

Minima Controlled Recursive Averaging Noise Reduction for Multi-aided Inertial Navigation of Ground Vehicles

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Abstract—Low-cost inertial measurement units (IMUs) are increasingly becoming commercially available and the use of IMUs in autonomous vehicle applications has increased rapidly in the past decade. IMUs are subject to various errors, such as biases, drifts, nonlinearities, scale factors and noise. The noise is produced by various sources, such as thermal and vibrational noise. Noise estimation is critical for accurate inertial navigation systems (INS). The main contribution of this paper is that a noise analysis of the raw accelerations measured by IMUs during signal (i.e. the acceleration caused by a specific force) presence or absence is carried out to reduce these noise components, which corrupt the inertial data. After the noise reduction, multi-aiding information from odometry, a single-axis gyroscope and vehicle constraints is utilized to bound the error growth of the inertial data and produce a reliable outdoor localization system. Experimental results are presented to show the effectiveness of the noise reduction method and the improved accuracy of the multi-aided INS.

I. INTRODUCTION

Contrary to fairly flat and structured indoor environments, it is much more difficult for mobile robots to localize in general outdoor locations. A straightforward solution to the problem of localization in outdoor environments is the use of Global Positioning System (GPS). However GPS is subject to multi-path errors and the problem of limited view of enough satellites. Further, GPS can be easily jammed and it may be totally unavailable, for example, in planetary exploration applications.

Due to the fragile GPS data, more robust methods of localization need to be found for ground vehicles in outdoor environments. Inertial navigation systems (INS) are non-jammable and self-contained and can provide pose estimation in 3D due to a triad of orthogonal accelerometers and gyroscopes. Low-cost solid state inertial measurement units (IMUs) are increasingly becoming commercially available to cater for autonomous vehicle applications. Hence, the utilization of INS in mobile robotics has increased rapidly in the past decade, [1], [8], [5], [6], [10] and [2].

In INS, since rate information has to be integrated to produce velocity, position and attitude measurements, the small errors in the rates will cause accumulated unbounded errors in the integrated measurements. For low-cost IMUs, the accelerations and angular rates are subject to noise and biases from various sources. Hence, it is typical to combine IMU with external sensors to produce effective vehicle pose information. GPS is mostly used to bound the

INS errors and many INS/GPS navigation systems for low speed autonomous vehicle applications have been developed successfully [12], [14], [15]. Under the consideration that GPS is unavailable, other methods to bound the errors of INS exist in the literature. In [8] and [9], data from odometry and gyroscopes have been fused together for localization. In [10], a method was presented for combining odometry and inertial information to provide an estimate of 6 degrees of freedom of a rough terrain rover. Gamini et al [6] put forward an in-flight alignment method by using the constraints that govern the motion of a vehicle to improve the accuracy of low-cost IMUs. They made the INS velocities observable by using these constraints and odometry information. In [2], a multi-aided INS has been developed to solve the outdoor localization problem and the multi-aiding information is from odometry, a single-axis gyroscope and vehicle constraints.

Almost all these inertial based localization methods combine inertial data with external data, while methods based on analysis of the noise and biases within IMUs are seldom found. In [1], Barshan *et al* modelled the biases and drifts of inertial sensors as exponential functions of time, and estimated an augmented mobile robot pose state, which contained these bias terms. In this paper, noise analysis during signal presence or absence is carried out to reduce the various noise, which corrupt the IMU data. It will be shown that commonly adopted noise filtering methods such as low pass filters or matched filters are inappropriate in this application. Hence, a noise analysis method, named the minima controlled recursive averaging (MCRA), which was originally developed by Cohen *et al* for robust speech enhancement [7], and has also been utilized by Jose *et al* to reduce the noise level for their Radar data successfully [4], is adopted. In this paper, after noise reduction, the multi-aided INS approach introduced in [2] is extended to make an accurate and reliable localization system for outdoor, uneven environments using the noise-reduced inertial data.

The paper is organized as follows. Noise estimation and reduction using MCRA will be introduced in section II. The concise INS prediction model and the multi-aiding method utilizing odometry, a single-axis gyroscope and vehicle constraint information will be presented in section III. In section IV, experimental results are presented.

II. NOISE ESTIMATION FOR THE IMU DATA

For typical outdoor environment localization applications, low-cost IMUs are subject to errors from diverse sources such as biases, drifts, nonlinearities, scale factors and noise. The noise is produced by various sources, such as thermal and vibrational noise. For low speed applications such as land-borne vehicle localization, the smaller accelerations and angular rates measured by low-cost IMUs can be indistinguishable from the noise. Figure 1 shows the frequency components of the Inertial Science DMARS-I IMU x -axis acceleration when the test vehicle is stationary. Since the vehicle is not in motion, these spikes in the figure denote noise and need to be filtered before further processing to obtain accurate vehicle pose. In this case, a low pass filter would not suppress the noise, as the noise also corrupts the low frequency accelerations.

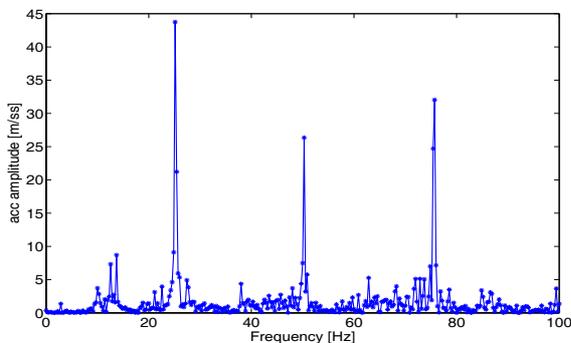


Fig. 1. Frequency components of acceleration in the Inertial Science DMARS-I IMU x -axis when the vehicle is stationary.

Matched filters are commonly adopted in digital signal processing. However, the exact frequency spectrum of the signal is needed (which is unknown for the measured acceleration of the moving vehicle), hence this is not a viable solution either. Therefore, a method called minima controlled recursive averaging (MCRA) [7] is now introduced to filter the noisy accelerations. Lower noise accelerations can be obtained by estimating the noise amplitude in the raw data. The noise estimate is performed by averaging past signal (acceleration) spectral amplitude and using a smoothing parameter. This smoothing parameter is adjusted by the signal (for example, for the acceleration term, the acceleration caused by a specific force) presence probability in the signal profile. The signal presence probability is obtained by taking the ratio between the local amplitude of signal spectra containing noise and its minimum. The estimated noise amplitude is then subtracted from the noisy signals to give a lower noise signal spectra.

Let $acc = a(n)$ denote a set of observed accelerations by one of the accelerometers in the IMU, where n is a discrete-time index and a is a function of n . It is now divided into overlapping frames by a w -point window function $b(n)$ whose length is $2w + 1 = N$ and analyzed using the discrete time Fourier transform. In the frequency domain,

the amplitude of the noisy signal spectra is given by,

$$\hat{A}(k, l) = \sum_{n=0}^{N-l} a(n + lM)b(n) e^{-j(\frac{2\pi}{N})nk} \quad (1)$$

where $\hat{A}(k, l)$ is the k -th amplitude value of l -th signal spectra, k is the frequency index, l is the frame index and M is the frame update step in time. For example, in one data set of our accelerations, the x -axis acceleration is $acc_x = a(n)$, with the discrete-time index $n \in (1, 42000)$. Then the acc_x is equally divided into 60 frames with 700 acceleration values in each frame. The sampling frequency of the accelerometers is 200 Hz and hence, in equation 1, the frequency index $k \in (1, 200)$, the frame index $l \in (1, 60)$ and the frame update step $M = 350$ (50% overlapping windows).

Smoothing is then performed by a first order recursive averaging technique:

$$\hat{A}(k, l) = \alpha_s \hat{A}(k, l-1) + (1 - \alpha_s) \hat{A}(k, l) \quad (2)$$

where α_s ($0 < \alpha_s < 1$) is a weighting parameter and has the value of 0.8 in our case. First a minimum and then a temporary value of the local acceleration amplitude is initialized to $A_{min}(k, 0) = A_{mp}(k, 0) = \hat{A}(k, 0)$. Then a signal wise comparison is performed with the present data l and the previous data $l - 1$.

$$A_{min}(k, l) = \min \{A_{min}(k, l-1), \hat{A}(k, l)\} \quad (3)$$

$$A_{mp}(k, l) = \min \{A_{mp}(k, l-1), \hat{A}(k, l)\} \quad (4)$$

When L frames have been recorded, the temporary variable, A_{mp} is initialized by

$$A_{min}(k, l) = \min \{A_{mp}(k, l-1), \hat{A}(k, l)\} \quad (5)$$

$$A_{mp}(k, l) = \hat{A}(k, l) \quad (6)$$

The resolution of the local minima search is determined by the frame parameter L . For example, in our case, $L = 15$, which means that all 60 frames are divided into 4 local parts to search for the local minima.

Let the signal-to-noise ratio, $A_{SNR}(k, l) = \frac{\hat{A}(k, l)}{A_{min}(k, l)}$ be the ratio between the local noisy signal amplitude and its derived minimum.

In the Neyman-Pearson test [13], the optimal decision (*i.e.* whether a signal is present or absent) is made by minimizing the probability of the type II error (the false detection error), subject to a maximum probability of type I error (the missing detection error), and is as follows.

The test, based on the *likelihood ratio*, is

$$\frac{p(A_{SNR} | H_1)}{p(A_{SNR} | H_0)} \underset{H_0}{\overset{H_1}{\geq}} \delta \quad (7)$$

where δ is a threshold and H_0 and H_1 designate hypothetical signal absence and presence respectively. $p(A_{SNR} | H_0)$ and $p(A_{SNR} | H_1)$ are the conditional probability density functions. The decision rule of equation 7 can be expressed as

$$A_{SNR}(k, l) \underset{H_0}{\overset{H_1}{\geq}} \delta \quad (8)$$

An indicator function, $I(k, l)$ is defined where, $I(k, l) = 1$ for $A_{SNR} > \delta$ and $I(k, l) = 0$ otherwise.

The estimate of the conditional signal presence probability, $\hat{p}'(k, l)$ is

$$\hat{p}'(k, l) = \alpha_p \hat{p}'(k, l-1) + (1 - \alpha_p) I(k, l) \quad (9)$$

where $\alpha_p (0 < \alpha_p < 1)$ is a smoothing parameter. The value of α_p is chosen in such a way that the probability of signal presence in the previous frame has very small correlation with the next frame and $\alpha_p = 0.1$ in our application.

The variance of the noise, $\hat{\lambda}'(k, l+1)$ in k -th signal frequency is then denoted by

$$\hat{\lambda}'(k, l+1) = \tilde{\alpha}_d(k, l) \hat{\lambda}'(k, l) + [(1 - \tilde{\alpha}_d)(k, l)] A_{SNR}(k, l) \quad (10)$$

where

$$\tilde{\alpha}_d(k, l) = \alpha_d + (1 - \alpha_d) \hat{p}'(k, l) \quad (11)$$

$\alpha_d (0 < \alpha_d < 1)$ is a smoothing parameter and $\alpha_d = 0.95$ in our case. Subtracting the estimated noise amplitude from the noisy spectra will give a noise reduced signal bin. The upper graph of figure 2 shows the IMU x-axis acceleration recorded on a pickup truck. After the MCRA noise estimation and reduction applied, the noise in the raw data is reduced, which is shown in the lower graph.

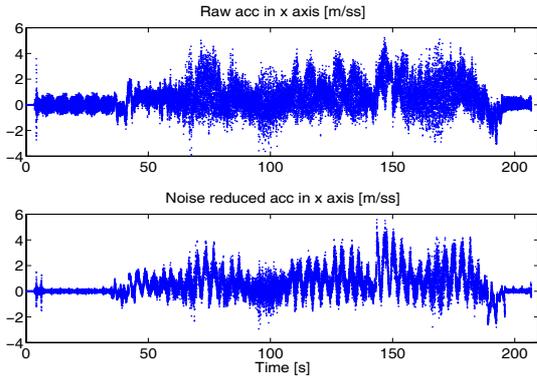


Fig. 2. Accelerations in the IMU x-axis, raw data (upper graph) and after noise reduction (lower graph).

The noise in the IMU's raw data can be reduced by this noise estimation method, which will be helpful in estimating the vehicle's pose more accurately. The biases in the inertial data are however not yet estimated and the accuracy of the vehicle's pose cannot be guaranteed. Hence, external information is still needed to aid in the accurate localization of the vehicle. The next section presents a multi-aiding observation method used for reliable and accurate INS.

III. THE INS PREDICTION MODEL AND THE MULTI-AIDING OBSERVATION MODEL

The last section aimed at reducing the noise in the raw accelerometer data. This section will present how the noise reduced inertial data can be used to produce the vehicle's position, velocity and attitude information accurately, with the multi-aiding information from external sensors.

Typical INS sensors contain a triad of orthogonal accelerometers (translatory rate sensors) as well as gyroscopes (angular rate sensors). By integrating the accelerations and angular rates from an IMU, the autonomous vehicles' pose, that is, the attitude, velocity and position can be computed. In this paper, the objective is to use a multi-aiding method to bound the errors in the estimated INS states after the noise is estimated and subtracted as in section II. A standard Kalman filter (KF) is used to combine the inertial information and the multi-aiding data.

The state vector \mathbf{X} of the KF is:

$$\mathbf{X} = [\delta \mathbf{P}_n^T, \delta \mathbf{V}_n^T, \delta \Psi_n^T]^T \quad (12)$$

where, $\delta \mathbf{P}_n$, $\delta \mathbf{V}_n$ and $\delta \Psi_n$ are position, velocity and attitude error vectors of the INS in the navigation frame (denoted by subscript n) respectively and Ψ_n consists of γ, β, θ , which are yaw, pitch and roll angles in Euler representation. The state equation is:

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}, \mathbf{u}) \quad (13)$$

where, \mathbf{f} will be defined in equation 15. The input is:

$$\mathbf{u} = [\mathbf{A}_b^T, \boldsymbol{\omega}_b^T]^T \quad (14)$$

where, \mathbf{A}_b and $\boldsymbol{\omega}_b$ are the acceleration vector and angular rate vector in the body frame¹ (denoted by subscript b).

The Pinson error model [11] is used here as the dynamic error propagation model of the INS. That is, the position, velocity and attitude error propagation equations can be written as

$$\begin{aligned} \delta \dot{\mathbf{P}}_n &= \delta \mathbf{V}_n \\ \delta \dot{\mathbf{V}}_n &= \mathbf{A}_n \times \delta \Psi_n + \mathbf{C}_b^n \delta \mathbf{A}_b \\ \delta \dot{\Psi}_n &= -\mathbf{C}_b^n \delta \boldsymbol{\omega}_b \end{aligned} \quad (15)$$

where it should be noted that \mathbf{A}_n and \mathbf{C}_b^n are functions of the input vector \mathbf{u} . In equation 15, $\delta \mathbf{A}_b$ and $\delta \boldsymbol{\omega}_b$ are the uncertainties in the accelerometers and gyroscopes in the body frame. These errors can be evaluated accurately when the vehicle is stationary and an initial alignment and calibration has been carried out with the IMU [12], [5]. Hence the state model in Eqn. 15 can be reduced to

$$\begin{bmatrix} \delta \dot{\mathbf{P}}_n \\ \delta \dot{\mathbf{V}}_n \\ \delta \dot{\Psi}_n \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{A}_n \times \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \mathbf{P}_n \\ \delta \mathbf{V}_n \\ \delta \Psi_n \end{bmatrix} = \mathbf{F} \begin{bmatrix} \delta \mathbf{P}_n \\ \delta \mathbf{V}_n \\ \delta \Psi_n \end{bmatrix} \quad (16)$$

where, $\mathbf{A}_n \times$ is the acceleration in the navigation frame represented in a skew-symmetric form. In equation 16, the bias components of the $\delta \mathbf{A}_b$ and $\delta \boldsymbol{\omega}_b$ are assumed to be removed from equation 15 after the initial alignment and calibration and it is further assumed that the remaining components in these error terms can be ignored. The effectiveness of the initial alignment and calibration is shown in [5], where the bias components of the inertial sensors are calibrated by tilt sensors when the vehicle is stationary.

¹Please refer to [3] for details on different attitude representations and frames of INS.

Equation 16 is the fundamental equation that enables the computation of the state \mathbf{X} of the vehicle from an initial state $\mathbf{X}(0)$ and the inputs \mathbf{A}_b and ω_b . The process model \mathbf{F} comprises time-varying terms, $\mathbf{A}_n \times$. Thus, numerical methods are used to determine it. Because the update frequency of \mathbf{F} is much larger than the frequency of the land-borne vehicle's dynamics during the sampling interval Δt , equation 16 can be discretized using the discrete transition matrix $\mathbf{F}(k)$

$$\begin{aligned} \mathbf{F}(k) &= \exp(\mathbf{F}\Delta t) \\ &= \mathbf{I} + \mathbf{F}\Delta t + \frac{(\mathbf{F}\Delta t)^2}{2!} + \dots \end{aligned} \quad (17)$$

The discretization is only taken to the first order term since any higher order terms are of negligible value for small Δt .

The discretization of the state prediction equation is approximated as

$$\hat{\mathbf{X}}(k|k-1) = \mathbf{F}(k)\hat{\mathbf{X}}(k-1|k-1) \quad (18)$$

Since the acceleration and angular rate input vector \mathbf{u} affects \mathbf{A}_n in the process model directly, there is no control vector in the state update equation. The resulting system is linear but $\mathbf{F}(k)$ must be updated with the input vector \mathbf{u} at each time step.

The IMU is initially aligned and calibrated by using Nebot's algorithm [5] and all the electrical and gravitational component biases are assumed removed, thus the initial prediction of the state errors $\mathbf{X}(1|0)$ is $\mathbf{0}$. However, a corresponding growth in uncertainty in the states due to the drift in the IMU should be evaluated by the predicted covariance matrix

$$\begin{aligned} \mathbf{P}(k|k-1) &= E \left[(\mathbf{X}(k) - \hat{\mathbf{X}}(k|k-1))(\mathbf{X}(k) - \hat{\mathbf{X}}(k|k-1))^T | \mathbf{Z}^{k-1} \right] \\ &= \mathbf{F}(k)\mathbf{P}(k-1|k-1)\mathbf{F}(k)^T + \mathbf{Q}(k) \end{aligned} \quad (19)$$

This 9×9 matrix represents the uncertainty in the IMU predicted errors, in which $\mathbf{Q}(k)$ is the process noise matrix and \mathbf{Z}^{k-1} represents all the observation information up to time step $k-1$.

The multi-aiding method for INS described in [2] is applied here to make a reliable INS based navigation system. The vehicle constraints, used as a "virtual sensor", together with the encoders and a single-axis gyroscope, will be integrated together to provide velocity and attitude observations for the INS.

In figure 3, P and Q define the offset distances of the IMU mounted on the vehicle to the center of the vehicle's rear axle. L is the length of the vehicle's wheelbase and it is assumed that the actual radii of the vehicle's rear wheels are the same, R . When the vehicle is performing a turn, R_{rr} and R_i are the radii of curvature of the path taken by the rear right wheel and the IMU respectively about the instantaneous center of rotation (ICR, denoted by "O" in the figure). According to the assumption that the vehicle is rigid, the angular rate at which any point on the vehicle rotates about "O" is the same, ω_v . The forward velocity

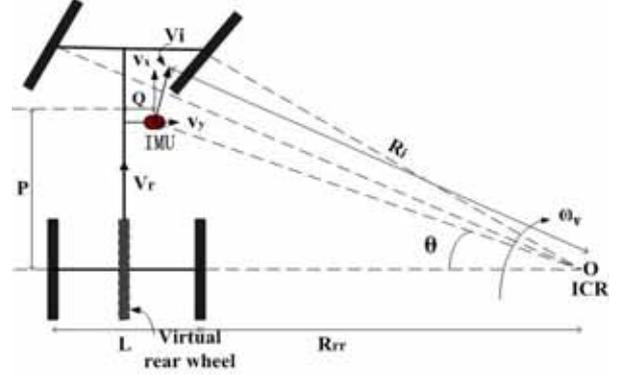


Fig. 3. Vehicle kinematics for a car-like vehicle with encoders and an IMU.

of the vehicle is v_r and the velocities in the IMU's body frame along the x and y axes are v_x and v_y respectively. It is possible to combine the pair of wheels on the rear axle and replace them with a single virtual wheel which lies at the center of the rear axle. The rear left, right and virtual wheels have angular velocities ω_{rl} , ω_{rr} and ω_r respectively and the first two angular velocities can be calculated from the encoders mounted on the pair of rear wheels directly. ω_r can simply be calculated as

$$\omega_r = \frac{1}{2}(\omega_{rl} + \omega_{rr}) \quad (20)$$

By geometry,

$$R_{rr} = \frac{\omega_{rr}L}{\omega_{rl} - \omega_{rr}} \quad (21)$$

Since,

$$\begin{aligned} V_r &= \omega_r R = \omega_v \left(R_{rr} + \frac{L}{2} \right) \\ \omega_v &= \frac{\omega_r R}{R_{rr} + \frac{L}{2}} \end{aligned}$$

the velocity of the IMU, \mathbf{V}_i is,

$$\mathbf{V}_i = \omega_v R_i = \frac{\omega_r R}{R_{rr} + \frac{L}{2}} \times \frac{R_{rr} + \frac{L}{2} - Q}{\cos \theta}$$

while θ is then given by,

$$\tan \theta = \frac{P}{R_{rr} + \frac{L}{2} - Q}$$

Hence, the velocities in the IMU's body frame along the x and y axes, v_x and v_y are,

$$v_x = V_i \cos \theta = \frac{\omega_r R}{R_{rr} + \frac{L}{2}} \times \left(R_{rr} + \frac{L}{2} - Q \right) \quad (22)$$

$$v_y = V_i \sin \theta = \frac{\omega_r P R}{R_{rr} + \frac{L}{2}} \quad (23)$$

where, ω_r and R_{rr} are defined in equations 20 and 21. Hence the observed velocities v_x and v_y of the IMU in the body frame, using the two encoders mounted on the rear wheels of the vehicle, can be estimated.

Under ideal conditions, when the vehicle moves on a surface, it does not leave the ground, which means there is

no motion normal to the road surface (z -axis in the body frame) [6]. In practical situations, an approximation can be made to model the constraint violations due to vibrations caused by the vehicle and road imperfections as follows:

$$v_z = \nu_z \quad (24)$$

where, ν_z is a Gaussian white noise source with zero mean and variance σ_z^2 .

In practical operation, when the odometry information is available, v_x and v_y are obtained by using equations 22 and 23. While at the same time, $v_z = \nu_z$ is provided by the “virtual sensor”. Hence the velocity observation can be made,

$$\mathbf{z}_V^{velocity}(k) = \mathbf{C}_b^n \begin{bmatrix} v_x(k) \\ v_y(k) \\ v_z(k) \end{bmatrix} = \begin{bmatrix} v_x(k) \cos \beta \cos \gamma + v_y(k) (-\cos \theta \sin \gamma + \sin \theta \sin \beta \cos \gamma) \\ v_x(k) \cos \beta \sin \gamma + v_y(k) (\cos \theta \cos \gamma + \sin \theta \sin \beta \sin \gamma) \\ -v_x(k) \sin \beta + v_y(k) \sin \theta \cos \beta \end{bmatrix} \quad (25)$$

Thus the observation vector of the KF is

$$\mathbf{z}(k) = \mathbf{z}_V^{inertial}(k) - \mathbf{z}_V^{aiding}(k) \quad (26)$$

In order to make the attitude also observable so that the error growth of the INS can be further reduced, a single-axis gyroscope is mounted, aligned with the center line of the vehicle to measure the its heading angle. This model is simply

$$\dot{\gamma} = \dot{\Gamma} \quad (27)$$

where, $\dot{\Gamma}$ is the angular rate reading from the gyroscope. It is beneficial to use this single-axis gyroscope to provide the heading observation for the INS if the gyroscope is more accurate than the yaw axis gyro within the INS. As is the case here, it is now possible to find commercially available, low-cost, relatively accurate gyroscopes and the one adopted in our work is the KVH DSP-5000 fiber optic gyro. The bias level of this model ($1^\circ/hr$) is much lower than the gyros contained in the IMU ($5^\circ/hr$).

The best estimate for the state vector \mathbf{X} can be obtained based on all the observations. When an observation from an aiding sensor, that is the encoders, the “virtual sensor” or the gyroscope, is available, the observation vector is

$$\mathbf{z}(k) = \begin{bmatrix} z_V^{inertial}(k) - z_V^{aiding}(k) \\ z_\gamma^{inertial}(k) - z_{\gamma}^{gyro}(k) \end{bmatrix} \quad (28)$$

The observation is the error between the velocities and yaw angle of the INS and those of the aiding sensors, and the uncertainty in this observation is reflected by the noise of the aiding observation.

Hence the observation matrix is

$$\mathbf{H}(k) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (29)$$

And now the observation equation is

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{X} + \boldsymbol{\nu} \quad (30)$$

where $\boldsymbol{\nu}$ is the observation noise vector, the value of which has been determined experimentally. The updating equations of the KF are standard and omitted here [11]. Once the observations are formed, the state vector can be updated. Hence, the vehicle constraints, odometry and gyroscope can be used to aid the INS to form a reliable localization system and produce position, velocity and attitude estimation of the vehicle.

IV. RESULTS

In this section, experimental results will be presented to prove the effectiveness of noise estimation and the multi-aiding method. Firstly, the initial calibration and alignment method in [5] is carried out with the IMU. Secondly the noise analysis in the section II is conducted and the estimated noise is subtracted from the accelerations in each of the IMU’s axes. Then the prediction model and observation model in section III is used to compute the vehicle’s position, velocity and attitude.

The testing pickup truck with all the mounted sensors used here, is shown in figure 4. The inertial sensor used in this work is a low-cost IMU from Inertial Science, DMARS-I. The IMU together with tilt sensors (for initial calibration) and the single-axis gyroscope were mounted on a rigid platform, which was placed on top of the pickup. A digital Honeywell compass was also used to provide the initial heading of the vehicle. The encoders were mounted on the rear wheels as depicted in the figure. A Trimble DGPS was also used in the experiments and the INS/GPS data was fused by using the algorithm in [12]. The INS/GPS result was the best ground truth available, since GPS alone produced large jumps in the estimated vehicle path due to multiple GPS reflections.

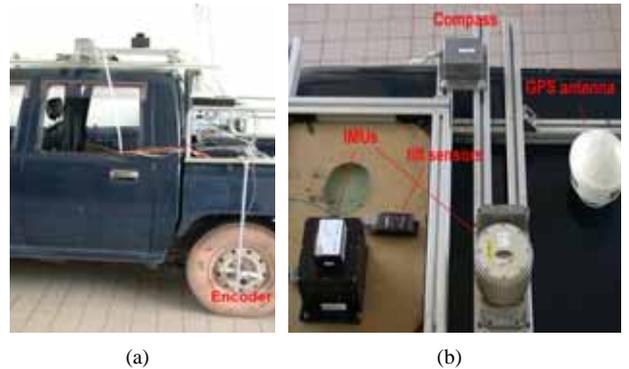


Fig. 4. The testing pickup truck with mounted sensors (each sensor is labelled). A DGPS is used to be fused with INS to produce the ground truth for the experiments.

In the experiments, we ran the utility vehicle in an outdoor, undulating environment (on the campus of Nanyang Technological University). The altitude of this environment ranges from 8 to 38 meters below the sea level. The whole path is approximately 1.6 km in length and the vehicle ran for approximately 4 minutes to complete one loop.

Figure 5 compares the velocities in the North and East directions from the free running INS using the raw data

and the free running INS using noise reduced data, with INS/GPS data as the ground truth, in one experiment. The velocities from these two free INS are close to each and deviate obviously from the ground truth, in both directions. Although the proposed MCRA method is effective in reducing the noise level of the raw IMU data (as shown in figure 2), it only slightly contributes in reducing the errors in the velocities and then the positions. The reason is that in the raw inertial data, *biases* rather than noise are dominant errors. Hence initial alignment and calibration or in-flight alignment is fundamental in correctly using the IMUs to provide vehicles' navigation information ([5], [6]). Yet the MCRA method still helps in terms of reducing the noise and hence it is still useful when the accelerations are used as direct inputs for the system in some applications.

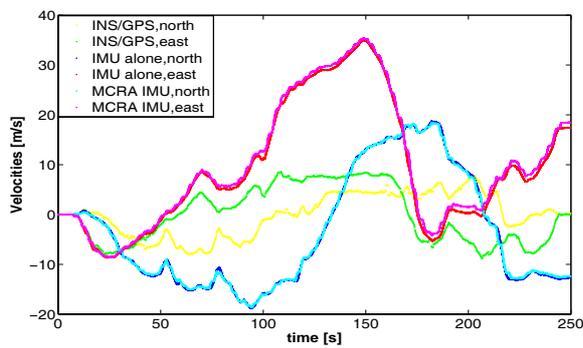


Fig. 5. The comparison of velocities in the North and East directions from the free running INS and the free running INS using noise reduced data, with INS/GPS data as the ground truth.

In the following, results from three different kinds of inertial navigation methods, namely the free running INS, the multi-aided INS using noise reduced data (MCRA aided INS in short) and the INS/GPS method, are used for comparison.

The map of the environment and the path is shown in figure 6.



Fig. 6. The red curve is the path from INS/GPS as the ground truth. The cyan one is the MCRA aided INS path.

In figure 6, the red curve shows the path generated from

the INS/GPS integration technique, while the cyan one is the path from the multi-aiding method using noise reduced inertial data by applying the MCRA method. Compared to the INS/GPS result, the MCRA aided INS path was a little offset from the ground truth.

Figure 7 shows the path from the free running INS, compared with the two paths in figure 6. Even after initial calibration, without any kind of aiding, the free INS can only function for a short period of time accurately as expected. The mean position errors of the free INS result are 461m and 814m in the North and East directions respectively, which are much larger than those of the MCRA aided INS, 7.3m and 5.5m.

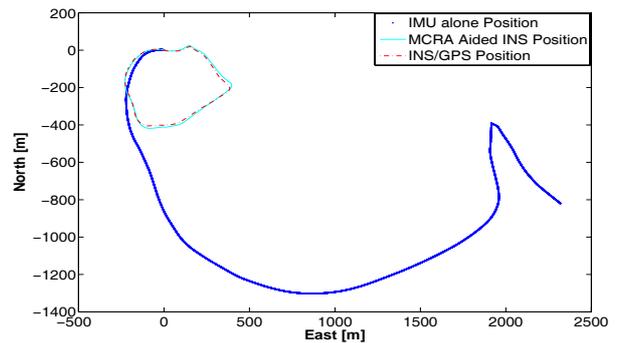


Fig. 7. The generated paths from the three localization methods. The red and cyan curves are from the INS/GPS and MCRA aided INS respectively. The large curve is the result of the free INS.

Figure 8 shows the velocity comparison from the 3 different INS methods. The velocities in the north and east directions have been plotted from each localization method. It is clear that after initial calibration, estimation of the velocities from the free running INS soon diverges while the MCRA aiding method followed the ground truth well throughout the process. The mean velocity errors of the free INS result are 7.9m/s and 9.4m/s in the North and East directions respectively, while those of the aided INS are reduced to 0.29m/s and 0.21m/s.

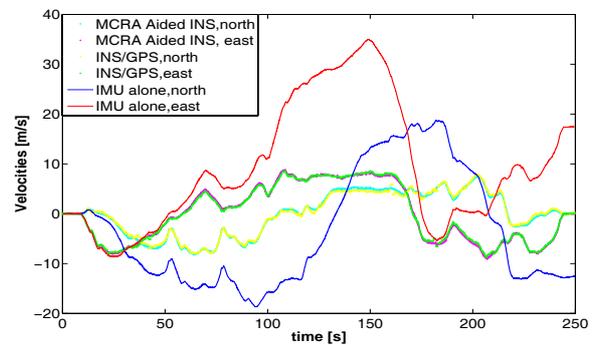


Fig. 8. The comparison of velocities in the North and East directions from the three localization methods.

Figure 9 shows the velocities as well as the positions in the “down” direction estimated using the 3 different INS

methods. Since the vehicle ran in a 3D environment, the pose estimation in this “down” direction is also important. It is seen that the velocity and position estimation from the MCRA aided INS follows the INS/GPS estimates much better than those estimated in the free running INS.

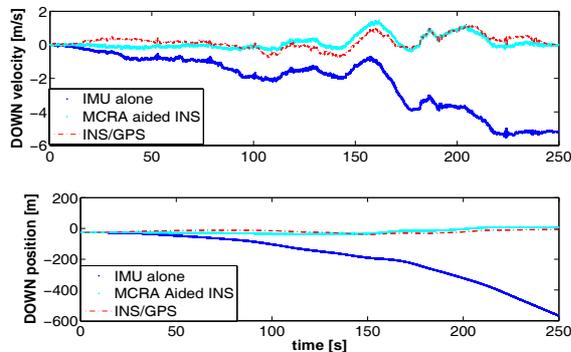


Fig. 9. The comparison of velocities and positions from three methods, in the “down” direction. The red and cyan curves are from the INS/GPS and MCRA aided INS respectively. The blue curves are from the free INS.

In figure 10 the headings of the vehicle from the INS/GPS and the multi-aiding method are shown. Due to the single-axis gyroscope, the heading estimation of the multi-aiding method is relatively accurate. The mean heading error is only 1° around.

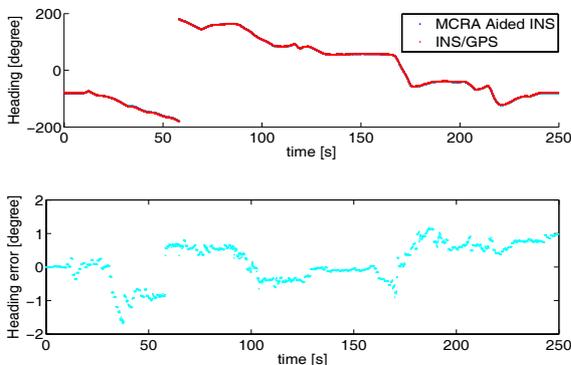


Fig. 10. The upper graph is the comparison of the headings of the vehicle from the INS/GPS (red curves) and the MCRA aided INS (cyan curves) and the lower graph is the heading errors of the MCRA aided INS.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, noise estimate on the accelerometers within IMUs and a multi-aided inertial based method has been presented for outdoor ground vehicles. The noise analysis method estimates the noise in the acceleration by averaging past signals and using a hypothesis test. Result shows that the noise level in the raw accelerations has been reduced. The multi-aiding information is from odometry, a single-axis gyroscope and vehicle constraints.

Experimental results from the noise-reduction and multi-aiding method have been compared with the standard INS/GPS fusion method and a free running INS method. From the results, it can be seen that the position and velocity estimates could be only slightly improved if applying the noise reduction method only to the raw inertial data. Results also show that even without the help of GPS, the proposed MCRA aided INS can still provide reasonable position, velocity and attitude estimation when a vehicle operates in outdoor non-flat environments.

In order to make this INS system more robust and accurate, it will be interesting to estimate the bias components in the IMU data as well. If the bias components can be estimated precisely, the INS accuracy can be further improved and less external sensors may be needed to aid the IMU.

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