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Robust Adaptive Control of Cooperating Mobile Manipulators with Relative Motion

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Abstract— **In this paper, coupled dynamics are presented for two cooperating mobile robotic manipulators manipulating an object with relative motion in the presence of system dynamics uncertainty and external disturbances. Centralized robust adaptive controls are introduced to guarantee the motion and force trajectories of the constrained object converge to the desired manifolds with prescribed performance. The stability of the closed-loop system and the boundedness of tracking errors are proved using Lyapunov stability synthesis. The tracking of the constraint trajectory/force up to an ultimately bounded error is achieved. The proposed adaptive controls are robust against relative motion disturbances and parametric uncertainties and validated by simulation studies.**

I. INTRODUCTION

The controls of multiple mobile manipulators present a significant increase in complexity over the single mobile manipulator case [1], [2], [3], [4], [5]. The difficulties of the control problem lie in the fact that, when multiple mobile manipulators coordinate each other, they form a closed kinematic chain mechanism. This will impose a set of kinematic and dynamic constraints on the position and velocity of coordinated mobile manipulators. As a result, the degrees of freedom of the whole system decrease, and internal forces are generated which need to be controlled.

Thus far, there are two categories of coordination schemes for multiple mobile manipulators reported in the literature. These methods can be classified into two categories as: (i) hybrid position force control by decentralized/centralized scheme, where the position of the object is controlled in a certain direction of the workspace, and the internal force of the object is controlled in a small range of the origin [1], [4]; (ii) leader-follower control for mobile manipulator, where one or a group of mobile manipulators or robotic manipulator play the role of the leader or master, which track a preplanned trajectory, and the rest of the mobile manipulators form the follow group which are moved in conjunction with the leader mobile manipulators [2], [6].

However, in the hybrid position force control of constrained coordinated multiple mobile manipulator, such as [1], [4], although the constraint object is moving, it is usually assumed, for the ease of analysis, to be held tightly and thus has no relative motion with respect to the end effectors of the mobile manipulators. These works have focused on dynamics based on pre-defined, fixed constraints among them. The assumption of these works are not applicable to some applications which require both the motion of the object and its relative motion with respect to the end effectors of the manipulators, such as sweeping tasks and cooperating assembly tasks by two or multiple mobile manipulators. The object/tool is required to move with respect to not only the world coordinates but also the end effectors of the mobile manipulators. The motion of the object with respect to the mobile manipulators can also be utilized to cope with the limited operational space and to increase task efficiency. Such tasks need simultaneously control of position and force in the given direction, therefore, the impedance control like [2], [6] may not be applicable.

Model-based and neural-network-based adaptive control were developed for a class of constrained robots where one robotic manipulator (manipulator I) performs constrained motion on the surface of an object which is held tightly by another robotic manipulator (manipulator II) [7]. Motivated by the results, we investigate, in this paper, a similar class of system where manipulators I and II are mobile manipulators. Mobile manipulator II is to be controlled such that the constraint object follows the planned trajectory, while mobile manipulator I is to be controlled such that its end effector follows a planned trajectory on the surface with the desired contact force. We first present the dynamics of two mobile robotic manipulators manipulating an object with relative motion. This will be followed by centralized robust adaptive control to guarantee the converge of of the motion/force trajectories tracking of the constraint object under the parameters uncertainties and the external disturbances.

II. DESCRIPTION OF INTERCONNECTED SYSTEM

The system under study is schematically shown in Fig. 1. The object is held tightly by the end effector of mobile manipulator II and can be moved as required in space. The end effector of mobile manipulator I follows a trajectory on the surface of the object, and at the same time exerts a certain desired force on the object. The dynamics of the constrained mobile manipulators can be described as

$$M_1(q_1)\ddot{q}_1 + C_1(q_1,\dot{q}_1)\dot{q}_1 + G_1(q_1) + d_1(t) = B_1\tau_1 + J_1^T\lambda_1 \quad (1)$$

$$M_2(q_2)\ddot{q}_2 + C_2(q_2,\dot{q}_2)\dot{q}_2 + G_2(q_2) + d_2(t) = B_2\tau_2 + J_2^T\lambda_2 \quad (2)$$

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where $M_i(q_i) = \begin{bmatrix} M_{ib} & M_{iva} \\ M_{iab} & M_{ia} \end{bmatrix}$, $C_i(\dot{q}_i, q_i) = \begin{bmatrix} C_{ib} & C_{iba} \\ C_{iab} & C_{ia} \end{bmatrix}$, $G_i(q_i) = \begin{bmatrix} G_{ib} \\ G_{ia} \end{bmatrix}$, $d_i(t) = \begin{bmatrix} d_{ib}(t) \\ d_{ia}(t) \end{bmatrix}$ $(i = 1, 2)$, $\lambda_1 = \begin{bmatrix} \lambda_{1n} \\ \lambda_c \end{bmatrix}$, $\lambda_2 = \begin{bmatrix} \lambda_{2n} \\ \lambda_c \end{bmatrix}$, $M_i(q_i) \in R^{n_i \times n_i}$ is the symmetric bounded positive definite inertia matrix, $C_i(\dot{q}, q)\dot{q} \in R^{n_i}$ denote the Centripetal and Coriolis forces; $G_i(q) \in R^{n_i}$ are the gravitational forces; $\tau_i \in R^{p_i}$ is the vector of control inputs; $B_i \in R^{n_i \times p_i}$ is a full rank input transformation matrix and is assumed to be known because it is a function of the fixed geometry of the system; $d_i(t) \in R^{n_i}$ is the disturbance vector; $q_i = [q_{ib}, q_{ia}]^T \in R^{n_i}$, and $q_{ib} \in R^{n_{iv}}$ describe the generalized coordinates for the mobile platform; $q_{ia} \in R^{n_{ia}}$ are the coordinates of the manipulator, and $n_i = n_{iv} + n_{ia}$; $F_i = J_i^T \lambda_i \in R^{n_i}$ denotes the vector of constraint forces; $A_i^T \lambda_{in} = 0$ satisfies the nonholonomic constraint; $\lambda_i = [\lambda_{in}, \lambda_c]^T \in R^{p_i}$ with λ_{in} being the Lagrangian multipliers with the nonholonomic constraints.

Assumption 2.1: The mobile manipulator is subject to known nonholonomic constraints.

Remark 2.1: In actual implementation, we can adopt the methods of producing enough friction between the wheels of the mobile platform and the ground such that this assumption holds.

Since $A_i \in R^{(n_{iv}-m)\times n_v}$, it is always realizable to find an m rank matrix $S_i \in R^{n_{iv}\times m}$ formed by a set of smooth and linearly independent vector fields spanning the null space of A_i, i.e. $S_i^T A_i^T = 0$. Since $S_i = [s_{i1}, \cdots, s_{im}]$ is formed by a set of smooth and linearly independent vector fields spanning the null space of A_i, define an auxiliary time function $v_{ib} = [v_{ib1}, \cdots, v_{ibm}]^T \in R^m$ such that $\dot{q}_{ib} = S_i v_{ib} = s_{i1}v_{ib1} + \cdots + s_{im}v_{ibm}$, which is the so-called the kinematics of nonholonomic system. Let $v_{ia} = \dot{q}_{ia}$. One can obtain

$$\dot{q}_i = R_i(q_i) v_i \tag{3}$$

where $v_i = [v_{ib}, v_{ia}]^T$, and $R_i(q_i) = \mathrm{diag}[S_i, I_{n_{ia}\times n_{ia}}]$.

Differentiating equation (3) yields

$$\ddot{q}_i = \dot{R}_i(q_i) v_i + R_i(q_i)\dot{v}_i \tag{4}$$

Substituting (4) into (1) and (2), and multiplying both sides with $R_i^T(q_i)$ to eliminate λ_{in}, yields:

$$M_{i1}(q_i)\dot{v}_i + C_{i1}(q_i, \dot{q}_i)v_i + G_{i1}(q_i) + d_{i1}(t) = B_{i1}(q_i)\tau + J_{i1}^T \lambda_i \tag{5}$$

where $M_{i1}(q_i) = R_i^T M_i(q_i) R_i$, $C_{i1}(q_i, \dot{q}_i) = R_i^T(q_i) M_i(q_i) \dot{R}_i(q_i) + R_i^T C_i(q_i, \dot{q}_i) R_i(q_i)$, $G_{i1}(q_i) = R_i^T(q_i) G_i(q_i)$, $d_{i1}(t) = R_i^T(q_i) d_i(t)$, $B_{i1} = R_i^1(q_i) B_i(q_i)$, $J_{i1}^T = R_i^T(q_i) J_i^T$, $\lambda_i = \lambda_c$.

There exist a coordinate transformation and a state feedback

$$\begin{aligned} \zeta_i &= [\zeta_{ib}, \zeta_{ia}]^T = T_1(q_i) \\ &= [T_{11}(q_{ib}), q_{ia}^1, T_{12}(q_{ib}, q_{ia}^1)]^T \end{aligned} \tag{6}$$

$$v_i = [v_{ib}, v_{ia}]^T = T_2(q_i)u_i = [T_{21}(q_{ib})u_{ib}, u_{ia}]^T \tag{7}$$

with $T_2(q) = \mathrm{diag}[T_{21}(q_{iv}), I]$ and $u = [u_{ib}, u_{ia}]^T$, $u_{ia} = \dot{q}_{ia}$, so that the kinematic system (3) could be locally or globally converted to the chained form [8], [9]

$$\begin{cases} \dot{\zeta}_{ib1} = u_{i1} \\ \dot{\zeta}_{ibj} = u_{i1}\zeta_{ib(j+1)} (2 \le j \le n_v - 1) \\ \dot{\zeta}_{ibn_v} = u_{i2} \\ \dot{\zeta}_{ia} = \dot{q}_{ia} = u_{ia} \end{cases} \tag{8}$$

Consider the above transformations, the dynamic system (1) and (2) could be correspondingly converted into the following canonical transformation

$$M_{i2}(\zeta_i)\dot{u}_i + C_{i2}(\zeta_i, \dot{\zeta}_i)u_i + G_{i2}(\zeta_i) + d_{i2}(t) = B_{i2}\tau_i + J_{i2}^T \lambda_i \tag{9}$$

$M_{i2}(\zeta_i) = T_2^T(q_i) M_{i1}(q) T_2(q_i)|_{q_i = T_1^{-1}(\zeta_i)}$, $C_{i2}(\zeta_i, \dot{\zeta}_i) = T_2^T(q_i)[M_{i1}(q_i)\dot{T}_2(q_i) + C_{i1}(q_i, \dot{q}_i)T_2(q_i)]|_{q_i = T_1^{-1}(\zeta_i)}$, $G_{i2}(\zeta_i) = T_2^T(q_i) G_{i1}(q_i)|_{q_i = T_1^{-1}(\zeta_i)}$, $d_{i2}(t) = T_2^T(q_i) s_{i1}(t)|_{q_i = T_1^{-1}(\zeta_i)}$, $B_{i2} = T_2^T(q_i) B_{i1}(q_i)|_{q_i = T_1^{-1}(\zeta_i)}$, $J_{i2}^T = T_2^T(q_i) J_{i1}^T|_{q = T_1^{-1}(\zeta_i)}$.

Assumption 2.2: The Jacobian matrix J_{i2} is uniformly bounded and uniformly continuous, if q_i is uniformly bounded and uniformly continuous.

Assumption 2.3: Each manipulator is redundant and operating away from any singularity.

Remark 2.2: Under Assumptions 2.2 and 2.3, the Jacobian J_{i2} is of full rank. The vector $q_{ia} \in R^{n_{ia}}$ can always be properly rearranged and partitioned into $q_{ia} = [q_{ia}^1, \ q_{ia}^2]^T$, $q_{ia}^1 = [q_{ia1}^1, \ \ldots, q_{ia\kappa_i}^1]$ describes the constrained motion of the manipulator and $q_{ia}^2 \in R^{n_{ia}-\kappa_i}$ denotes the remaining joints variables which make the arm redundant such that the possible breakage of contact could be compensated.

Therefore, we have $J_{i2}(q_i) = [J_{i2b}, J_{i2a}^1, J_{i2a}^2]$. Considering the object trajectory and relative motion trajectory as holonomic constraints, we could obtain $\dot{q}_{ia}^2 = -(J_{i2a}^2)^{-1}[J_{i2b}\dot{q}_{ib} + J_{i2a}^1 \dot{q}_{ia}^1]$.

From (6) and (7), $\dot{q}_{ib} = S_i(q_{ib})T_{21}(q_{ib})u_{ib}$, we have

$$\begin{aligned} u_i &= \begin{bmatrix} u_{ib} \\ \dot{q}_{ia}^1 \\ -(J_{i2a}^2)^{-1}[J_{i2b}S_i(q_{ib})T_{21}(q_{ib})u_{ib} + J_{i2a}^1\dot{q}_{ia}^1] \end{bmatrix} \\ &= L_i u_i^1 \end{aligned} \tag{10}$$

where

$$\begin{aligned} &L_i \\ &= \begin{bmatrix} I & 0 \\ 0 & I \\ -(J_{i2a}^2)^{-1}J_{i2b}S_i(q_{ib})T_{21}(q_{ib}) & -(J_{i2a}^2)^{-1}J_{i2a}^1 \end{bmatrix} \\ u_i^1 &= \begin{bmatrix} u_{ib} & \dot{q}_{ia}^1 \end{bmatrix}^T \end{aligned}$$

It is easy to know that $L_i^T J_i^{2T} = 0$.

Combining (9) and (10), we can obtain the following compact dynamics:

$$M\dot{u}^1 + Cu^1 + G + d = B\tau + J^T\lambda \tag{11}$$

where $M = \begin{bmatrix} M_{12}L_1 & 0 \\ 0 & M_{22}L_2 \end{bmatrix}$, $L = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}$, $C = \begin{bmatrix} M_{12}\dot{L}_1 + C_{12}L_1 & 0 \\ 0 & M_{22}\dot{L}_2 + C_{22}L_2 \end{bmatrix}$, $G = \begin{bmatrix} G_{12} \\ G_{22} \end{bmatrix}$,

$d = \begin{bmatrix} B_{12} & 0 \\ 0 & B_{22} \end{bmatrix}$, $d = \begin{bmatrix} d_{12}(t) \\ d_{22}(t) \end{bmatrix}$, $\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$, $J^T = \begin{bmatrix} J_{12}^T \\ J_{22}^T \end{bmatrix}$, $\lambda = \lambda_c$.

Property 2.1: Matrices $\mathcal{M} = L^T M$, $\mathcal{G} = L^T G$ are uniformly bounded and uniformly continuous if $\zeta = [\zeta_1, \zeta_2]^T$ is uniformly bounded and continuous, respectively. Matrix $\mathcal{C} = L^T C$ is uniformly bounded and uniformly continuous if $\dot{\zeta} = [\dot{\zeta}_1, \dot{\zeta}_2]^T$ is uniformly bounded and continuous.

Property 2.2: $\forall \zeta \in R^{2n}$, $0 < \lambda_{min}(\mathcal{M}) I \le \mathcal{M}(\zeta) \le \beta I$ where λ_{min} is the minimal eigenvalue of $\mathcal{M}$ and $\beta > 0$.

III. Robust Adaptive Controls Design

Let $r_o^d(t)$ be the desired trajectory of the object, $r_{co}^d(t)$ be the desired trajectory on the object and $\lambda_c^d(t)$ be the desired constraint force. The first control objective is to drive the mobile manipulators such that $r_o(t)$ and $r_{co}(t)$ track their desired trajectories $r_o^d(t)$ and $r_{co}^d(t)$ respectively. The second objective is to make $\lambda_c(t)$ its desired trajectory $\lambda_c^d(t)$. The centralized control is used for two mobile manipulators.

Assumption 3.1: Time varying positive functions δ_k and α_ς converge to zero as $t \to \infty$ and satisfy $\lim_{t\to\infty} \int_0^t \delta_k(\omega) d\omega = a_k < \infty$, and $\lim_{t\to\infty} \int_0^t \alpha_\varsigma(\omega) d\omega = b_\varsigma < \infty$ with finite constants a_k with $k = 1, \ldots, 6$, and b_ς with $\varsigma = 1, \ldots, 5$ [11].

For the given ζ^d, the tracking errors are denoted as $e = \zeta - \zeta^d = [e_1, e_2]^T$, $e_i = \zeta_i - \zeta_i^d$ and $e_\lambda = \lambda_c - \lambda_c^d$. Define the reference signals $z = [z_1, z_2]^T$ and $z_i = [z_{ib}, z_{ia}]^T$ as

$$z_{ib} = \begin{bmatrix} u_{id1} + \eta_i \\ u_{id2} - s_{i(n_{iv}-1)} u_{id1} - k_{n_{iv}} s_{in_{iv}} \\ + \sum_{j=0}^{n_{iv}-3} \frac{\partial(e_{in_{iv}} - s_{in_{iv}})}{\partial u_{id1}^{(j)}} u_{id1}^{(j+1)} \\ + \sum_{j=2}^{n_{iv}-1} \frac{\partial(e_{in_{iv}} - s_{in_{iv}})}{\partial e_{ij}} e_{i(j+1)} \end{bmatrix} \quad (12)$$

$$z_{ia} = \dot{q}_{ia}^{1d} - K_{1a}(q_{ia}^1 - q_{ia}^{1d}) \quad (13)$$

$$s_i = \begin{bmatrix} e_{i1} \\ e_{i2} \\ \vdots \\ e_{in_{iv}} + s_{i(n_{iv}-2)} + k_{n_{iv}-1} s_{i(n_{iv}-1)} u_{id1}^{2l-1} \\ -\frac{1}{u_{id1}} \sum_{j=0}^{n_{iv}-4} \frac{\partial(e_{i(n_{iv}-1)} - s_{i(n_{iv}-1)})}{\partial u_{id1}^{(j)}} u_{id1}^{(j+1)} \\ - \sum_{j=2}^{n_{iv}-2} \frac{\partial(e_{i(n_{iv}-1)} - s_{i(n_{iv}-1)})}{\partial e_{ij}} e_{i(j+1)} \end{bmatrix} \quad (14)$$

$$\dot{\eta}_i = -k_0 \eta_i - k_1 s_{i1} - \sum_{j=2}^{n_{iv}-1} s_{ij} \zeta_{i(j+1)} + \sum_{k=3}^{n_{iv}} s_{ik} \sum_{j=2}^{k-1} \frac{\partial(e_{ik} - s_{ik})}{\partial e_{ik}} \zeta_{i(k+1)} \quad (15)$$

and $l = n_{iv} - 2$, $u_{id1}^{(l)}$ is the lth derivative of u_{id1} with respect to t, and k_j is positive constants, K_{ia} is diagonal positive.

Denote $\tilde{u} = [\tilde{u}_b, \tilde{u}_a]^T = u - z$ and define a filter tracking error $\sigma = \tilde{u} + K_u \int_0^t \tilde{u} ds$ with $K_u = \mathrm{diag}[0_{m\times m}, K_{u1}] > 0$, $K_{u1} \in R^{(n_{ia}-\kappa_i)\times(n_{ia}-\kappa_i)}$. We could obtain $\dot{\sigma} = \dot{\tilde{u}} + K_u \tilde{u}$ and $u = \nu + \sigma$ with $\nu = z - K_u \int_0^t \tilde{u} ds$.

We could rewrite (11) as

$$M\dot{\sigma} + C\sigma + M\dot{\nu} + C\nu + G + d = B\tau + J^T\lambda \quad (16)$$

If the system is certain, we could choose the control law given by

$$B\tau = M(\dot{\nu} - K_\sigma \sigma) + C(\nu + \sigma) + G + d - J^T \lambda_h \quad (17)$$

with diagonal matrix $K_\sigma > 0$. The force control input λ_h as $\lambda_h = \lambda_d - K_\lambda \tilde{\lambda} - K_I \int_0^t \tilde{\lambda} dt$, where $\tilde{\lambda} = \lambda_c - \lambda_c^d$, K_λ is a constant matrix of proportional control feedback gains, and K_I is a constant matrix of integral control feedback gains. However, since $\mathcal{M}(\zeta)$, $\mathcal{C}(\zeta, \dot{\zeta})$, $\mathcal{G}(\zeta)$ are uncertain, to facilitate the control formulation, the following assumption is required.

Assumption 3.2: There exist some finite positive constants b, $c_\varsigma > 0$ $(1 \le \varsigma \le 4)$, and finite non-negative constant $c_\varsigma \ge 0$ $(\varsigma = 5)$ such that $\forall \zeta \in R^{2n}$, $\forall \dot{\zeta} \in R^{2n}$, $\|\Delta M\| = \|\mathcal{M} - \mathcal{M}_0\| \le c_1$, $\|\Delta C\| = \|\mathcal{C} - \mathcal{C}_0\| \le c_2 + c_3\|\dot{\zeta}\|$, $\|\Delta G\| = \|\mathcal{G} - \mathcal{G}_0\| \le c_4$, and $\sup_{t\ge0} \|d_L(t)\| \le c_5$, where M_0, C_0 and G_0 are nominal parameters of the system.

Let $\mathcal{B} = L^T B$, the proposed control for the system is given as

$$\mathcal{B}\tau = \mathrm{U}_1 + \mathrm{U}_2 \quad (18)$$

where U_1 is the nominal control, $\mathrm{U}_1 = \mathcal{M}_0(\dot{\nu} - K_\sigma \sigma) + \mathcal{C}_0(\nu + \sigma) + \mathcal{G}_0$, and U_2 is designed to compensate for the parameter errors arising from estimating the unknown function $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{G}$ and the disturbance, respectively.

$$\begin{aligned}
\mathrm{U}_2 &= \mathrm{U}_{21} + \mathrm{U}_{22} + \mathrm{U}_{23} + \mathrm{U}_{24} + \mathrm{U}_{25} + \mathrm{U}_{26} \quad (19) \\
\mathrm{U}_{21} &= -\frac{\beta}{\lambda_{min}} \frac{\hat{c}_1^2 \|K_\sigma \sigma - \dot{\nu}\|^2 \sigma}{\hat{c}_1 \|K_\sigma \sigma - \dot{\nu}\| \|\sigma\| + \delta_1} \\
\mathrm{U}_{22} &= -\frac{\beta}{\lambda_{min}} \frac{\hat{c}_2^2 \|\sigma + \nu\|^2 \sigma}{\hat{c}_2 \|\sigma + \nu\| \|\sigma\| + \delta_2} \\
\mathrm{U}_{23} &= -\frac{\beta}{\lambda_{min}} \frac{\hat{c}_3^2 \|\dot{\zeta}\|^2 \|\sigma + \nu\|^2 \sigma}{\hat{c}_3 \|\dot{\zeta}\| \|\sigma + \nu\| \|\sigma\| + \delta_3} \\
\mathrm{U}_{24} &= -\frac{\beta}{\lambda_{min}} \frac{\hat{c}_4^2 \sigma}{\hat{c}_4 \|\sigma\| + \delta_4} \\
\mathrm{U}_{25} &= -\frac{\beta}{\lambda_{min}} \frac{\hat{c}_5^2 \|L\|^2 \sigma}{\hat{c}_5 \|L\| \|\sigma\| + \delta_5} \\
\mathrm{U}_{26} &= -\beta \frac{\|\tilde{u}_b\| \|\Lambda\|^2 \sigma}{\|\Lambda\| \|\sigma\|^2 + \delta_6}
\end{aligned}$$

where δ_k $(k = 1, \ldots, 6)$ satisfying Assumption 3.1, and $\hat{c}_\varsigma$ denoting the estimate c_ς, which are adaptively tuned according to

$$\dot{\hat{c}}_1 = -\alpha_1 \hat{c}_1 + \frac{\gamma_1}{\lambda_{min}} \|\sigma\| \|K_\sigma \sigma - \dot{\nu}\|, \hat{c}_1(0) > 0 \quad (20)$$

$$\dot{\hat{c}}_2 = -\alpha_2 \hat{c}_2 + \frac{\gamma_2}{\lambda_{min}} \|\sigma\| \|\sigma + \nu\|, \hat{c}_2(0) > 0 \quad (21)$$

$$\dot{\hat{c}}_3 = -\alpha_3 \hat{c}_3 + \frac{\gamma_3}{\lambda_{min}} \|\sigma\| \|\dot{\zeta}\| \|\sigma + \nu\|, \hat{c}_3(0) > 0 \quad (22)$$

$$\dot{\hat{c}}_4 = -\alpha_4 \hat{c}_4 + \frac{\gamma_4}{\lambda_{min}} \|\sigma\|, \ \hat{c}_4(0) > 0 \quad (23)$$

$$\dot{\hat{c}}_5 = -\alpha_5 \hat{c}_5 + \frac{\gamma_5}{\lambda_{min}} \|L\| \|\sigma\|, \ \hat{c}_5(0) > 0 \quad (24)$$

where $\alpha_\varsigma > 0$ satisfies Assumption 3.1 and $\gamma_\varsigma > 0$, ($\varsigma = 1, \ \ldots, 5$).

Theorem 3.1: Consider the mechanical system described by (9), under Assumption 2.4, using the control law (18), the following can achieved:

(i) $\zeta, \dot{\zeta}, \lambda_c$ converge to $\zeta_d, \dot{\zeta}_d, \lambda_c^d$ as $t \to \infty$; and
(ii) all the signals in the closed-loop are bounded for all $t \geq 0$.

Proof: Integrating the dynamic equation (16) together with (14), (15) and (18), the close-loop system dynamics can be written as

$$
\begin{aligned}
\dot{s}_{i1} &= \eta_i + \tilde{u}_{i1} && (25)\\
\dot{s}_{i2} &= (\eta_i + \tilde{u}_{i1})\zeta_{i3} + s_{i3}u_{id1} - k_2 s_{i2} u_{id1}^{2l} && (26)\\
&\vdots \\
\dot{s}_{in_{iv}} &= (\eta_i + \tilde{u}_{i1}) \sum_{j=2}^{n_{iv}-2} \frac{\partial(e_{in_{iv}} - s_{in_{iv}})}{\partial e_{ij}} \zeta_{i(j+1)} \\
&\quad -k_{n_{iv}} s_{in_{iv}} - s_{i(n_{iv}-1)} u_{id1} + \tilde{u}_{i2} && (27)\\
\dot{\eta}_i &= -k_0 \eta_i - \Lambda_{i1} && (28)\\
M\dot{\sigma} &= -M\dot{\nu} - C(\nu + \sigma) - G - d + B\tau \\
&\quad +J^T \lambda && (29)
\end{aligned}
$$

Let $\mathcal{D} = L^T d$. Multiplying L^T on both sides of (29), using (18), one can obtain

$$
\begin{aligned}
\dot{\sigma} &= -K_\sigma \sigma + \mathcal{M}^{-1}\Delta M(K_\sigma \sigma - \dot{\nu}) - \mathcal{M}^{-1}\Delta C(\nu + \sigma) \\
&\quad -\mathcal{M}^{-1}\Delta G - \mathcal{M}^{-1}\mathcal{D} + \mathcal{M}^{-1} \sum_{i=1}^{6} \mathrm{U}_{2i} && (30)
\end{aligned}
$$

Let $\tilde{c}_\varsigma = \hat{c}_\varsigma - c_\varsigma$, $\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2 \end{bmatrix}^T$, and $\Lambda_i = \begin{bmatrix} k_1 s_{i1} + \sum_{j=2}^{n_v-1} s_{ij}\zeta_{i(j+1)} \\ -\sum_{j=3}^{n_v} s_j \sum_{k=2}^{j-1} \frac{\partial(e_{ik}-s_{ik})}{\partial e_{ik}} \zeta_{i(k+1)} \\ s_{in_v} \\ 0 \end{bmatrix}$.

Consider the following positive definite functions:

$$
\begin{aligned}
V &= V_1 + V_2 && (31)\\
V_1 &= \frac{1}{2}\sum_{i=1}^{2}\sum_{j=2}^{n_{iv}} s_{ij}^2 + \frac{1}{2}\sum_{i=1}^{2} k_{i1}s_{i1}^2 + \frac{1}{2}\sum_{i=1}^{2}\eta_i^2 \\
V_2 &= \frac{1}{2}\sigma^T\sigma + \sum_{\varsigma=1}^{5}\frac{1}{2\gamma_\varsigma}\tilde{c}_\varsigma^2
\end{aligned}
$$

Taking the time derivative of V_1 with (25) - (28) results

$$
\begin{aligned}
\dot{V}_1 &= \sum_{i=1}^{2}\sum_{j=2}^{n_{iv}-1} s_{ij}\dot{s}_{ij} + \sum_{i=1}^{2} k_{i1}s_{i1}\dot{s}_{i1} + \sum_{i=1}^{2}\eta_i\dot{\eta}_i \\
&= -\sum_{i=1}^{2}\sum_{j=2}^{n_{iv}-1} k_{ij}s_{ij}^2 u_{id1}^{2l} - \sum_{i=1}^{2} k_{in_{iv}} s_{in_{iv}}^2 \\
&\quad -\sum_{i=1}^{2} k_0\eta_i^2 + \tilde{u}_b^T\Lambda && (32)
\end{aligned}
$$

Taking the time derivative of V_2 and integrating (30) result

$$
\begin{aligned}
\dot{V}_2 &= -\sigma^T K_\sigma \sigma \\
&+ \left[\sigma^T\mathcal{M}^{-1}\Delta M(K_\sigma\sigma - \dot{\nu}) + \sigma^T\mathcal{M}^{-1}\mathrm{U}_{21} + \frac{1}{\gamma_1}\tilde{c}_1\dot{\hat{c}}_1\right] \\
&+ \left[-\sigma^T\mathcal{M}^{-1}\Delta C(\sigma + \nu) + \sigma^T\mathcal{M}^{-1}\mathrm{U}_{22} + \frac{1}{\gamma_2}\tilde{c}_2\dot{\hat{c}}_2\right] \\
&+ \left[\sigma^T\mathcal{M}^{-1}\mathrm{U}_{23} + \frac{1}{\gamma_3}\tilde{c}_3\dot{\hat{c}}_3\right] \\
&+ \left[-\sigma^T\mathcal{M}^{-1}\Delta G + \sigma^T\mathcal{M}^{-1}\mathrm{U}_{24} + \frac{1}{\gamma_4}\tilde{c}_4\dot{\hat{c}}_4\right] \\
&+ \left[-\sigma^T\mathcal{M}^{-1}\mathcal{D} + \sigma^T\mathcal{M}^{-1}\mathrm{U}_{25} + \frac{1}{\gamma_5}\tilde{c}_5\dot{\hat{c}}_5\right] \\
&+\sigma^T\mathcal{M}^{-1}\mathrm{U}_{26} && (33)
\end{aligned}
$$

The second right-hand term of (33) is bounded by

$$
\begin{aligned}
&\sigma^T\mathcal{M}^{-1}\Delta M(K_\sigma\sigma - \dot{\nu}) + \sigma^T\mathcal{M}^{-1}u_{21} + \frac{1}{\gamma_1}\tilde{c}_1\dot{\hat{c}}_1 \\
&\leq \frac{1}{\lambda_{min}}\delta_1 - \frac{\alpha_1}{\gamma_1}(\hat{c}_1 - \frac{1}{2}c_1)^2 + \frac{\alpha_1}{4\gamma_1}c_1^2 && (34)
\end{aligned}
$$

Similarly, the third right-hand term of (33) is bounded by

$$
\begin{aligned}
&-\sigma^T M^{-1}\Delta C(\sigma + \nu) + \sigma^T M^{-1}u_{22} + \sigma^T M^{-1}u_{23} \\
&+\frac{1}{\gamma_2}\tilde{c}_2\dot{\hat{c}}_2 + \frac{1}{\gamma_3}\tilde{c}_3\dot{\hat{c}}_3 \\
&\leq \frac{1}{\lambda_{min}}\delta_2 - \frac{\alpha_2}{\gamma_2}(\hat{c}_2 - \frac{1}{2}c_2)^2 + \frac{\alpha_2}{4\gamma_2}c_2^2 \\
&+\frac{1}{\lambda_{min}}\delta_3 - \frac{\alpha_3}{\gamma_3}(\hat{c}_3 - \frac{1}{2}c_3)^2 + \frac{\alpha_3}{4\gamma_3}c_3^2 && (35)
\end{aligned}
$$

Similarly, the fourth right-hand term of (33) is bounded by

$$
\begin{aligned}
&\sigma^T M^{-1}\Delta G + \sigma^T M^{-1}u_{24} + \frac{1}{\gamma_4}\tilde{c}_4\dot{\hat{c}}_4 \\
\leq\ &\frac{1}{\lambda_{min}}\delta_4 - \frac{\alpha_4}{\gamma_4}(\hat{c}_4 - \frac{1}{2}c_4)^2 + \frac{\alpha_4}{4\gamma_4}c_4^2 && (36)
\end{aligned}
$$

Similarly, the fifth right-hand term of (33) is bounded by

$$
\begin{aligned}
&\sigma^T\mathcal{M}^{-1}\mathcal{D} + \sigma^T\mathcal{M}^{-1}u_{25} + \frac{1}{\gamma_5}\tilde{c}_5\dot{\hat{c}}_5 \\
\leq\ &\frac{1}{\lambda_{min}}\delta_5 - \frac{\alpha_5}{\gamma_5}(\hat{c}_5 - \frac{1}{2}c_5)^2 + \frac{\alpha_5}{4\gamma_5}c_5^2 && (37)
\end{aligned}
$$

Integrating (32) and (33), it is easily to obtain that

$$
\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^{2}\sum_{j=2}^{n_{iv}-1} k_{ij}s_{ij}^2 u_{id1}^{2l} - \sum_{i=1}^{2} k_{in_{iv}}s_{in_{iv}}^2 \\
&\quad -\sum_{i=1}^{2} k_0\eta_i^2 + \tilde{u}_b^T\Lambda - \sigma^T K_\sigma\sigma - \sum_{\varsigma=1}^{5}\frac{\alpha_\varsigma}{\gamma_\varsigma}(\hat{c}_\varsigma - \frac{1}{2}c_\varsigma)^2 \\
&\quad +\frac{1}{\lambda_{min}}\sum_{k=1}^{5}\delta_k + \sum_{\varsigma=1}^{5}\frac{\alpha_\varsigma}{4\gamma_\varsigma}c_\varsigma^2 + \sigma^T\mathcal{M}^{-1}\mathrm{U}_{26} && (38)
\end{aligned}
$$

The fourth and ninth right-hand term of (38) is bounded by

$$
\begin{aligned}
\tilde{u}_b^T\Lambda + \sigma^T\mathcal{M}^{-1}\mathrm{U}_{26} &\leq \|\tilde{u}_b\|\|\Lambda\| - \frac{\|\tilde{u}_b\|\|\Lambda\|^2\|\sigma\|^2}{\|\Lambda\|\|\sigma\|^2 + \delta_6} \\
&\leq \delta_6 && (39)
\end{aligned}
$$

Therefore, we can rewrite (38) as

$$\begin{aligned}\dot{V} \quad &\leq -\sum_{i=1}^{2}\sum_{j=2}^{n_{iv}-1} k_{ij}s_{ij}^2 u_{id1}^{2l} - \sum_{i=1}^{2} k_{in_v}s_{in_{iv}}^2 \\ &-\sum_{i=1}^{2} k_0\eta_i^2 - \sigma^T K_\sigma \sigma - \sum_{\varsigma=1}^{5}\frac{\alpha_\varsigma}{\gamma_\varsigma}(\hat{c}_\varsigma - \frac{1}{2}c_\varsigma)^2 \\ &+\frac{1}{\lambda_{min}}\sum_{k=1}^{5}\delta_k + \delta_6 + \sum_{\varsigma=1}^{5}\frac{\alpha_\varsigma}{4\gamma_\varsigma}c_\varsigma^2 \end{aligned} \quad (40)$$

Noting Assumption 3.1, we have $\mathcal{B} = \frac{1}{\lambda_{min}}\sum_{k=1}^{5}\delta_k + \sum_{\varsigma=1}^{5}\frac{\alpha_\varsigma}{4\gamma_\varsigma}c_\varsigma^2 + \delta_6 \to 0$ as $t \to \infty$.

Let $\mathcal{A} = \sum_{i=1}^{2} k_0\eta_i^2 + \sum_{i=1}^{2} k_{in_{iv}}s_{in_{iv}}^2 + \sum_{i=1}^{2}\sum_{j=2}^{n_{iv}-1} k_{ij}s_{ij}^2 u_{id1}^{2l} + \lambda_{min}(K_\sigma)\|\sigma\|^2 + \sum_{\varsigma=1}^{5}\frac{\alpha_\varsigma}{\gamma_\varsigma}(\hat{c}_\varsigma - \frac{1}{2}c_\varsigma)^2$, and it is easy to know $\mathcal{A} > 0$.

Integrating both sides of (40) gives $V(t) - V(0) \leq -\int_0^t \mathcal{A}ds + \int_0^t \mathcal{B}ds < -\int_0^t \mathcal{A}ds + \mathcal{C}$, where $\mathcal{C} = \sum_{k=1}^{5}\frac{a_k}{\lambda_{min}} + \sum_{\varsigma=1}^{5}\frac{b_\varsigma}{4\gamma_\varsigma}c_\varsigma^2 + a_6$ is a finite constant from Assumption 3.1, we have $V(t) < V(0) - \int_0^t \mathcal{A}ds + \mathcal{C}$. Thus V is bounded and $V(t)$ is no increasing and $\dot{V}(t) < 0$.

Substituting the control (18) into the reduced order dynamics (11) yields $J^T[(K_\lambda+1)e_\lambda + K_I\int_0^t e_\lambda dt] = M(\dot{\sigma} + \dot{\nu}) + C(\nu+\sigma) + G + d - L(L^TL)^{-1}(\mathrm{U}_1 + \mathrm{U}_2)$. Since $\dot{\sigma}$, σ, $\dot{\nu}$, ν, c_i, α_i, $\dot{\zeta}$, γ_i, Λ, δ_i are all bounded, the right hand side of (III) is bounded, i.e., $J^T[(K_\lambda+1)e_\lambda + K_I\int_0^t e_\lambda dt] = \Gamma(\dot{\sigma},\sigma,\dot{\nu},\nu,c_i,\alpha_i,\dot{\zeta},\gamma_i,\Lambda,\delta_i)$, $\Gamma(*) \in L_\infty$.

Let $\int_0^t e_\lambda dt = E_\lambda$, then $\dot{E}_\lambda = e_\lambda$. By appropriate choosing $K_\lambda = \mathrm{diag}[K_{\lambda,i}]$, $K_{\lambda,i} > -1$ and $K_I = \mathrm{diag}[K_{I,i}]$, $K_{I,i} > 0$ to make $E_i(p) = \frac{1}{(K_{\lambda,i}+1)p + K_{I,i}}, p = d/dt$ a strictly proper exponential stable transfer function, it can be concluded that $\int_0^t e_\lambda dt \in L_\infty$, $e_\lambda \in L_\infty$, and the size of e_λ can be adjusted by choosing the proper gain matrices K_λ and K_I.

Since $\dot{\sigma}$, σ, $\dot{\nu}$, ν, c_i, α_i, $\dot{\zeta}$, γ_i, Λ, δ_i, e_λ and $\int_0^t e_\lambda dt$ are all bounded, it is easy to conclude that τ is bounded.

IV. Simulation Studies

Consider the two 3-DOF mobile manipulators systems shown in Fig. 2. The parameters are set as as $m_p = 5kg$, $m_1 = 1.0kg$, $m_2 = m_3 = 0.5kg$, $I_w = 1.0kgm^2$, $I_p = 2.5kgm^2$, $I_1 = 1.0kgm^2$, $I_2 = 0.5kgm^2$, $I_3 = 0.5kgm^2$, $d = l = r = 0.5m$, $2l_1 = 1.0m$, $2l_2 = 0.5m$, $2l_3 = 0.5m$. The mass of the object is $m_{obj} = 0.5kg$. Let us set the desired trajectories of the object as $r_{od} = [x_{od}, y_{od}, z_{od}]^T$, $x_{od} = 1.7\cos(t)$, $y_{od} = 1.7\sin(t)$, $z_{od} = 2l_1$. Therefore, we could obtain the corresponding desired trajectory of mobile manipulator II as $q_{2d} = [x_{2d}, y_{2d}, \theta_{2d}, \theta_{21d}, \theta_{22d}]^T$ with $x_d = 1.0\cos(t)$, $y_d = 1.0\sin(t)$, $\theta_d = t$, $\theta_{21d} = \pi/2rad$, θ_{i2}, θ_{i3} are redundant joints to control force and compensate the task space errors. The end-effector hold tightly on the top point of surface. The constraint relative motion by the mobile manipulator I is an arc with the center on the joint 2 of the mobile manipulator I, angle$= \pi/2 - \pi/6\cos(t)$, $R = 0.7m$ and $z_c^d = 1.0m$, and the constraint force is set as $\lambda_c^d = 15.0N$. Therefore, from the constraint relative motion, we can obtain the desired trajectory of mobile manipulator I is $q_{1d} = [x_{1d}, y_{1d}, \theta_{1d}, \theta_{11d}, \theta_{12d}]^T$ with the corresponding trajectories $x_{1d} = 2.4\cos(t)$, $y_{1d} = 4.0\sin(t)$, $\theta_{1d} = t$, $\theta_{11d} = \pi/2 - \pi/6\cos(t)$, and θ_{22}, θ_{23} are used to compensate the position errors of the mobile platform.

The initial conditions selected for mobile manipulator I are $x_1(0) = 2.5m$, $y_1(0) = 0.0m$, $\theta_1(0) = 0.0rad$, $\theta_{11}(0) = 0.5253rad$, $\theta_{12}(0) = -0.9273rad$, $\theta_{13}(0) = 1.8546rad$, $\lambda(0) = 0.0N$ and $\dot{x}_1(0) = 0.5m/s$, $\dot{y}_1(0) = \dot{\theta}_1(0) = \dot{\theta}_{11}(0) = \dot{\theta}_{12}(0) = \dot{\theta}_{13(0)} = 0.0$, and the initial conditions selected for mobile manipulator II are $x_2(0) = 1.2m$, $y_2(0) = 0m$, $\theta_2(0) = 0.0rad$, $\theta_{21}(0) = 1.57rad$, $\theta_{22}(0) = 2.62rad$, $\theta_{23}(0) = 1.05rad$, and $\dot{x}_2(0) = \dot{y}_2(0) = \dot{\theta}_2(0) = \dot{\theta}_{12}(0) = \dot{\theta}_{22}(0) = \dot{\theta}_{23}(0) = 0.0$.

In the simulation, the design parameters for each mobile manipulator are set as $k_0 = 5.0$, $k_1 = 180.0$, $k_2 = 5.0$, $k_3 = 5.0$, $\eta(0) = 0.0$, $K_{a1} = \mathrm{diag}[2.0]$, $K_\lambda = 0.3$, $K_I = 1.5$, $K_\sigma = \mathrm{diag}[0.5]$, $K_u = \mathrm{diag}[1.0]$. The design parameters in u_2 of (19) are $\gamma_i = 0.1$, $\alpha_i = \delta_i = 1/(1+t)^2$, and $\hat{c}_i(0) = 1.0$. The disturbances on each joint of each mobile manipulator are set to a time varying form as $0.5\sin(t)$, $0.5\sin(t)$, $0.1\sin(t)$ and $0.1\sin(t)$, respectively. If using the control law (18), we can obtain Fig. 3 to describe the trajectory of the mobile platforms of both mobile manipulators. Fig. 4, Fig. 7, Fig. 5 and Fig. 8 show the trajectory tracking $(\zeta - \zeta_d)$ of the joints with the disturbances for both mobile manipulators. Fig. 10 shows the contact force tracking $\lambda_c - \lambda_c^d$. Therefore, the validity of the proposed controls are confirmed by these simulation results.

V. Conclusion

In this paper, dynamics and control of two mobile robotic manipulators manipulating a constrained object have been investigated. In addition to the motion of the object with respect to the world coordinates, its relative motion with respect to the mobile manipulators is also taken into consideration. Robust adaptive controls have been developed which can guarantee the convergence of positions and boundedness of the constraint force.

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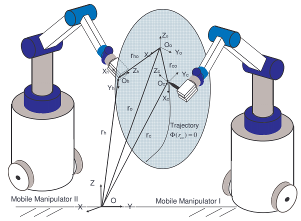

Fig. 1. Coordinated operation of two robots

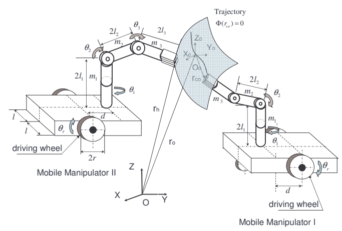

Fig. 2. Cooperating 2-DOF mobile manipulators

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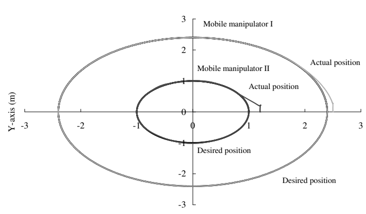

Fig. 3. Tracking trajectories of both mobile platforms

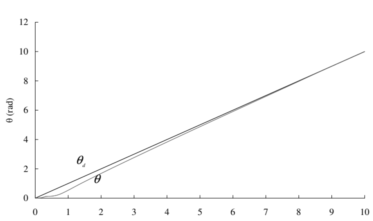

Fig. 4. Tracking of θ for mobile manipulator I

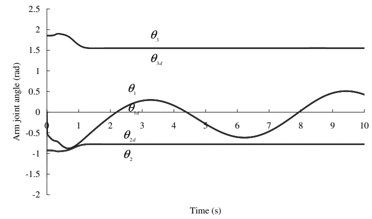

Fig. 5. Tracking of arm joint angles of mobile manipulator I

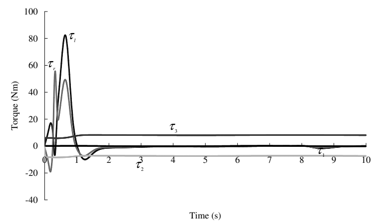

Fig. 6. Input torques for mobile manipulator I

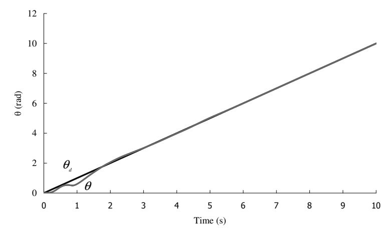

Fig. 7. Tracking of θ for mobile manipulator II

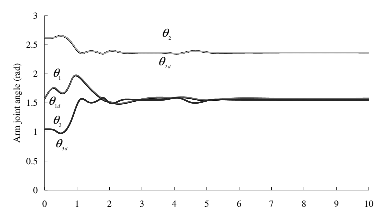

Fig. 8. Tracking of arm joint angles of mobile manipulator II

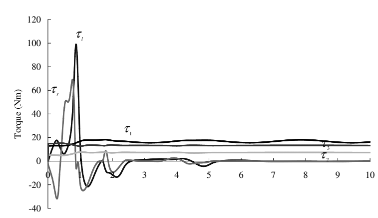

Fig. 9. Torques of mobile manipulator II

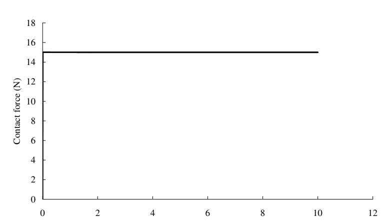

Fig. 10. Contact force of relative motion