $p_{\text{FA}}(\boldsymbol{\theta})$. Figure 2 shows the number points n as a function of the predicted number of points $n_p(\boldsymbol{m}, \boldsymbol{x}_k)$ in one such environment. This data was obtained by taking LIDAR scans in an environment with known circular sectioned features determined by independent means [17]. As can be seen in Figure 2, the number of range values n varies approximately linearly with its prediction $n_p(\boldsymbol{m}, \boldsymbol{x}_k)$. This suggests that the difference $n - n_p(\boldsymbol{m}, \boldsymbol{x}_k)$ should be modelled, since it is expected to have close to zero mean. Figure 3 shows a histogram of the difference between the predicted number of points and the actual number of points for detections made in the same dataset and feature detector as in [17].

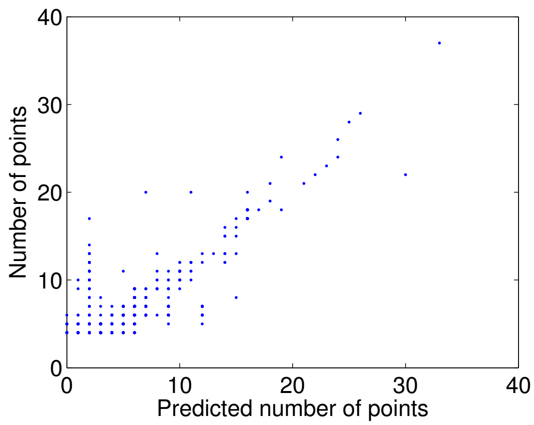

Fig. 2: Number of points measured n as a function of the predicted number of points $n_p(\boldsymbol{m}, \boldsymbol{x}_k)$. The dependency of n on $n_p(\boldsymbol{m}, \boldsymbol{x}_k)$ can be observed in this figure.

To model the distribution of the number of range values used to generate a detection, given the predicted number of unoccluded points based on the current vehicle pose and feature location estimates, a Normal distribution is used:

$$p(n|n_p(\boldsymbol{m}, \boldsymbol{x}_k)) = \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\frac{(n-n_p(\boldsymbol{m}, \boldsymbol{x}_k)-\mu_n)^2}{\sigma_n^2}} \tag{35}$$

In Equation (35) the mean and covariance of the distribution are estimated using a dataset with known features.

For the case of false alarms, the distribution of the number of points has to be modelled using known false alarms from a specific dataset. Data from a park environment was used for this purpose and the exponential distribution was selected as the closest fit to the data. Figure 4 shows the histogram for the number of points, for the case of false alarms.

V. Simulated SLAM Results

To confirm the potential of applying the concept of [9] to the SLAM problem, simulations were carried out. Both the regular RB-PHD-SLAM from [19] and RB-PHD-SLAM with the modified PHD filter (33), which incorporates target descriptor information were executed in a simulated environment. In both simulations the false alarm $\kappa(afhfh)$ was increased until one of the filters diverged. Each result was averaged over 5 independent simulations. In the simulation, values for n where randomly generated based on the distributions estimated from

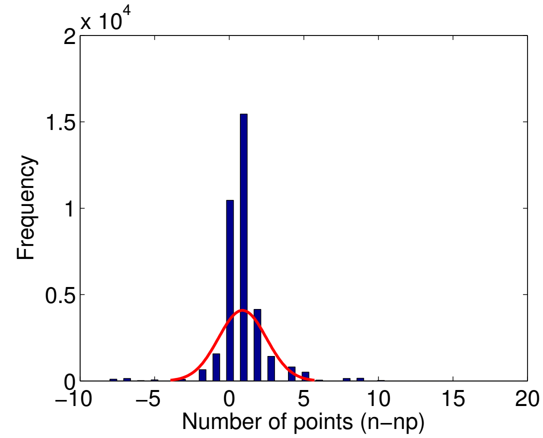

Fig. 3: Number of points minus predicted number of points distribution.

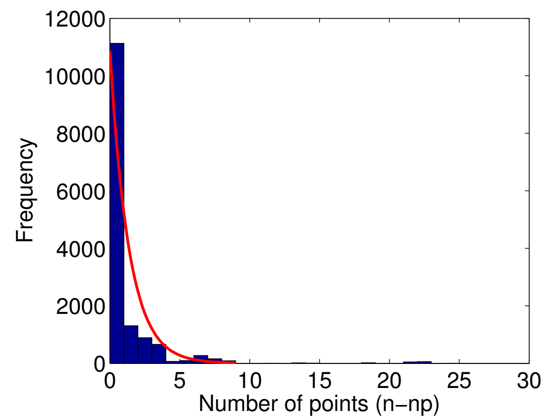

Fig. 4: Number of points distribution for false alarms resembles an exponential distribution.

the dataset, this is Normal and exponential for detections and false alarms respectively.

Results for both the regular RB-PHD solution without feature descriptor likelihood information and the proposed solution with this information are displayed in Figures 5 and 6 respectively. As is qualitatively observable in the figures, the

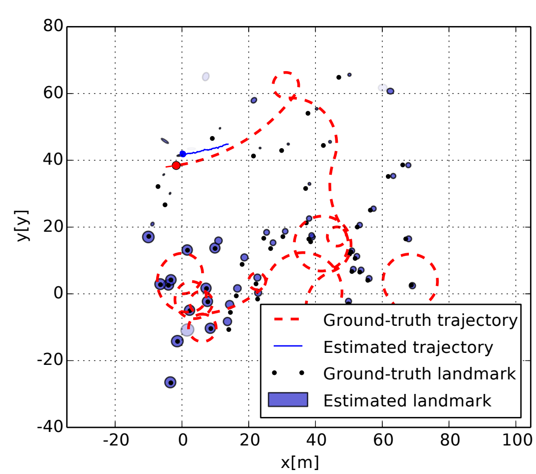

Fig. 5: RB-PHD-SLAM simulation results.

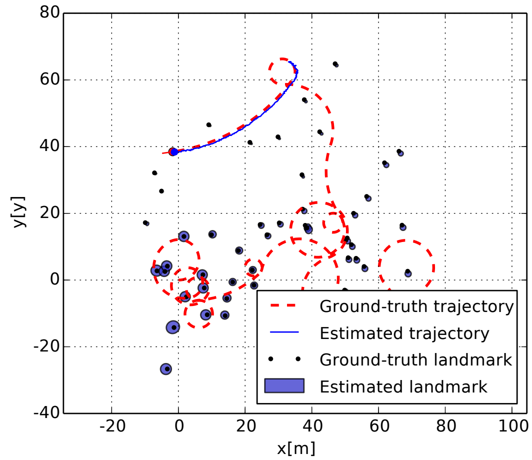

Fig. 6: RB-PHD-SLAM simulation results with target amplitude feature.

addition of target descriptor information improves the solution in terms of trajectory and map estimates. The mapping error was more precisely evaluated using the Optimal Sub-pattern Assignment (OSPA) metric [20], as shown in Figure 7.

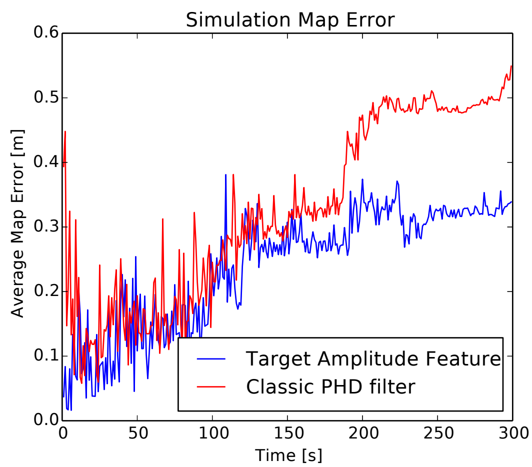

Fig. 7: Average OSPA distance between the ground truth map and the map estimates. Errors were averaged over 5 independent simulations.

The metric shows an improved performance of the modified RB-PHD-SLAM implementation, compared to the solution which ignores feature descriptor likelihoods. In this graph the mapping error reduction resulting from the proposed method is evident.

VI. Summary

In this article, a modified PHD filter which incorporates feature descriptor likelihoods into its update equations was applied to the SLAM problem. In a manner similar to the technique proposed in [9], the measurement likelihoods of a feature descriptor representing the centers and radii of trees, were modelled using real data from a park environment with known feature locations. The modified PHD SLAM algorithm was then simulated using the estimated likelihoods and was shown to outperform RB-PHD-SLAM which used assumed constant detection statistics.

The code of the various SLAM algorithms used in this paper, and the actual parameter settings used, are a part of the RFS-SLAM C++ library that can be obtained at: `https://github.com/kykleung/RFS-SLAM`.

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