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Adaptive Robust Output-feedback Motion/Force Control of Electrically Driven Nonholonomic Mobile Manipulators

Zhijun Li, Shuzhi Sam Ge, Martin Adams, and Wijerupage Sardha Wijesoma

Abstract— **In this paper, adaptive robust output-feedback force/motion control strategies are presented for mobile manipulators under both holonomic and nonholonomic constraints in the presence of uncertainties and disturbances. The controls are developed on structural knowledge of the dynamics of the robot, and the dynamics of actuators. The system stability and the boundedness of tracking errors are proved using Lyapunov stability synthesis. Simulation results validate that not only the states of the system asymptotically converge to the desired trajectory, but also the constraint force asymptotically converges to the desired force.**

I. INTRODUCTION

Mobile manipulators refer to robotic manipulators mounted on mobile platforms. Such systems combine the advantages of mobile platforms and robotic arms and reduce their drawbacks. Mobile manipulators possess complex and strongly coupled dynamics of mobile platforms and manipulators. Input-output feedback linearization was investigated to control the mobile platform such that the manipulator is always positioned at the preferred configurations measured by its manipulability [1]. Similarly, through nonlinear feedback linearization and decoupling dynamics in [6], force/position control of the end-effector along the same direction for mobile manipulators was proposed and applied to nonholonomic cart pushing. In [2], the effect of the dynamic interaction between the arm and the vehicle of a mobile manipulator on the tracking performance was studied, and nonlinear feedback control for the mobile manipulator was developed to compensate for the dynamic interaction. In [3], coordination and control of mobile manipulators were presented with two basic task-oriented controls: end-effector task control and platform self posture control.

To solve for the unknown parameters cases, adaptive schemes have been investigated to deal with dynamics uncertainty of mobile manipulators. In [4], adaptive neural network controls had been developed for the motion control of mobile manipulators subject to kinematic constraints. In [5], adaptive control was proposed for trajectory/force control of mobile manipulators subjected to holonomic and nonholonomic constraints with unknown inertia parameters, which ensures the state of the system to asymptotically converge to the desired trajectory and force.

Z. Li and S. S. Ge are with Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576 Email:elegesz@nus.edu.sg

Z. Li, M. Adams and W. S. Wijesoma are with School of Electrical and Electronics Engineering, Nanyang Technological University, Singapore 639798

Z. Li is also with Department of Automation, Shanghai JiaoTong University, Shanghai

As demonstrated in [8], actuator dynamics constitute an important component of the complete robot dynamics, especially in the case of high-velocity movement and highly varying loads. Many control methods have therefore been developed to take into account the effects of actuator dynamics[9], [11]. In this paper, we will treat actuator input voltages as control inputs and only use the position and the driving current of actuator to address adaptive robust output-feedback control of force/motion for a class of mobile manipulator systems that are electrically driven by DC motors. The systems are under both holonomic and nonholonomic constraints in the parameter uncertainties and external disturbances.

II. SYSTEM DESCRIPTION

Consider an n DOF mobile manipulator with nonholonomic mobile base, the dynamics can be described as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d(t) = B(q)\tau + f \tag{1}$$

where $q = [q_v, q_r]^T \in R^n$ with $q_v \in R^v$ denoting the generalized coordinates for the mobile platform and $q_r \in R^r$ denoting the coordinates of the manipulator, $n = v + r$, and the symmetric positive definite inertia matrix $M(q) \in R^{n\times n}$, the Centripetal and Coriolis torques $C(\dot{q}, q) \in R^{n\times n}$, the gravitational torque vector $G(q) \in R^n$, the external disturbances $d(t) \in R^n$ and the control inputs $\tau \in R^m$ could be represented as, respectively

$$M(q) = \begin{bmatrix} M_v & M_{vr} \\ M_{rv} & M_r \end{bmatrix}, C(\dot{q}, q) = \begin{bmatrix} C_v & C_{vr} \\ C_{rv} & C_a \end{bmatrix}$$
$$G(q) = \begin{bmatrix} G_v \\ G_r \end{bmatrix}, d(t) = \begin{bmatrix} d_v(t) \\ d_r(t) \end{bmatrix}, \tau = \begin{bmatrix} \tau_v \\ \tau_r \end{bmatrix}$$

$B(q) = \mathrm{diag}(B_v,\ B_r) \in R^{n\times m}$ is a full rank input transformation matrix and is assumed to be known because it is a function of fixed geometry of the system; $f = J^T\lambda \in R^m$ denotes the vector of constraint forces; $J^T \in R^{n\times m}$ is Jacobian matrix; and $\lambda = [\lambda_n, \lambda_h]^T \in R^m$ are Lagrangian multipliers corresponding to the nonholonomic and holonomic constraints.

Assume that the l_n non-integrable and independent velocity constraints on the mobile platform can be viewed as restricting the dynamics on the manifold Ω_n

$$\Omega_n = \{(q_v, \dot{q}_v) | A(q_v)\dot{q}_v = 0\} \tag{2}$$

where $A = [A_1^T(q_v),\ \ldots,\ A_{l_n}^T(q_v)]^T : R^v \to R^{l_n\times v}$ is the kinematic constraint matrix which is assumed to have full rank l_n. The effect of the nonholonomic constraints can be viewed as the generalized constraint forces given by $f_v =$

$A^T(q)\lambda_n$, where $\lambda_n \in R^{l_n}$ is known as force on the contact point between the rigid body and environmental surface.

Assume that the annihilator of the co-distribution spanned by the covector fields $A_1(q_v), \ \ldots, \ A_{l_n}(q_v)$ is an $(v - l_n)$-dimensional smooth nonsingular distribution Δ on R^v. This distribution Δ is spanned by a set of $(v - l_n)$ smooth and linearly independent vector fields $H_1(q_v), \ \ldots, \ H_{v-l_n}(q_v)$, i.e., $\Delta = \text{span}\{H_1(q_v), \ \ldots, \ H_{v-l_n}(q_v)\}$. Thus, $H^T(q_v)A^T(q_v) = 0$, $H(q_v) = [H_1(q_v), \ \ldots, \ H_{v-l_n}(q_v)] \in R^{v\times(v-l_n)}$. Note that H^TH is of full rank. Constraints (2) implies the existence of vector $\dot{\eta} \in R^{v-l_n}$, such that

$$\dot{q}_v = H(q_v)\dot{\eta} \tag{3}$$

Integrating the nonholonomic constraints (2) and the transformation (3) and their derivatives, the dynamics of a mobile manipulator can be expressed as

$$\begin{aligned} M_1(q_1)\ddot{q}_1 + C_1(q_1,\dot{q}_1)\dot{q}_1 + G_1(q_1) + d_1(t) &= B^1(q_1)\tau \\ &+ f_1 \end{aligned} \tag{4}$$

$$\begin{aligned}
M_1(q_1) &= \begin{bmatrix} H^TM_vH & H^TM_{vr} \\ M_{rv}H & M_r \end{bmatrix}, q_1 = \begin{bmatrix} \eta \\ q_r \end{bmatrix} \\
C_1(q_1,\dot{q}_1) &= \begin{bmatrix} H^TC_v\dot{H} + H^TC_vH & H^TC_{va} \\ M_{av}\dot{H} + C_{av}H & C_r \end{bmatrix} \\
G_1(q_1) &= \begin{bmatrix} H^TG_v \\ G_r \end{bmatrix}, d_1(t) = \begin{bmatrix} H^Td_v \\ d_r \end{bmatrix} \\
B^1(q_1)\tau &= \begin{bmatrix} H^TB_v\tau_v \\ B_r\tau_r \end{bmatrix}, f_1 = \begin{bmatrix} 0 & 0 \\ J_v & J_r \end{bmatrix}^T \begin{bmatrix} 0 \\ \lambda_h \end{bmatrix}
\end{aligned}$$

Assume that the k independent holonomic constraints on the system (4) can be written as

$$h(q) = 0, \ h(q) \in R^k \tag{5}$$

Assume that $h(q)$ is of full rank, then we have $J(q) = \frac{\partial h(q)}{\partial q}$ and $J(q)\dot{q} = 0$. The effect of the holonomic constraints can be viewed as the generalized constraint forces f_h converted to the joint space as $f_h = J^T\lambda_h$. Hence, the holonomic constraint on the robot's end-effector can be viewed as restricting only the dynamics on the constraint manifold Ω_h defined by $\Omega_h = \{(q,\dot{q})|h(q) = 0, J(q)\dot{q} = 0\}$.

Assume that the mobile manipulator is a series-chain multi-link manipulator with holonomic constraints (i.e. geometric constraints). According to the implicit function theorem, the vector q_r can be properly rearranged and partitioned into the form $q_r = [q_r^1, q_r^2]^T$, $q_r^1 \in R^{r-k}$ describes the constrained motion of the manipulator, $q_r^2 \in R^k$ denotes the remaining joint variable. Then, $J(q) = [\frac{\partial h}{\partial \eta}, \frac{\partial h}{\partial q_r^1}, \frac{\partial h}{\partial q_r^2}]$ Moreover, there is a unique function such that the q is expressed explicitly as the function of $\zeta = [\eta, q_r^1]^T$, that is, $q_1 = q_1(\zeta)$ [7], and we have $\dot{q}_1 = L(\zeta)\dot{\zeta}$, where $L(\zeta) = \frac{\partial q_1}{\partial \zeta}$, $\ddot{q}_1 = L(\zeta)\ddot{\zeta} + \dot{L}(\zeta)\dot{\zeta}$, and $L(\zeta)$, $J^1(\zeta) = J(q_1(\zeta))$ satisfies the relationship $L^T(\zeta)J^{1T}(\zeta) = 0$. The dynamics (4) restricted to the constraint surface, can be transformed into the reduced order model:

$$M_2\ddot{\zeta} + C_2\dot{\zeta} + G_2 + d_2(t) = u + J^{1T}\lambda_h \tag{6}$$

where $M_2 = M_1(q_1)L(\zeta)$, $C_2 = M_1(q_1)\dot{L}(\zeta) + C_1(q_1,\dot{q}_1)L(\zeta)$, $G_2 = G_1(q_1)$, $d_2(t) = d_1(t)$ and $u = B^1(q_1)\tau$.

Multiplying L^T on both sides of (6), we can obtain

$$M_L(\zeta)\ddot{\zeta} + C_L(\zeta,\dot{\zeta})\dot{\zeta} + G_L + d_L(t) = L^Tu \tag{7}$$

where $M_L(\zeta) = L^T(\zeta)M_1L(\zeta)$, $C_L(\zeta,\dot{\zeta}) = L^T(\zeta)C_2$, $G_L = L^T(\zeta)G_2$, $d_L = L^T(\zeta)d_2$.

The force multipliers λ_h can be obtained by (6)

$$\lambda_h = Z(C_2\dot{\zeta} + G_2 + d_2(t) - u) \tag{8}$$

where $Z = (J^1(M_1)^{-1}J^{1T})^{-1}J^1(M_1)^{-1}$.

Property 2.1: The matrix $M_L(\zeta)$ is symmetric and positive definite.

Property 2.2: The matrix $\dot{M}_L(\zeta) - 2C_L(\zeta,\dot{\zeta})$ is skew-symmetric. Moreover, for the $C_L(\zeta,\dot{\zeta})$ satisfies $C_L(\zeta,x)y = C_L(\zeta,y)x$ and $C_L(\zeta,z+kx)y = C_L(\zeta,z)y + kC_L(\zeta,x)y$, $\forall x,y$, k is scalar.

Property 2.3: For holonomic systems, matrices $J^1(\zeta)$, $L(\zeta)$ are uniformly bounded and uniformly continuous if ζ is uniformly bounded and continuous, respectively.

Property 2.4: There exist some finite positive constants $c_i > 0$ $(1 \leq i \leq 4)$ and finite non-negative constant $c_i \geq 0$ $(i = 5)$ such that $\forall\zeta \in R^n$, $\forall\dot{\zeta} \in R^n$, $||M_L(\zeta)|| \leq c_1$, $||C_L(\zeta,\dot{\zeta})|| \leq c_2 + c_3||\dot{\zeta}||$, $||G_L(\zeta)|| \leq c_4$, and $\sup_{t\geq 0}||d_L(t)|| \leq c_5$.

III. ACTUATOR DYNAMICS

The joints of the mobile manipulators are assumed to be driven by DC motors. Consider the following notations used to model a DC motor: $\nu \in R^m$ represents the control input voltage vector; I denotes an m-element vector of motor armature current; $K_N \in R^{m\times m}$ is a positive definite diagonal matrix which characterizes the electromechanical conversion between current and torque; $L_a = \text{diag}\,[L_{a1}, L_{a2}, L_{a3}, ..., L_{am}]$, $R_a = \text{diag}\,[R_{a1}, R_{a2}, R_{a3}, ..., R_{am}]$, $K_e = \text{diag}\,[K_{e1}, K_{e2}, K_{e3}, ..., K_{em}]$, $\omega = [\omega_1, \omega_2, ..., \omega_m]^T$ represent the equivalent armature inductances, resistances, back EMF constants, angular velocities of the driving motors, respectively; $G_r = \text{diag}(g_{ri}) \in R^{m\times m}$ denotes the gear ratio for m joints; τ_m are the torque exerted by the motor. In order to apply the DC servomotors for actuating an n-DOF mobile manipulator, assuming no energy losses, a relationship between the ith joint velocity $\dot{q}_i$ and the motor shaft velocity ω_i can be presented as $g_{ri} = \frac{\omega_i}{\dot{q}_i} = \frac{\tau_i}{\tau_{mi}}$ with the gear ratio of the ith joint g_{ri}, the ith motor shaft torque τ_{mi}, and the ith joint torque τ_i. The motor shaft torque is proportional to the motor current $\tau_m = K_NI$. The back EMF is proportional to the angular velocity of the motor shaft, then we can obtain

$$L_a\frac{dI}{dt} + R_aI + K_e\omega = \nu \tag{9}$$

In the actuator dynamics (9), the relationship between ω and $\dot{\zeta}$ is dependent on the type of mechanical system and can be generally expressed as

$$\omega = G_rT\dot{\zeta} \tag{10}$$

where T will be defined later.

The structure of T depends on the mechanical systems to be controlled. For instance, in the simulation example, a two-wheel differential drive 2-DOF mobile manipulator is used to illustrate the control design. From [11], we have

$$\begin{aligned} v &= (r\dot{\theta}_l + r\dot{\theta}_r)/2 \\ \dot{\theta} &= (r\dot{\theta}_r - r\dot{\theta}_l)/2l \\ \dot{\theta}_1 &= \dot{\theta}_1 \\ \dot{\theta}_2 &= \dot{\theta}_2 \end{aligned}$$

where $\dot{\theta}_l$ and $\dot{\theta}_r$ are the angular velocities of the left and right wheels, respectively, v is the linear velocity of the mobile platform, as shown in Fig. 1. Since $\dot{y} = v\cos\theta$, we have

$$\begin{bmatrix} \dot{\theta}_l & \dot{\theta}_r & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T = T \begin{bmatrix} \dot{y} & \dot{\theta} & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T \tag{11}$$

$$T = \begin{bmatrix} \frac{1}{r\cos\theta} & \frac{l}{r} & 0 & 0 \\ \frac{1}{r\cos\theta} & -\frac{l}{r} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

where r and l are shown in Fig. 1.

Eliminating ω from the actuator dynamics (9) by substituting (10), one obtains

$$\begin{aligned} L^T B^1 G_r K_N I &= M_L(\zeta)\ddot{\zeta} + C_L(\zeta,\dot{\zeta})\dot{\zeta} + G_L \\ &\quad + d_L(t) \qquad (13) \\ \lambda_h &= Z(\zeta)(C_2\dot{\zeta} + G_2 + d_2(t) \\ &\quad - B^1 G_r K_N I) \qquad (14) \\ \nu &= L_a \frac{dI}{dt} + R_a I + K_e G_r T\dot{\zeta} \qquad (15) \end{aligned}$$

Until now we have brought the kinematics (2), dynamics (13), (14) and actuator dynamics (15) of the considered nonholonomic system from the generalized coordinate system $q \in R^n$ to feasible independent generalized velocities $\zeta \in R^{n-l_n-k}$ without violating the nonholonomic constraint (2).

Assumption 3.1: The actuator parameters L_a, R_a and K_e are considered unknown for control design such that $||L_a|| \leq \alpha_1,\ ||R_a|| \leq \alpha_2,\ ||K_e|| \leq \alpha_3$ with finite positive constants $\alpha_i, (i = 1,\ \ldots, 3)$.

Remark 3.1: In reality, these constants c_i $(1 \leq i \leq 5)$ and α_i $(1 \leq i \leq 3)$ cannot be obtained easily. Although any fixed large c_i and α_i can guarantee good performance, it is not recommended in practice as large c_i and α_i simply, in general, high noise amplification and high cost of control. Therefore, it is necessary to develop a control law which does not require the knowledge of c_i $(1 \leq i \leq 5)$ and α_i $(1 \leq i \leq 3)$, and adaptively tune the values of the parameters as necessary.

IV. PROBLEM STATEMENT

Given a desired motion trajectory $\zeta_d(t) = [\eta_d\ q^1_{r\,d}]^T$ and a desired constraint force $f_d(t)$, or, equivalently, a desired multiplier $\lambda_h^d(t)$, the trajectory and force tracking control is to determine a control law such that for any $(\zeta(0), \dot{\zeta}(0)) \in \Omega$, $\zeta, \dot{\zeta}$, λ_h asymptotically converge to a manifold Ω_d specified as Ω where

$$\Omega_d = \{(\zeta, \dot{\zeta}, \lambda_h) | \zeta = \zeta_d, \dot{\zeta} = \dot{\zeta}_d, \lambda_h = \lambda_h^d\} \tag{16}$$

The control design consists of two stages: (i) a virtual input I_d is designed so that the subsystems (13) and (14) converge to the desired values, and (ii) the actual control input ν is designed in such a way that $I \to I_d$. In turn, this allows $\zeta - \zeta_d$ and $\lambda_h - \lambda_h^d$ to be stabilized to the origin.

Assumption 4.1: The desired reference trajectory $\zeta_d(t)$ is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order. The desired Lagrangian multiplier $\lambda_h^d(t)$ is also bounded and uniformly continuous.

V. ADAPTIVE ROBUST OUTPUT-FEEDBACK CONTROL

Lemma 5.1: For $x > 0$ and $\delta \geq 1$, we have $\ln(\cosh(x)) + \delta \geq x$.

Proof: See Appendix.

Remark 5.1: Lemma 5.1 is used to facilitate the control design.

A. Kinematic & Dynamic Subsystems

Consider the following signals:

$$\begin{aligned} \dot{\zeta}_r &= \dot{\zeta}_d - K_\zeta(\hat{\zeta} - \zeta) = \dot{\zeta}_d - K_\zeta e_\zeta + K_\zeta \tilde{\zeta} \qquad (17) \\ r_1 &= \dot{\zeta} - \dot{\zeta}_r = \dot{e}_\zeta + K_\zeta e_\zeta - K_\zeta \tilde{\zeta} \qquad (18) \\ \dot{\zeta}_o &= \dot{\hat{\zeta}} - K_\zeta \tilde{\zeta} \qquad (19) \\ r_2 &= \dot{\zeta} - \dot{\zeta}_o = \dot{\tilde{\zeta}} + K_\zeta \tilde{\zeta} \qquad (20) \\ r &= r_1 + r_2 \qquad (21) \\ e_\lambda &= \lambda_h - \lambda_h^d \qquad (22) \end{aligned}$$

where $e_\zeta = \zeta - \zeta_d$, $\tilde{\zeta} = \zeta - \hat{\zeta}$ with $\hat{\zeta}$ denoting the estimate of ζ and K_ζ is diagonal positive.

The linear observer [10] for velocity estimation is introduced to the system

$$\begin{aligned} \dot{\hat{\zeta}} &= z + K_\zeta \tilde{\zeta} + k_d \tilde{\zeta} \qquad (23) \\ \dot{z} &= \ddot{\zeta}_r + k_d K_\zeta \tilde{\zeta} \qquad (24) \end{aligned}$$

where k_d is a positive constant.

Let $I = [I_a, I_b]^T$, $I_a \in R^{n-l_n-k}$ for the position control joints and $I_b \in R^k$ for force control joints, and consider the control u of the following form:

$$\begin{aligned} u &= L^{+T} u_a - J^{1T} u_b \qquad (25) \\ u_a &= B_a^1 G_r K_{Na} I_a \\ u_b &= B_b^1 G_r K_{Nb} I_b \end{aligned}$$

where $B^1 = [B_a^1\ B_b^1]^T$, $u_a, B_a^1 \in R^{n-l_n-k}$ and $u_b, B_b^1 \in R^k$ and $L^+ = (L^T L)^{-1} L^T$. Then, equations (7) and (8) can be rewritten as

$$B_a^1 G_r K_{Na} I_a = M_L(\zeta)\ddot{\zeta} + C_L(\zeta,\dot{\zeta})\dot{\zeta} + G_L + d_L(t) \tag{26}$$

$$\begin{aligned} \lambda_h &= Z(\zeta)(C_2\dot{\zeta} + G_2 + d^1(t) \\ &\quad - B_a^1 G_r K_{Na} I_a) + B_b^1 G_r K_{Nb} I_b \qquad (27) \end{aligned}$$

Consider the following virtual control laws:

$$
\begin{aligned}
I_d &= [I_{ad}, I_{bd}]^T && (28)\\
I_{ad} &= (B_a^1 G_r K_{Na})^{-1}(-K_p(r_1 - r_2) - K_i(\tilde{\zeta} + e_\zeta) \\
&\quad -\mathrm{sgn}(r)[\ln(\cosh(\hat{\Phi})) + \delta]) && (29)\\
\hat{\Phi} &= \hat{C}^T \Psi && (30)\\
\dot{\hat{C}}^T &= -\Lambda \hat{C}^T + \Gamma \Psi ||r|| && (31)\\
I_{bd} &= (B_b^1 G_r K_{Nb})^{-1}([\ln(\cosh(\hat{\chi}) + \delta] L^{+T} \ddot{\zeta}_d \\
&\quad + \lambda_h^d - K_f e_\lambda) && (32)\\
\hat{\chi} &= \hat{c}_1 \|Z^*\| && (33)
\end{aligned}
$$

where $I_d = [I_{ad}, I_{bd}]^T$ is the desired motor current. $\hat{C} = [\hat{c}_1 \ \hat{c}_2 \ \hat{c}_3 \ \hat{c}_4 \ \hat{c}_5]^T$; $\Psi = [\|\frac{d}{dt}[\dot{\zeta}_r]\| \ \|\dot{\zeta}_r\| \ \|\zeta\|\|\dot{\zeta}_r\| \ 1 \ 1]^T$; K_p, K_i, K_f are diagonal positive; if $r > 0$, $\mathrm{sgn}(r) = 1$, else $\mathrm{sgn}(r) = -1$; $\delta \geq 1$ is constant, Γ and Λ are positive diagonal matrices, $Z^* = (J^1(M^*)^{-1}J^{1T})^{-1}J^1(M^*)^{-1}$ with $\|L^T M^* L\| = \hat{c}_1$.

B. Control Design at the Actuator Level

We can design the control input ν such that I converges to I_d, then $(\zeta - \zeta_d) \to 0$ and $(\lambda_h - \lambda_h^d) \to 0$.

Define

$$
\begin{aligned}
e_I &= \int (I - I_d)dt\\
\dot{e}_I &= I - I_d\\
I_r &= I_d - K_r e_I\\
s &= \dot{e}_I + K_r e_I
\end{aligned}
$$

Substituting I and $\dot{\zeta}$ of (15), one gets

$$
L_a \dot{s} + R_a s + K_e G_r T \dot{e}_\zeta = -L_a \dot{I}_r - R_a I_r - K_e G_r T \dot{\zeta}_d + \nu
$$

Consider the control law

$$
\nu = -\mathrm{sgn}(s)[\ln(\cosh(\hat{\varphi})] + \delta) - K_v s \tag{34}
$$

where $\hat{\varphi} = \hat{\alpha}^T \mu$, $\hat{\alpha} = [\hat{\alpha}_1 \ \hat{\alpha}_2 \ \hat{\alpha}_3]^T$, $\dot{\hat{\alpha}} = -\kappa\hat{\alpha} + \beta\mu||s||$, and $\mu = [\|\dot{I}_d\| \ \|I_d\| \ G_r\|T\dot{\zeta}_d\|]^T$ where κ, β are positive diagonal.

C. Stability Analysis for the System

Theorem 5.1: Consider the mechanical system described by (1), (2) and (5), using the control law (29), (32) and (34), the following hold for any $(q(0), \dot{q}(0)) \in \Omega_n \cap \Omega_h$:

(i) r and s converge to a set containing the origin as $t \to \infty$;
(ii) e_q and $\dot{e}_q$ asymptotically converge to 0 as $t \to \infty$; and
(iii) e_λ and τ are bounded for all $t \geq 0$.

Proof (i) By combining (13) with (18), the closed-loop system dynamics can be rewritten as

$$
\begin{aligned}
M_L(\zeta)\dot{r}_1 &= B_a^1 G_r K_{Na} I_{ad} + B_a^1 G_r K_{Na} s - \gamma \\
&\quad -(C_L(\zeta, \dot{\zeta}) + C_L(\zeta, \dot{\zeta}_r))r_1 && (35)
\end{aligned}
$$

where $\gamma = M_L(\zeta)\ddot{\zeta}_r + C_L(\zeta, \dot{\zeta}_r)\dot{\zeta}_r + G_L + d_L$.

Substituting (29) into (35) and consider Property 2.2, the closed-loop dynamics is obtained

$$
\begin{aligned}
M_L(\zeta)\dot{r}_1 &= -C_L(\zeta, \dot{\zeta})r_1 - K_p(r_1 - r_2) - K_i(\tilde{\zeta} + e_\zeta) \\
&\quad -\mathrm{sgn}(r)[\ln(\cosh(\hat{\Phi})) + \delta] \\
&\quad -\gamma - C_L(\zeta, \dot{\zeta}_r)r_1 + B_a^1 G_r K_{Na} s && (36)
\end{aligned}
$$

Differentiating (23) and considering (24), one has

$$
\ddot{\hat{\zeta}} = \ddot{\zeta}_r + k_d \dot{\tilde{\zeta}} + k_d K_\zeta \tilde{\zeta} + K_\zeta \dot{\tilde{\zeta}} \tag{37}
$$

which leads to

$$
\dot{r}_2 + k_d r_2 = \dot{r}_1 \tag{38}
$$

Subsituting (38) into (36) and consider Property 2.2, we have

$$
\begin{aligned}
M_L(\zeta)\dot{r}_2 &= -C_L(\zeta, \dot{\zeta})r_2 - (k_d M_L(\zeta) - K_p)r_2 - K_p r_1 \\
&\quad +C_L(\zeta, \dot{\zeta}_r + r_1)r_2 - C_L(\zeta, r_1)(r_1 + 2\dot{\zeta}_r) \\
&\quad -K_i(\tilde{\zeta} + e_\zeta) - \mathrm{sgn}(r)[\ln(\cosh(\hat{\Phi})) + \delta] \\
&\quad -\gamma + B_a^1 G_r K_{Na} s && (39)
\end{aligned}
$$

Consider the Lyapunov function candidate

$$
\begin{aligned}
V &= V_1 + V_2 && (40)\\
V_1 &= \frac{1}{2}X^T \mathcal{M} X && (41)\\
V_2 &= \frac{1}{2}s^T L_a s + \frac{1}{2}\tilde{\alpha}^T K_{Na} \beta^{-1} \tilde{\alpha} && (42)
\end{aligned}
$$

where $X = [r_1^T \ e_\zeta^T \ r_2^T \ \tilde{\zeta}^T \ \tilde{C}^T]^T$ and $\mathcal{M} = \mathrm{diag}[M_L \ K_i \ M_L \ K_i \ \Gamma^{-1}]$.

Differentiating V_1 with respect to time, we have

$$
\begin{aligned}
\dot{V}_1 &= r_1^T(M_L \dot{r}_1 + \frac{1}{2}\dot{M}_L r_1) \\
&\quad + r_2^T(M_L \dot{r}_2 + \frac{1}{2}\dot{M}_L r_2) + \tilde{C}^T \Gamma^{-1} \dot{\tilde{C}} \\
&\quad + e_\zeta^T K_e \dot{e}_\zeta + \tilde{\zeta}^T K_i \dot{\tilde{\zeta}} && (43)
\end{aligned}
$$

From Property 2.1 and Property 2.2, considering $\dot{e}_\zeta = r_1 - K_\zeta e_\zeta + K_\zeta \tilde{\zeta}$, $\dot{\tilde{\zeta}} = r_2 - K_\zeta \tilde{\zeta}$ from (18) and (20), the time derivative of V_1 along the trajectory of (36) and (39) is

$$
\begin{aligned}
\dot{V}_1 &= -r_1^T K_p r_1 - r_2^T(k_d M_L(\zeta) - K_p)r_2 \\
&\quad -r_2^T K_i e_\zeta + \tilde{\zeta}^T K_i K_\zeta e_\zeta - r_1^T K_i \tilde{\zeta} \\
&\quad -e_\zeta^T K_i K_\zeta e_\zeta - \tilde{\zeta}^T K_i K_\zeta \tilde{\zeta} \\
&\quad -r^T \mathrm{sgn}(r)[\ln(\cosh(\hat{\Phi})) + \delta] - r^T \gamma \\
&\quad -r_1^T C_L(\zeta, \dot{\zeta}_r)r_1 + r_2^T C_L(\zeta, \dot{\zeta}_r + r_1)r_2 \\
&\quad -r_2^T C_L(\zeta, r_1)(r_1 + 2\dot{\zeta}_r) \\
&\quad +\tilde{C}^T \Gamma^{-1} \dot{\tilde{C}} + r^T B_a^1 G_r K_{Na} s
\end{aligned}
$$

Considering Lemma 5.1, we have $\ln(\cosh(\hat{\Phi})) + \delta \geq \hat{\Phi}$, and $\|r\|[\ln(\cosh(\hat{\Phi})) + \delta] \geq \|r\|\hat{\Phi}$,

Since

$$
\begin{aligned}
\|K_i\|\|e_\zeta\|\|r_2\| &\leq \frac{1}{2}\|(K_i\|\|e_\zeta\|^2 + K_i\|r_2\|^2) \\
\|K_i K_\zeta\|e_\zeta\|\|\tilde{\zeta}\| &\leq \frac{1}{2}(K_i K_\zeta \|e_\zeta\|^2 + K_i K_\zeta \|\tilde{\zeta}\|^2) \\
\|K_i\|\|\tilde{\zeta}\|\|r_1\| &\leq \frac{1}{2}(K_i\|\tilde{\zeta}\|^2 + K_i\|r_1\|^2)
\end{aligned}
$$

From Property 2.4, the following relationships are valid:

$$\|C_L(\zeta,\dot{\zeta}_r)\| \leq \mu_1 \quad (44)$$

$$\|C_L(\zeta,\dot{\zeta}_r+r_1)\| \leq \mu_2 \quad (45)$$

$$\|C_L(\zeta,r_1)(r_1+2\dot{\zeta}_r)\| \leq \mu_3 \quad (46)$$

where μ_1, μ_2 and μ_3 are known constants.

Therefore, we have

$$\begin{aligned}\dot{V}_1 \leq & -r_1^T\left(K_p-\frac{1}{2}\|K_i\|I-\mu_1 I-\frac{1}{2}\mu_3 I\right)r_1\\ & -r_2^T\left(k_d M_L(\zeta)-K_p-(\frac{1}{2}\|K_i\|+\mu_2+\frac{1}{2}\mu_3)I\right)r_2\\ & -e_\zeta^T\left(K_iK_\zeta-\frac{1}{2}\|K_iK_\zeta\|I-\frac{1}{2}\|K_i\|I\right)e_\zeta\\ & -\tilde{\zeta}^T\left(K_iK_\zeta-\frac{1}{2}\|K_iK_\zeta\|I-\frac{1}{2}\|K_i\|I\right)\tilde{\zeta}\\ & +\frac{1}{4}C^T\Gamma^{-1}\Lambda C+r^T B_a^1 G_r K_{Na} s \quad (47)\end{aligned}$$

Differentiating $V_2(t)$ and using (15), one has

$$\begin{aligned}\dot{V}_2 = & -s^T K_{Na}[-\nu+(L_a\dot{I}_r+R_aI_r+K_eG_rT\dot{\zeta}_d)\\ & +R_a s+K_eG_rT\dot{e}_\zeta]+\tilde{\alpha}^T K_{Na}\beta^{-1}\dot{\tilde{\alpha}} \quad (48)\end{aligned}$$

Substituting ν in (48) by the control law (34) and noting $\|s\|[\ln(\cosh(\hat{\varphi}))+\delta]\geq\|s\|\hat{\varphi}$, we have

$$\begin{aligned}\dot{V}_2 \leq & -s^T(K_v+R_a)s\\ & -s^TK_eG_rT(r_1-K_\zeta e_\zeta+K_\zeta\tilde{\zeta})\\ & +\frac{1}{4}\alpha^T\beta^{-1}\kappa\alpha \quad (49)\end{aligned}$$

Since the last term in (47)

$$\begin{aligned}& r^T B_a^1 G_r K_{Na} s \leq \frac{1}{2}\|B_a^1G_rK_{Na}\|\|r_1\|^2\\ & +\frac{1}{2}\|B_a^1G_rK_{Na}\|\|r_2\|^2+\|B_a^1G_rK_{Na}\|\|s\|^2 \quad (50)\end{aligned}$$

and

$$\begin{aligned}& -s^TK_eG_rT(r_1-K_\zeta e_\zeta+K_\zeta\tilde{\zeta})\leq\\ & \frac{3}{2}\|K_eG_rT\|\|s\|^2+\frac{1}{2}\|K_eG_rT\|\|r_1\|^2\\ & +\frac{1}{2}\|K_eG_rK_\zeta\|\|e_\zeta\|^2+\frac{1}{2}\|K_eG_rK_\zeta\|\|\tilde{\zeta}\|^2 \quad (51)\end{aligned}$$

Integrating (47), (49), (50) and (51), we obtain

$$\dot{V}\leq\frac{1}{4}(C^TK_{Na}\Gamma^{-1}\Lambda C+\alpha^TK_{Na}\beta^{-1}\kappa\alpha) \quad (52)$$

(ii) Since V is bounded, which implies that $r_1, r_2, s\in L_\infty^{n-k}$. From the definitions of r_1 and r_2 and s, it can be obtained that $e_\zeta, \dot{e}_\zeta, e_I, \dot{e}_I\in L_\infty^{n-k}$. As we have established $e_\zeta, \dot{e}_\zeta, e_I, \dot{e}_I\in L_\infty$, from Assumption 4.1, we conclude that $\zeta(t), \dot{\zeta}(t), \dot{\zeta}_r(t), \ddot{\zeta}_r(t), I, \dot{I}, I_r, \dot{I}_r\in L_\infty^{n-k}$ and $\dot{q}\in L_\infty^n$.

Therefore, all the signals on the right hand side of (36) and (39) are bounded and we can conclude that $\dot{r}_1$, $\dot{r}_2$ and $\dot{s}$ and therefore $\ddot{\zeta}$ and $\dot{I}_r$ are bounded. Thus, $r, s\to 0$ as $t\to\infty$ can be obtained. Consequently, we have $e_\zeta, e_I\to 0, \dot{e}_\zeta, \dot{e}_I\to 0$ as $t\to\infty$. It follows that $e_q, \dot{e}_q\to 0$ as $t\to\infty$.

(iii) Substituting the control (29) and (32) into the reduced order dynamics (27) yields

$$\begin{aligned}& (1+K_f)e_\lambda = Z(C^1(\zeta,\dot{\zeta})\dot{\zeta}+G^1+d^1(t)\\ & -L^{+^T}B_a^1G_rK_{Na}I_a)+B_b^1G_rK_{Nb}I_{br}+B_b^1G_rK_{Nb}s\\ & =-ZL^{+^T}M_L(\zeta)\ddot{\zeta}+[\ln(\cosh(\hat{\chi}))+\delta]L^{+^T}\ddot{\zeta}_r\\ & +B_b^1G_rK_{Nb}s \quad (53)\end{aligned}$$

For the joints in the force space, since $\dot{\zeta}=0$, $I\in R^k$, (15) could be written as $L_a\frac{dI_b}{dt}+R_aI_b=\nu_b$. Therefore, $r=0$ and $s=0$ in the force space, (49) could be written as $\dot{V}_2=-s^TK_{Nb}(K_v+R_a)s$. Since K_{Nb} is bounded, $\dot{V}<0$, we can obtain $s\to 0$ as $t\to\infty$. The proof is completed by noticing that since $\ddot{\zeta}$, Z, K_{Nb} and s are bounded. Moreover, $\zeta\to\zeta_d$, and $-ZL^{+^T}M_L(\zeta)\ddot{\zeta}_d+[\ln(\cosh(\hat{\chi}))+\delta]L^{+^T}\ddot{\zeta}_d$ is bounded, $s\to 0$, the right-hand side terms of (53) tend uniformly asymptotically to zero, then it follows that $e_\lambda\to 0$, then $\lambda_h\to\lambda_h^d$.

Since r, ζ, $\dot{\zeta}$, ζ_r, $\dot{\zeta}_r$, $\ddot{\zeta}_r$, e_λ and s are all bounded, it is easy to conclude that τ is bounded from (25).

VI. SIMULATIONS

Consider the mobile manipulator system shown in Fig. 1. Let the desired trajectory $q_d=[x_d,y_d,\theta_d,\theta_{1d},\theta_{2d}]^T$ and the end-effector is subject to the geometric constraint: $\Phi=l_1+l_2\sin(\theta_2)=0$, and $y_d=1.5\sin(t)$, $\theta_d=1.0\sin(t)$, $\theta_{1d}=\pi/4(1-\cos(t))$, $\lambda_h^d=10.0N$. In the simulation, we assume that the parameters $m_p=m_1=m_2=1.0kg$, $I_w=I_p=1.0kg/m^2$, $2I_1=I_2=1.0kg/m^2$, $I=0.5kg/m^2$, $d=l=r=1.0m$, $2l_1=1.0m$, $2l_2=0.6m$, $q(0)=[0,2.0,0.6,0.5]^T$, $\dot{q}(0)=[0.0,0.0,0.0,0.0]^T$, $K_N=\mathrm{diag}[0.01]$, $G_r=\mathrm{diag}[100]$, $L_a=[0.005,0.005,0.005,0.005]^T$, $R_a=[2.5,2.5,2.5,2.5]^T$, and $K_e=[0.02,0.02,0.02,0.02]^T$. The disturbances on the mobile base are set $0.1\sin(t)$ and $0.1\cos(t)$. By Theorem 5.1, the observer gain is selected as $k_d=\mathrm{diag}[50]$, the control gains are selected as $K_p=\mathrm{diag}[10.0,10.0,10.0]$, $K_\zeta=\mathrm{diag}[1.0,1.0,1.0]$,$K_v=\mathrm{diag}[10.0,10.0,10.0,10.0]$, $K_i=\mathrm{diag}[0.5]$ and $K_f=0.995$, $C(0)=[1.0,1.0,1.0,1.0,1.0]^T$, $K_N=0.1$, $G_r=50$, $K_v=\mathrm{diag}[10,10,10,10]$, $\Lambda=\kappa=\mathrm{diag}[1/(1+t)^2]$, $\Gamma=\mathrm{diag}[4.8]$, $\beta=\mathrm{diag}[0.2]$, $\alpha(0)=[0.001,1.0,0.01]$. The disturbance on the mobile base is set $0.1\sin(t)$ and $0.1\cos(t)$. The simulation results for motion/force are shown in Figs. 2 and Fig. 4. The input voltages on the motors are shown in Fig. 3 and Fig. 5. The simulation results show that: the trajectory and force tracking errors asymptotically tend to zero.

VII. CONCLUSIONS

In this paper, adaptive robust control integrating an observer has been presented systematically to control the holonomic constrained noholonomic mobile manipulator in the presence of uncertainties and disturbances and actuator dynamics has been considered in the control.

APPENDIX

Proof: If $x \geq 0$, we have $\int_0^x \frac{2}{e^{2s}+1} ds < \int_0^x \frac{2}{e^{2s}} ds = 1 - e^{-2x} < 1$. Therefore, $\ln(\cosh(x)) + \delta \geq \ln(\cosh(x)) + \int_0^x \frac{2}{e^{2s}+1} ds$ with $\delta \geq 1$. Let $f(x) = \ln(\cosh(x)) + \int_0^x \frac{2}{e^{2s}+1} ds - x$, we have $\dot{f}(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + \frac{2}{e^{2x}+1} - 1 = 0$. From the Mean Value Theorem, we have $f(x) - f(0) = \dot{f}(x)(x - 0)$. Since $f(0) = 0$, we have $f(x) = 0$ that is, $\ln(\cosh(x)) + \int_0^x \frac{2}{e^{2s}+1} ds = x$, then, we have $\ln(\cosh(x)) + \delta \geq x$. This completes the proof.

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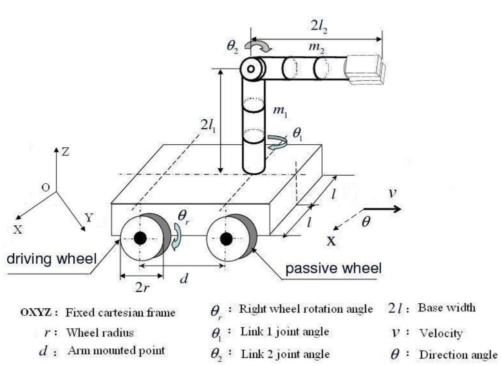

Fig. 1. The 2-DOF Mobile Manipulator

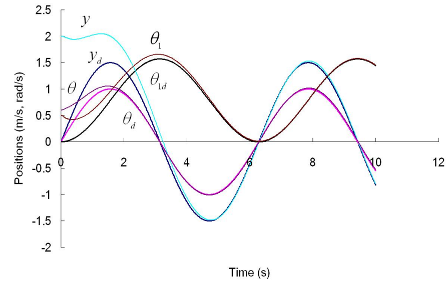

Fig. 2. The positions of the joints

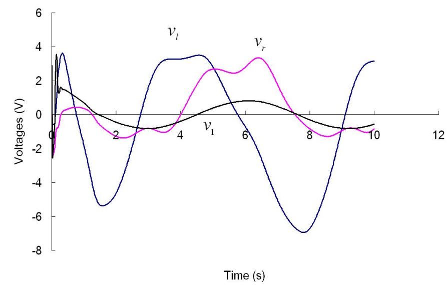

Fig. 3. The input voltages

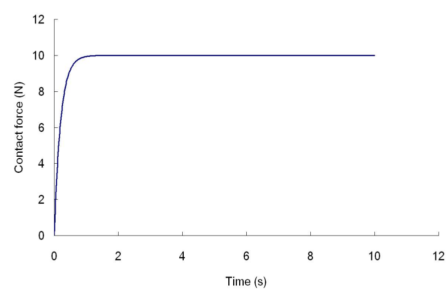

Fig. 4. The constraint force

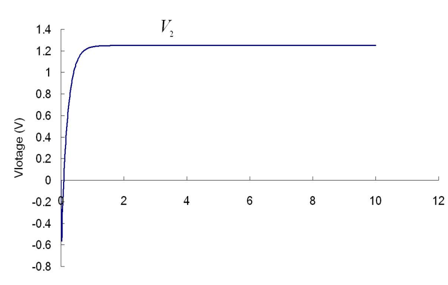

Fig. 5. The input voltage of the joint 2