A Relative Information Metric For Vehicle Following Systems

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Abstract—**Vehicle following can be achieved by minimizing the relative information (Kullback-Leibler or K-L distance), between the estimated poses of leader and follower vehicles by formulating the vehicle following system as an optimization problem. The aim is to search for an optimal control action for the follower vehicle in the admissible control command space. Relative information is used as a metric in the search space and for evaluating the expected performance of vehicle following. With this metric, and based on the assumption that both vehicle pose (position and orientation) distributions are Gaussian functions, the K-L distance of the vehicle following system can be computed. With a series of admissible actions, such as steering and velocity commands, for the follower vehicle at each pose prediction step, and by minimizing the K-L distance, an optimized action for the follower vehicle can be obtained. The proposed vehicle following algorithm has been tested and the performance of the follower vehicle when the leader undergoes various kinds of maneuvers has been analyzed. Results using this new method, as compared to classical methods, have shown the advantages of this method.**

I. INTRODUCTION

For autonomous vehicle following, both longitudinal and lateral controllers [1][2] are widely implemented. Longitudinal controllers regulate the desired spacing and time headway between the two vehicles. On the other hand, lateral controllers are commonly implemented as a lane following system. These two control strategies have been successfully demonstrated on roads with large radii of curvature, such as motorways [1][2]. However, for vehicle following in urban environments [3][4], the radius of curvature can be significantly lower, and the performance of these two controllers may be degraded. Vehicle following in urban environments requires the follower vehicle to trail the *trajectory* of the leader for safety purposes to avoid the cutting of corners in low radius of curvature areas.

In the vehicle following function proposed by Ng et. al [5], the pose states of both the follower and leader vehicles can be obtained through an estimation process. Alternatively, vehicle states, such as the dynamics and pose of the leader vehicles, can be transmitted to the follower vehicle via inter-vehicle communication systems [1][2][6]. Other vehicle following systems instead use on board sensors, such as laser scanners [7][8][9] and cameras [10], as the main perception tools. However, real sensors introduce noise into the vehicle following system. The sensor data uncertainty may affect the reliability of the vehicle following system if it is not addressed properly.

On the other hand, information theoretic frameworks have been used extensively in mobile robotics applications. Typical applications are surveillance systems using Unmanned Aerial Vehicles (UAVs) [11] and active exploration of an area using Unmanned Ground Vehicles (UGVs) [12]. These systems aim to maximize the knowledge, or information, gained by the robot, through optimized control actions [13][14][15]. The strategy also aims at minimizing the uncertainties of the system state through the selection of a sequence of control actions. Moreover, information theoretic frameworks have been used in the machine vision community as a tool in image association. For example, the K-L distance was used as a measure for optimal feature selection such that the feature was selected by maximizing the K-L distance between target classes [16].

In anticipation of the above challenges in urban environments and the advantages of information theoretic frameworks in minimising system state uncertainties, this paper focuses on the generation of control commands for a follower vehicle to pursue a leader vehicle. This is achieved by taking into consideration the kinematic constraints of the vehicle and the uncertainty in the measurement data obtained by the follower vehicle. The control commands to the follower vehicle are computed based on the minimization of the relative information (K-L distance) between the two vehicles. By formulating the vehicle following system as a Bayesian representation, we obtain two probabilistic distributions describing the uncertainties of the states of the leader and the follower vehicles. Before issuing an action to the follower vehicle, a series of achievable actions is identified. This series of achievable actions acts as the input to the pose estimation filter of the follower vehicle. Then a set of predictions of pose uncertainty for the follower vehicle, based on this series of actions, can be obtained. The relative information can be computed based on this series of expected pose uncertainties with respect to the uncertainty of the state of the leader vehicle. An optimized action for the follower vehicle can be selected from the actions that yield minimum relative information for the system.

The intended application is for vehicle following in urban environments. As the cost/performance ratio of the electronic components and sensors has significantly reduced in recent years, the implementation of this system is now feasible.

II. VEHICLE FOLLOWING SYSTEM: PROBLEM FORMULATION

The vehicle following function can be defined in an analytical manner as follows:

Definition 1: Conventionally, vehicle following is achieved when the follower vehicle attains the pose of the leader vehicle some instant of time later, that is,

$$\mathbf{x}_F(t) = \mathbf{x}_L(t - m), \qquad m > 0 \quad \forall\, t > 0 \tag{1}$$

where t is the time and m is some time elapsed for a follower vehicle to reach the position of, and align with, the leader. $\mathbf{x}_F(t)$ and $\mathbf{x}_L(t)$ are the poses of follower and leader vehicles at time t respectively.

For safe vehicle following, the following criteria must be fulfilled:

$$\| \mathbf{x}_F(t) - \mathbf{x}_L(t) \| > \mathbf{d_m} \tag{2}$$

where $\mathbf{d_m} > 0$ is defined as a minimum safety separation distance between the two vehicles at any time.

Furthermore, the velocities and steering angles of the leader and follower vehicles at all times are constrained as

$$
\begin{aligned}
0 &< v_L &\le V_{Lmax} \\
-\alpha_{Lmax} &\le \phi_L &\le \alpha_{Lmax} \\
0 &< v_F &\le V_{Fmax} \\
-\alpha_{Fmax} &\le \phi_F &\le \alpha_{Fmax}
\end{aligned}
\tag{3}
$$

where V_{Lmax}, V_{Fmax}, α_{Lmax} and $\alpha_{Fmax} \in \mathbf{R}^+$ are the maximum achievable velocities and steering angles of the leader (indicated as subscript L) and follower (indicated as subscript F) respectively.

However, as presented in our earlier work [17], it was demonstrated that vehicle following can be achieved by implementing a virtual trailer link model. In that model, the leader vehicle is modelled as the towing vehicle and the follower as the trailer. As the model suggested, the leader vehicle (towing) is effectively pulling a follower vehicle (trailer) via a virtual trailer link. Also, it has been proven that the length of the virtual trailer link must equal the length of the follower vehicle itself for an intrinsically safe vehicle following system (figure 1) [17]. If a chain of vehicles is to follow a leader, vehicles further down the chain suffer from a phenomenon known as string stability [2]. This issue can be addressed using this virtual trailer link model [17].

Definition 2: With the virtual trailer link model [17], vehicle following is redefined as:

$$\|\mathbf{x}_F(t + \delta t) - \mathbf{x}_T(t)\| = 0 \quad \forall\, t > 0 \tag{4}$$

where $\mathbf{x}_T(t)$ is the pose of the virtual trailer at time t and δt is the time increment between measurement.

With reference to figure 1, the motion model, in discrete time state space, of the follower vehicle can be represented
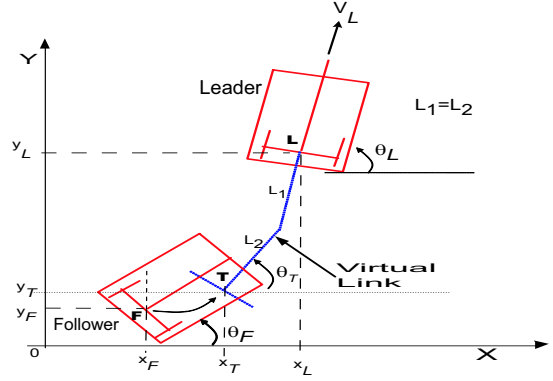

Fig. 1. Virtual trailer link model for vehicle following. At any time instance, the follower vehicle perceives the pose of the leader with an on board sensor. The pose, $\mathbf{T}$, (x_T, y_T) of the virtual trailer link is then computed. The follower vehicle will be commanded to the new position, $\mathbf{T}$. The whole process will be repeated at the next time instant.

as[1]:

$$\mathbf{x}_{F,(k+1)} = \mathbf{f}(\mathbf{X}_{L,k}, \mathbf{X}_{T,k}, \mathbf{U}_k, \omega_k) \tag{5}$$

where $\mathbf{X}_{F,k}$, $\mathbf{X}_{L,k}$ and $\mathbf{X}_{T,k}$ are the histories of the state of the follower vehicle, leader and virtual trailer respectively. $\mathbf{U}_k$ is a vector of motion control signals input to the follower vehicle, ω_k is the motion uncertainty and $\mathbf{f}(\cdots)$ is a non-linear function representing the motion of the follower.

The sensor is used to acquire the noisy observation $\mathbf{z}_k$ of the leader vehicle taken from the follower. The sensor model is described as:

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_{L,k}, \mathbf{x}_{F,k}, \nu_k) \tag{6}$$

where ν_k is the sensor noise and $\mathbf{h}(\cdots)$ is a non-linear function representing the sensor model.

Both the sensor and motion uncertainties will be modelled as random variables. The sequences $\{\nu_0, \nu_1, \ldots, \nu_k\}$ and $\{\omega_0, \omega_1, \ldots, \omega_k\}$ are assumed independent, zero mean, white processes with known covariances.

As formulated in [5], a complete vehicle following system can be formulated as a probability density function (pdf):

$$
\begin{aligned}
& P(\mathbf{x_{F,k}}, \mathbf{x_{L,k}} | \mathbf{U_k}, \mathbf{Z_k}) \\
& \propto \underbrace{P(\mathbf{z_k^p} | \mathbf{x_{F,k}}) P(\mathbf{x_{F,k}} | \mathbf{U_k}, \mathbf{Z_{k-1}^P})}_{localization\ of\ follower} \\
& \times \underbrace{P(\mathbf{z_k^e} | \mathbf{x_{F,k}}, \mathbf{x_{L,k}}) P(\mathbf{x_{L,k}} | \mathbf{Z_{k-1}^e})}_{Tracking\ of\ leader\ vehicle\ w.r.t\ follower}
\end{aligned}
\tag{7}
$$

where $\mathbf{z}_k^p$ and $\mathbf{z}_k^e$ are the current observations made by the proprioceptive and exteroceptive sensors onboard the follower. In vehicle following, the follower can predict its pose iteratively with the on-board sensors. Also, the follower

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[1]The lowercase notation, eg $\mathbf{x}_{L,k}$ denotes the current state and the uppercase notation, eg $\mathbf{X}_{L,k}$ denotes the entire history of the state up to and including time k. The state in discrete time space is represented by subscript k, eg $\mathbf{x}_{L,k}$. For continuous time space, it is denoted in the form of $\mathbf{x_L}(t)$.

can make observations of the leader and predict the relative pose of the leader. The advantage in representing the vehicle following system under a Bayesian framework is that the uncertainties in the system and sensor models are considered in the formulation.

With the Bayesian framework as shown in equation (7), the history of observations and control signals are recorded and the poses of both vehicles are estimated. By collating this information into an information vector, $\mathbf{I}_k$,

$$\mathbf{I}_k = \{\mathbf{x}_{F,k}, \mathbf{x}_{L,k}, \mathbf{Z}_k, \mathbf{U}_{k-1}\} \tag{8}$$

The control (heading and speed) commands for the follower, are the input to the vehicle following system. However, because of the vehicle's kinematic constraints, there are limits to the steering and heading commands which are achievable.

Definition 3: The admissible control signals at time k are the collection of all available control signals $\mathbf{A}_k$. Therefore,

$$\mathbf{A}_k = \{\mathbf{a}_0(\mathbf{I}_k), \mathbf{a}_1(\mathbf{I}_k), \ldots, \mathbf{a}_{N-1}(\mathbf{I}_k)\} \tag{9}$$

where $\mathbf{a}_i$ is defined as a function of $\mathbf{I}_k$ at time k and N is the total number of admissible control signals.

From *definitions 2* and *3* and the constraints of equation (3), it is possible to define the problem of vehicle following as finding an optimized control action from the admissible control signals (in *definition 3*) such that the condition in *definition 2* (equation (4)) is fulfilled under the constraints defined in *definition 1* (equation (3)). Hence, the problem of vehicle following can be defined as:

$$\mathbf{a}_{k+1}^* = arg \min_{\mathbf{A}_k} \|\hat{\mathbf{x}}_{F,(k+1)} - \hat{\mathbf{x}}_{T,k}\| \tag{10}$$

where $\mathbf{a}_{k+1}^*$ is the optimized control action for the follower. Equation (10) can be viewed as an optimization problem. The aim is to search for an optimized control signal, that is to be input to the controller of the follower, in the admissible command space. A metric (or objective function) can be formulated for this purpose.

A. Information Theoretic Vehicle Following

For our vehicle following, it has been shown that it is possible to estimate the poses of both the leader, hence the pose of the virtual trailer, and follower vehicles using the onboard sensors as in equation (7). However, two main issues need to be considered:

- **Sensor uncertainty**, which affects the performance of the vehicle following system. The uncertainty in the pose estimates of the leader vehicle and the virtual trailer must be considered by the follower when determining its next control action. Furthermore, the possible consequences of sensor uncertainty, which might cause vehicle following operation failure, has to be considered during implementation.
- **Vehicle Constraints**: Typically, a command is sent to the follower so that it can maneuver towards the pose of the virtual trailer at time k. This is based on the estimations of the leader and virtual trailer pose

relative to the follower. However, at any given time k, in practice, the pose of the follower may not allow it to attain the pose of the virtual trailer, due to the violation of the kinematic constraints.

To minimize the effects of sensor uncertainty and vehicle kinematic constraints, the concept of relative information is used to determine the control actions for the follower. This is made possible by equation (7). Two probabilistic distributions, representing the uncertainty of the poses of the vehicles, can be obtained in the recursive estimation process and then be used in the computation of relative information.

Relative Information

As the relative information formulation will be used in this paper, a summary of the concept is included here.

Relative information (K-L distance) [18] is a metric that quantifies the "goodness of fit" or "closeness" of two probability density functions.

For the case of two Gaussian distributions [19], [20],

$$H(Q\|P) = \frac{1}{2} \log \frac{|\Sigma_P|}{|\Sigma_Q|} + \frac{1}{2} Tr\{\Sigma_P^{-1}(\Sigma_Q - \Sigma_P)\}$$
$$+ \frac{1}{2}(\mu_Q - \mu_P)^T \Sigma_P^{-1}(\mu_Q - \mu_P) \tag{11}$$

where (μ_P, Σ_P) and (μ_Q, Σ_Q) are the mean and covariance matrix pairs for Gaussian distributions P and Q respectively. The first term on the right hand side, of equation (11) represents the information gained, the second term represents mutual information and the last term is actually the Mahalanobis distance of the two pdfs. From equation (11), if the covariance matrices of the two distributions to be compared are of the same magnitude, the K-L distance is exactly the same as the measure of the Manalanobis distance. Whereas, in the case of the two distributions having the same mean values, the K-L distance measures the information gained and the mutual information. Hence, the K-L distance formulation compares both the mean values and covariance matrices of the two distributions under consideration.

III. GENERALIZED INFORMATION THEORETIC VEHICLE FOLLOWING IN A FINITE TIME WINDOW

In general, the vehicle following algorithm can be formulated in a finite time horizon $[k, k + M]$, where k is the current time step and M is the finite time window size in the time horizon.

Suppose that the follower is controlled by a set of actions at each time step denoted by

$$\mathbf{U} = \{\mathbf{u}_{(k+i)}\}_{|i=0,1,2,\ldots,M} \tag{12}$$

where $\mathbf{u}_{(k+i)}$ is the vector of actions specifying the control command issued to the follower at time $k + i$.

At every time step, the follower makes observations about the leader vehicle. The observation is denoted as

$$\mathbf{Z} = \{\mathbf{z}_{k+i}\}_{|i=0,1,2\ldots,M} \tag{13}$$

Let $\mathcal{N}_F$ and $\mathcal{N}_T$ denote the normal distribution functions representing the mean poses and covariance matrices of the

follower and virtual trailer respectively for all time steps $j \subset [k, k+M]$ defined in the time horizon.

$$\mathcal{N}_T = \{\mathcal{N}_{T,j}\}_{|j=k,k+1,.....,k+M} \qquad (14)$$

$$\mathcal{N}_F = \{\mathcal{N}_{F,j}\}_{|j=k,k+1,.....,K+M} \qquad (15)$$

The terms $\mathcal{N}_{F,j}$ and $\mathcal{N}_{T,j}$ denote the distribution functions computed at time step j.

Let the admissible control command, at time step k, for the follower be denoted as

$$\mathbf{A}_k = \{\mathbf{a}_n\}_{|n=0,1,2,...,N-1} \qquad (16)$$

where $\mathbf{a}_n$ is the control command to be input to the follower and N is the total number of admissible commands available for the follower.

The information theoretic vehicle following problem can now be formulated as follows:

$$\mathbf{a}^*{}_{(k+1)} =$$
$$arg\min_{\mathbf{A}}\{C(H_j(\mathcal{N}_{F,j+1}\|\mathcal{N}_{T,j})\}_{|j=(k,k+1,...,k+M)} \qquad (17)$$

subject to the constraints

$$\mathbf{g}(\mathbf{x}_k, \mathbf{x}_{(k+1)}, ..., \mathbf{x}_{(k+N-1)},$$
$$\mathbf{u}_k, \mathbf{u}_{(k+1)}, ..., \mathbf{u}_{(k+N-1)})) \leq \mathbf{g}_{th} \qquad (18)$$

where $C(.)$ is the composite scalar function representing the K-L distance, $H_j(.)$ is the K-L distance computed at time j, $\mathbf{x_k}$ is the augmented state vector of both the virtual trailer and follower, $\mathbf{g}(.)$ is the nonlinear constraint vector function and $\mathbf{g}_{th}$ is the constraint threshold vector. The constraints include the maximum allowable steering angle of the vehicle, safe following distance and the allowable following speed.

Equation (17) provides an unique decision-theoretic solution to the vehicle following problem. In general, a control command, such as velocity or steering angle, for the follower can be generated by analyzing the relative information between the two vehicles over a certain time horizon. However, optimization of equation (17) involves complex computation, which involves multiple iterations. The iteration scales in the order of $\mathcal{O}(N^{M+1})$. Hence, for implementation, the look-ahead time horizon for optimization is limited to one time step, which is also known as the greedy method [12].

IV. IMPLEMENTATION AND EXPERIMENTAL RESULTS

Figure 2 shows the simplified block diagram for our vehicle following function. There are 4 major modules and each of the functionalities are described as follow:

- **Perception Module (PM)**: The follower vehicle is assumed to have on board sensors. In our implementation, the odometry data and the information from a gyroscope were used to localize the follower. The pose of the leader vehicle can be detected using a laser scanner, camera or fusion of both images.
- **Pose Estimation Module (PEM)**: With the observation received from **PM**, both the poses of the leader and follower can be obtained using equation (7).
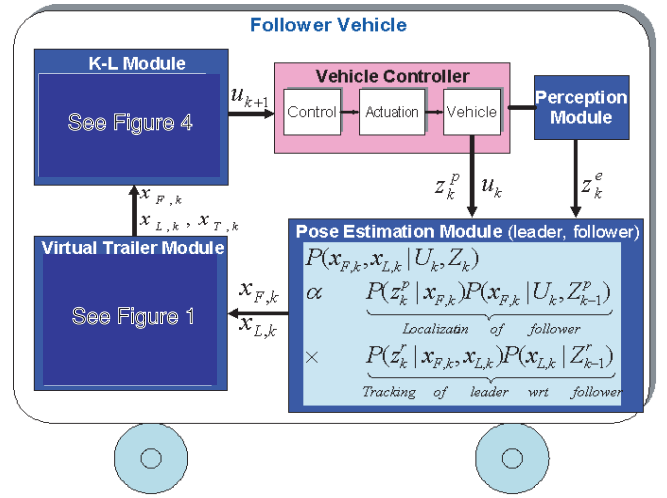

Fig. 2.   Control block diagram for the proposed vehicle following system.

TABLE I

ALGORITHM FOR VEHICLE FOLLOWING FUNCTION

Steps	Actions	Formulation
1	Current Pose estimates	$\hat{\mathbf{x}}_{F,k|k}, \hat{\mathbf{x}}_{T,k|k}$
2	Predict follower pose from achievable actions	$\hat{\mathbf{x}}_{F,k+1|k}$
3	Compute KL distance	$H_j \; \forall j = [1, M]$
4	Choose input and move follower	$\mathbf{a}^*(k+1)$
5	Observe pose of leader	$\mathbf{z}(k+1)$
6	Estimate new poses	$\hat{\mathbf{x}}_{F,k+1|k+1}, \hat{\mathbf{x}}_{T,k+1|k+1}$
7	Loop steps 1 to 6	-

- **Virtual Trailer Module (VTM)**: This module receives the estimated poses of the leader and follower from **PEM** and generates the estimated pose of the virtual trailer.
- **K-L Module (KLM)**: The greedy method presented in [12] is implemented to determine the control actions for the vehicle following function. A series of possible steering commands are used as input to compute the predicted poses of the follower and virtual trailer at the next time step. The K-L distances are then computed and the control action resulting from the minimum K-L distance is then selected.

The entire algorithm is summarized in table I.

A. Experimental Setup

The proposed method was validated using simulations. To make the simulation results comparable to an actual system, the standard deviation settings for the simulated sensors were set as close to real known values as possible[2]. For simulation purpose, the desired forward velocity and heading angle of the leader vehicle were generated at regular intervals based on the desired leader vehicle's trajectory. The leader vehicle was modelled as a line and a line fitting algorithm was

implemented for vehicle detection using range scanner [5]. Detailed implementation issues of leader vehicle detection, such as its false or failed detection, have been presented in our earlier publication [5]. Nevertheless, a camera can also be used for vehicle detection [4][10]. However, vision related issues would need to be solved before reliable implementation can be achieved, and is beyond the scope of this paper. The leader vehicle was controlled by a standalone program during the simulation. Its position was recorded as ground truth. The follower was controlled by the K-L algorithm embedded in another program[3].

To test the feasibility of the new vehicle following theory, a S-Curve trajectory for the leader vehicle is generated. The purpose is to test if the proposed vehicle following method can cope with sharp curves turning in both directions, i.e, sharp left and right turns. The trajectory represents constraints found in typical urban road environments and attempts to challenge the controller's response.
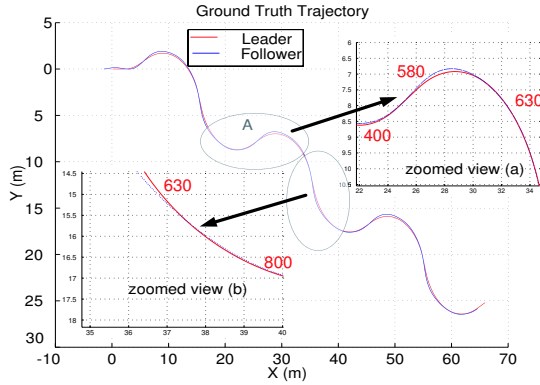
B. Performance Analysis

Fig. 3. The ground truth of both the leader and follower vehicle trajectories. Zoomed views (a) and (b) show the detailed trajectories of the vehicles at various time steps.

As shown in figure 3, the leader vehicle is commanded to manoeuvre in a straight path for a short period of time, then to make a series of left and right turns.

Figure 4 shows the KL distances computed based on the estimated poses and equation (17) when the leader vehicle is moving straight (top figure), making a left turn (middle figure) and making a right turn (bottom figure). For our experiment, the total number of admissible steering angles, $N = 40$, with the angular resolution of 1 degree. As observed in figure 4, the minimum K-L distance can be obtained, and hence the optimized steering angle, can be chosen.

Figure 5 shows the path deviation between the vehicles and the corresponding KL distances. The path deviation was computed off-line based on the closest positions of the two vehicles. The small path deviation as shown in figure 5 has suggested that the information theoretic based vehicle following algorithm presented is robust to various kinds of

[3]Details of the system setup can be found in our earlier publication [5]. The maximum speed of the leader vehicle is set at 2m/s, simulating slow speed vehicle following in urban environments.
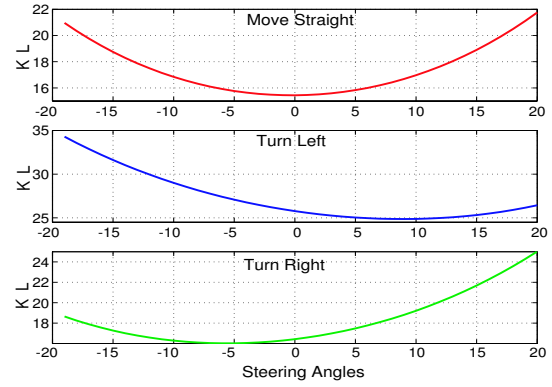

Fig. 4. Plot of K-L distance when the vehicles are moving in a straight line (top), turn left (middle), turn right (bottom).
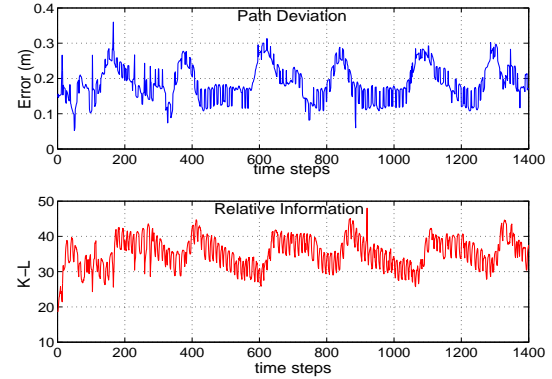

Fig. 5. Path deviation and corresponding KL distance.

maneuvers. Furthermore, from figure 5, the trend in the path deviation plot resembles the trend in the K-L distance plot. For example, this can be particularly observed from the figure between time steps 400 to 800 (which corresponds to the location marked 'A' in figure 3). From time steps 400 to 580 (zoomed view (a) in figure 3), the leader is making a gradual left turn. As the turn rate is gradual, the orientation difference between the two vehicles will eventually decrease, while the inter-vehicle separation distance between the two vehicles will remain constant. Therefore, the KL distance decreased. From time steps 580 to 630 (zoomed view (a) in figure 3), the leader vehicle is making a sharp right turn. As the turn becomes sharp, the orientation difference between the vehicle will be large and hence the K-L distance increased. From time steps 630 to 800 (zoomed view (b) in figure 3), the leader vehicle is making gradual left turn. Similarly, the K-L distance decreased gradually. The above scenarios have verified that the K-L distance can be used as a metric for evaluating the performance of the vehicle following function.

Figure 6 shows the steering angles computed from the K-L metric using a pure pursuit algorithm [21]. It can be observed from the figure that the pure pursuit algorithm has computed the steering angles to be greater than 20 degrees, when the leader vehicle is making sharp turns. These angles have exceeded the maximum allowable angle for a typical vehicle. Also, the transition of the steering angles from the
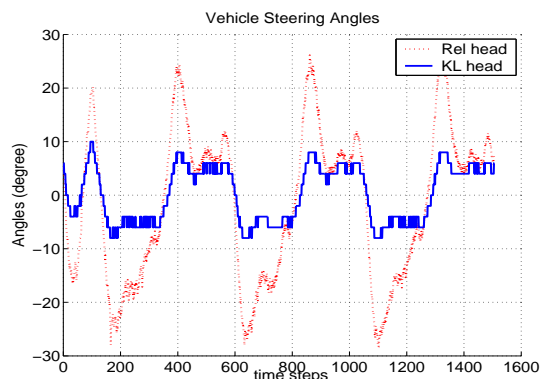
Fig. 6. Plot of optimized steering angles computed from K-L distance metric and a pure pursuit algorithm.

TABLE II
PATH DEVIATION RESULTS COMPARISON

	Stefan[10]	Wang[9]	Lu[7]	K-L
Avg(m)	0.2	0.3	0.15	0.2
max(m)	0.7	0.5	0.35	0.35

left to the right turn is large for a pure pursuit algorithm. This may cause discomfort to the driver. On the other hand, the steering angles as generated by the K-L metric is within the allowable steering angle of the vehicle. The transition of the steering angles as generated by K-L metric is gradual.

Table II shows a direct comparison of the path deviation achieved by the proposed system with the experiments carried out by Stefan [10], Wang [9] and Lu [7]. The vehicle following algorithm implemented by Stefan [10] achieved a maximum path deviation (between the two vehicles) of approximately *70 cm*. The system implemented by Lu [7] achieved a maximum path deviation of *35 cm*. The vehicle following system by Wang [9] has achieved a maximum path deviation of *50cm*. As for our case, the maximum path deviation is *35 cm*. Our system has achieved an average path deviation of *20cm* throughout the whole trajectory. Overall, the K-L algorithm is able to optimize the control actions for the follower to achieve close pursuit of the leader vehicle and at the same time provide a smooth input to the follower.

V. CONCLUSIONS

Autonomous vehicle following capabilities have been achieved using an information theoretic framework based on the K-L distance metric. It optimizes the control inputs for the follower vehicle so as to minimize the pose error between the follower and the leader vehicles. Both the follower vehicle's constraints and the uncertainties in the estimation of the poses of both vehicles have been considered. Under this framework, the relative information or K-L distance has been used as a metric to evaluate a sequence of control actions, which are used as inputs to the follower. The method has been simulated and the results have shown that the information theoretic vehicle following system is robust to estimation errors and the safe separation of the vehicles has been considered. The system is robust as uncertainties in the

estimation of the poses of both vehicles are considered and taken into account as part of the vehicle following function. The inter-vehicle distance is maintained as desired and thus it is possible to warrant that the follower is in a position to stop safely in case of emergencies.

An extension of our approach is to use a priori information, by predicting the future trajectory of the leader vehicle. This is possible if the curvature information of the road that lies ahead is made available. This information has already been made available in standard car navigation systems and can be incorporated as an additional observation which could complement our information theoretic framework.

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