# Outdoor Radar Mapping Using Measurement Likelihood Filtering

John Mullane, Martin D. Adams, Wijerupage Sardha Wijesoma

School of Electrical & Electronic Engineering Nanyang Technological University Singapore john.mullane@pmail.ntu.edu.sg, eadams@ntu.edu.sg, eswwijesoma@ntu.edu.sg

Abstract— This paper presents an outdoor radar mapping algorithm, using an occupancy grid approach. It is shown that the occupancy mapping problem is directly coupled with the signal detection processing which occurs in a range sensor, and that the required measurement likelihoods are those commonly encountered in both the target detection and data association hypotheses decisions. Furthermore, these measurement likelihoods are highly correlated both with the environment and the non-linear target detection algorithm used. The classical binary Bayes filter occupancy grid mapping technique generally treats the measurement likelihoods as fully known deterministic values, whereas they are in fact estimated likelihoods which are dependant mainly on the feature's signal-to-noise ratio.

In this paper, contrary to standard sensor models, the measurement likelihoods are therefore treated as random states which must be recursively filtered for each region of the map, and are not unknown intrinsic sensor parameters which can be learned from training data. As the measurement likelihoods do not provide direct measurements of the state of interest (the occupancy random variable), but are required to generate the estimate, maximum likelihood estimates of the unknown measurement likelihoods are used to generate an estimate on the state of the map at each update. The ideas presented in this paper are demonstrated in the field robotics domain using a millimeter wave radar sensor.

Index Terms—Radar target detection, Occupancy grid, Measurement likelihoods

# I. INTRODUCTION

Autonomous outdoor navigation is still a very active topic of research due to the presence of unstructured objects and rough terrain in realistic situations. One of the core reasons for failure is the difficulty in the consistent detection and association of unstructured targets present in the environment. Mobile robot navigation is typically formulated as a dynamic state estimation process where predicted vehicle and target locations are fused with sensor readings. Reliable target detection from noisy sensor data is critical to the successful convergence of any such algorithm.

Most methods are concerned only in the location of detected targets, thus the noise in the sensor readings is typically 2 dimensional i.e. in range and bearing. For range/bearing sensors commonly used in robot navigation, such as the polaroid sonar or SICK laser, the target detection algorithm is performed internally resulting in a single  $(r, \theta)$  output to the *first* signal considered detected. No other information is returned about the world along the bearing angle  $\theta$ , however it is typical in the case of most sensor models to assume empty space up to range r [1]. This signal may or may not correspond to a target, depending on the environmental properties. These ambiguities are typically resolved in the data association stage by applying a threshold to some statistical distance metric based on the covariance of the predicted and observed feature locations.

Sensor noise in range/bearing measuring sensors however is in fact 3 dimensional, since an added uncertainty exists in the detection process itself. Whilst this observation is considered in such mapping techniques as occupancy grids [2], most localisation algorithms will disregard this probability and assume an ideal detector. Using this assumption, the distribution of the target coordinates can be conveniently modeled with probability density functions (typically Gaussian), where the probabilistic sum under the distribution is unity. That is, complete certainty is assumed that a target exists *somewhere* within that area, thus readily allowing for numerous stochastic filtering techniques to be applied. For most occupancy grid maps, the occupancy is distributed in a Gaussian manner as a function of the range returned, the intensity of the returned signal is rarely considered, resulting in *discrete* observations of occupancy in each cell. The binary Bayes filter is then used as a solution, which is possible as it subtly uses a completely known occupancy measurement model to update the posterior occupancy probability.

For most sensors, users do not have access to the signal detection parameters, however this is not the case for sensors such as the Frequency Modulated Continuous Wave (FMCW) radar<sup>1</sup> and certain underwater sonar devices where the output data is a complete signal power profile along the direction of beam projection, without any signal detection being performed. At each range bin, a power value is returned thus giving information at multiple ranges for a single bearing angle. FMCW radar sensors are typically applied to outdoor sensing applications as they can operate in hazardous outdoor environments where other sensors will fail. This is due to the radar's ability to penetrate dust, fog, and rain [3].

<sup>&</sup>lt;sup>1</sup>Due to the modulating techniques, a Fast Fourier Transform can be used to return a power value at discrete range steps. Range resolution, beamwidth, and maximum range are dependant on the particular sensor.

#### II. RELATED WORK

In rugged outdoor or underwater environments where there can be numerous false alarms (incorrectly declared features) and/or outliers (features which are "infrequently" observed), so called "feature management" techniques are often used to identify "unreliable" features and delete them from the map. This is in order to reduce the possibility of false data association hypothesis decisions. From the literature, two common methods of identifying true features from noisy measurements is by using the binary Bayes filter [4], [5], which propagate a feature existence variable obtained from a sensor model and the "geometric feature track quality" measure [6], [7] which is a function of the innovation for that feature. The binary Bayes filter approach is more commonly used in an occupancy grid framework for map building applications.

Signal processing problems are not new to the field of autonomous mapping and target detection but are generally treated in a simplified manner. In the underwater domain, sonars also return a power versus range vector which is difficult to interpret. In his thesis [8], S. Williams outlined a simple target detection technique for autonomous navigation in a coral reef environment. A constant noise power threshold is used and the maximum signal to noise ratio is chosen as the point target. Clearly this method of extraction results in a large loss of information, which is not desirable for the construction of well defined maps. S. Majumder attempts to overcome this loss by fitting a sum of Gaussian probability density function to the raw sensor data [9], however this represents a likelihood distribution in range of a single point target which is misleading as the data can contain multiple targets, leading to the association of non-corresponding points.

In field robotics, standard noise power thresholding<sup>2</sup> was again used by S. Clark [10] using an FMCW radar. The range and bearing measurements of the detected point were then propagated through an Extended Kalman Filter framework to perform navigation and mapping. The method was shown to work in an environment containing a small number of well separated, highly reflective beacons. The method was extended slightly in [11] where, *even bounce specularities* were used to extract pose invariant features. Again the environment contained reflective, metallic containers.

This paper further explores the problem of signal detection within a robotics framework to perform mapping. It is shown that by using signal detection theory, the occupancy random variable has an exact (but unknown) measurement likelihood. Furthermore, it is shown that the binary Bayes filter is no longer applicable to the propagation of this variable, as the measurement likelihood itself is not deterministic. A new particle filter based method is therefore developed to estimate the posterior distribution of the occupancy variable and perform map building.

The paper is organized as follows: Section III outlines the general occupancy grid problem, showing how the exact

<sup>2</sup>Fixed threshold detection is indeed the optimal detector in the case of spatially uncorrelated and homogenous noise distributions of known mean.

occupancy variable measurement likelihood can be used when signal detection theory is considered. The problems with a binary Bayes filter solution are also discussed. Section IV presents the problem formulation while section V discusses a particle filter solution to the recursion. Section VI then presents some results of the proposed method using real radar data collected from outdoor field experiments and comparisons are made to a ground truth generated by SICK laser range finders.

## III. OCCUPANCY GRID MAPPING

Occupancy grid mapping is generally solved by assuming each grid cell to be independent so that the occupancy variable in each cell can independently estimated [2]. The independent state of interest in each cell is regarded as being discrete where,

$$\sum_{X \in \Theta} X = 1. \tag{1}$$

The set  $\Theta$  can consist of an arbitrary number of hypotheses but usually contains {*Occupied*, *Empty*} in the case of a binary Bayesian approach [5] and {*Occupied*, *Empty*, *Unknown*} in the case of a Dempster-Shafer approach [12]. As seen in [5], the 'inverse' Bayesian approach recursively estimates the probability of each hypothesis using the computationally efficient log-odds approach,

$$\log \frac{P(m|\mathbf{z}^{t})}{1 - P(m|\mathbf{z}^{t})} = \log \frac{P(m|z_{t})}{1 - P(m|z_{t})} + \log \frac{1 - P(m)}{P(m)} + \log \frac{P(m|\mathbf{z}^{t-1})}{1 - P(m|\mathbf{z}^{t-1})}$$
(2)

where *m* denotes the hypothesis X = Occupied and  $\mathbf{z}^t$  represents a history of range measurements up to time *t* at which the sensor hypothesized the presence of a landmark. This is referred to as the inverse model as  $P(m|z_t)$  inversely maps from the measurement at time *t* to the state. Inverse models are also required by Dempsters update rule,

$$m(X_3) = \frac{\sum_{X_1 \cap X_2 = X_3} m_k(X_1) m_m(X_2)}{1 - \sum_{X_1 \cap X_2 = \emptyset} m_k(X_1) m_m(X_2)}.$$
 (3)

Here  $m_k(\cdot)$  and  $m_m(\cdot)$  represent mass functions respectively containing the sensor and prior evidences in support of each hypothesis,  $\{X_1, X_2, X_3\} \subset X$ . That is, a direct mapping from the sensor measurement to the evidence in support of each hypothesis. However, this approach requires 'intuitive' models as it is contrary to the way in which the sensor operates. This may result in inconsistent maps as shown in [5].

Approaches using the 'forward' sensor model,  $P(z_t|m)$ , are also proposed [13]. Using the standard conditional independence assumptions the occupancy posterior can be obtained from,

$$P(m|\mathbf{z}^t) = \gamma P(z_t|m) P(m|\mathbf{z}^{t-1})$$
(4)

where  $\gamma$  is the normalizing constant. Sensor models *spatially* distribute the measurement likelihood about the detected range, *d*, typically using Gaussian spread functions with the sensor range covariance,  $\sigma^2$ . In the case of a 1D measurement,

$$p(z_t|m) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(z_t-d)^2}{2\sigma^2}}.$$
 (5)

Note in this model, the measurement  $z_t$  is a range reading. The range at which a sensor reports the presence of a landmark can be used in the filtering of its *location* estimate. However, whilst this may be correlated with the sensor's ability to correctly detect the landmark, the reported range at which the landmark is hypothesised to exist does not provide a measurement of the occupancy random variable. Therefore in the context of occupancy variable filtering the measurement,  $z_t$ , should not be a range reading but should in fact be the sensor's output hypothesis decision on the presence or absence of a landmark. That is, the measurement space should be redefined as  $z_t \in \{Detection, No \ Detection\}$ . As a result of this subtlety, previous occupancy sensor models typically assume complete knowledge of the sensors' detection characteristics (probabilities of detection and false alarm), and the occupancy measurements become deterministic. The signal processing and measurement intensity information that may be available, are usually ignored. Consequently, this assumption allows for each cell to contain a deterministic occupancy measurement which can be updated using the binary log-odds equation (or Dempsters equation in the case of evidential measurements). This is in contrast to the location measurements which are stochastically modeled and propagated.

### A. Occupancy Mapping from Detection Space

Once the occupancy measurement,  $z_t$ , is defined in detection space rather than polar space, the measurement likelihoods (for both detection and non-detection) become real signal processing parameters. A simple expansion of eqn 4 shows how the occupancy measurement likelihoods can be obtained when the signal processing stage is considered. Consider the probability of occupancy given a history of measurements,

$$P(m|\mathbf{z}^t). \tag{6}$$

The measurement history  $\mathbf{z}^t$  can now be considered as a set of hypothesis decisions on the presence or absence of a target (derived through some function of the measured signal intensity) given by the measurement model. Thus each measurement,  $z_t$ , can be denoted as D if a detection was made, or  $\overline{D}$  if no detection was made. We can then expand about both measurement hypotheses to get,

$$P(m|z_t = D, \mathbf{z}^{t-1}) = \gamma_D^{-1} P(z_t = D|m) P(m|\mathbf{z}^{t-1})$$
(7)

$$\gamma_D = P(z_t = D|m)P(m|\mathbf{z}^{t-1}) + P(z_t = D|\bar{m})P(\bar{m}|\mathbf{z}^{t-1})$$
(8)

$$P(m|z_{t} = \bar{D}, \mathbf{z}^{t-1}) = \gamma_{\bar{D}}^{-1} P(z_{t} = \bar{D}|m) P(m|\mathbf{z}^{t-1})$$
(9)

$$\gamma_{\bar{D}} = P(z_t = \bar{D}|m)P(m|\mathbf{z}^{t-1}) + P(z_t = \bar{D}|\bar{m})P(\bar{m}|\mathbf{z}^{t-1})$$

where m denotes occupancy and  $\bar{m}$  denotes emptiness in a given grid cell. These equations calculate in closed form a statistically correct posterior of the occupancy random variable, where the measurement likelihoods  $P(z_t = D|m)$ ,  $P(z_t = D|\bar{m})$ ,  $P(z_t = \bar{D}|m)$  and  $P(z_t = \bar{D}|\bar{m})$  are those frequently encountered in target detection algorithms. A graphical representation of the target detection hypothesis is shown in figure 1. Here, p(x|m) and  $p(x|\bar{m})$  represent the received signal fluctuation densities under both target present, m, and target absent,  $\bar{m}$ , situations respectively and are further discussed in section V-A.



Fig. 1. A graphical representation of the received signal classification problem. T represents the decision threshold,  $\mu$  is the mean noise power and  $\overline{S}$  is the mean target signal-to-noise ratio. The hypothesis decisions are  $H_0$ : Target absent, and  $H_1$ : Target present. The measurement likelihoods required to calculate the posterior probability of occupancy are also indicated.

The four probabilities present in the detection hypothesis problem, which are also required by equations 7 and 9, are typically referred to as,

$P(z_t = D m) - $ Probability of Target Detection
$P(z_t = D \bar{m}) - Probability of False Alarm$
$P(z_t = \bar{D} m)$ – Probability of Missed Detection
$P(z_t = \overline{D} \overline{m})$ – Probability of "Noise" Detection

Note that m can always be updated, given a detection, or no detection hypothesis. These likelihoods can generally only be calculated exactly when two a priori assumptions are made, that is - a known mean target signal to noise ratio (SNR), and known target power fluctuation likelihood. Under the further assumption of identical and independently distributed (IID) noise power (again of known mean), a suitable power threshold can be calculated which will exactly obtain the theoretically derived detection (and hence occupancy) likelihood. In this case, observations required to calculate the posterior occupancy probability,  $P(m|\mathbf{z}^t)$ , become deterministic and thus the standard deterministic update of eqn 4 is valid. However, when these assumptions are relaxed (the strongest being the known SNR assumption), the above measurement likelihoods become estimated properties and thus the propagation of the occupancy random variable must be carried out using stochastic filtering methods (EKF, Particle Filter, ...) as opposed to a binary filter. As the measurement likelihoods are two complimentary sets,  $\{P(z_t = D|m), P(z_t = \bar{D}|m)\}$ 

and  $\{P(z_t = D|\bar{m}), P(z_t = D|\bar{m})\}$ , we only need estimate one likelihood from each set.

Furthermore a binary filter approach will equally weight both the measurement and the prior, as they are considered to have equal covariances. Occupancy measurements are in fact highly correlated with the vehicle location, and should not be treated equally. For example, specular reflections are likely to occur at high angles of incidences, and clutter free observations should be treated with greater confidence than those with interfering signals.

# **IV. PROBLEM FORMULATION**

This section outlines the proposed system to jointly estimate both the occupancy random variable and the measurement likelihoods.

# A. Data Format

From a radar perspective, the environment can be considered to consist of an unknown number of spatially distributed signal probability density functions (pdf) of both unknown distribution with unknown moments. A single sensor sweep therefore acquires samples from these underlying environmental pdfs and returns them in the form of a power-range spectrum at each bearing angle. A sample of such a spectrum, collected from an outdoor field test, can be seen in figure 2. This spectrum, therefore contains a single signal sample at each the K discrete range increments from the sensor for a given bearing angle. Each range at which a signal sample is acquired by the sensor is referred to as a *range bin*. To model such data, assumptions are typically made



Fig. 2. A sample spectrum, from a frequency modulated continuous wave radar, containing K (assumed independent) signal measurements,  $s_k$ , at K range bins for a single sensor bearing angle.

on the signal distributions under both target presence and absence hypotheses. Let S(M) represent a single power-range spectrum, with  $p(s_k | \Omega_m, m_k)$  being the target presence signal pdf and  $p(s_k | \Omega_{\bar{m}}, \bar{m}_k)$  being the target absence (noise) signal pdf, in the  $k_{th}$  range bin.  $\Omega_m$  and  $\Omega_{\bar{m}}$  are the unknown distribution moments. The noise pdf is assumed IID,  $\forall k \in$  $\{1, \ldots, K\}$ , and the moments of  $p(s_k | m_k = 1)$  are a function of the target's mean SNR,  $\Re$ . The spatial distribution are typically modeled by point spread Gaussian functions using the sensor's range and bearing covariances.

### B. Mapping Algorithm Overview

Figure 3 shows a block diagram of the estimation problem under consideration. The system input is the true occupancy state, m, which is a vector of K binary numbers indicating the presence or absence of a target in the environment, in each of the K range bins. The corresponding SNR's,  $\Re$ , for each target are also required. The sensor model block then uses the targets range, SNR,  $p(s_k|m_k = 1)$  and  $p(s_k|m_k = 0) \forall$ k to generate a raw noisy power-range spectrum, S(M), of figure 2.



Fig. 3. A block diagram of the proposed algorithm. The aim is to estimate the posterior on the occupancy vector, M. This however requires estimates of the measurement likelihoods  $\hat{\Theta}$ . y represents the measurements used in the Bayes filter.

#### C. Constant False Alarm Rate (CFAR) Detector

This block contains the signal detection algorithm which has constant false alarm rate property. Its input the a single power-range spectrum, and its output, Z, is a vector of K binary numbers indicating the detection, or non-detection. A more in-depth explanation of this block can be seen in [14].

## D. Measurement Likelihood Estimator (MLE)

This block provides estimates of *both* measurement likelihoods,  $\theta_d$  and  $\theta_{fa}$ . They are estimated by assuming both  $m_k = 1$  and  $m_k = 0$  respectively  $\forall k \in \{1, \dots, K\}$ .  $\theta_d$ requires estimates of  $\Re$  for each range bin. This will be further discussed in section V-A.

## E. Bayes Filter

The problem is therefore to evaluate the joint likelihood on the occupancy and measurement likelihood random variables at each time t,

$$p(m_t, \Theta_t | y^t) \tag{11}$$

where the measurement,  $y^t$  consists a history of all measurements likelihoods,  $\Theta^t$ , which have been selected given the history of binary detection hypotheses,  $Z^t$ . Note that  $m_t$  now represents the occupancy estimate an arbitrary bin,  $m_k$  at time t. As explained in the previous section, only two measurement likelihoods are required, thus

$$\Theta_t = \begin{bmatrix} \theta_d \\ \theta_{fa} \end{bmatrix}$$

with  $\theta_d$  being the detection measurement likelihood and  $\theta_{fa}$  being the false alarm likelihood. Using Bayes rule to expand (11) we get,

$$p(m_t, \Theta_t | y^t) \propto p(y_t | m_t, \Theta_t) p(m_t, \Theta_t | m_{t-1}, \Theta_{t-1}).$$
(12)

Assuming independence between the components of the measurement likelihood we get,

$$p(y_t|m_t, \Theta_t) = p(Z|m_t, \Theta_t)p(\Theta_t|m_t, \Theta_t).$$
(13)

and as both  $Z_t$  and  $m_t$  are binary,

$$p(Z = 1|m_t = 1, \Theta_t) = \theta_d$$

$$p(Z = 0|m_t = 1, \Theta_t) = 1 - \theta_d$$

$$p(Z = 1|m_t = 0, \Theta_t) = \theta_{fa}$$

$$p(Z = 0|m_t = 0, \Theta_t) = 1 - \theta_{fa}.$$

Also a static process model implies,

$$p(m_t, \Theta_t | m_{t-1}, \Theta_{t-1}) = p(m_{t-1}, \Theta_{t-1} | y^{t-1}).$$
(14)

The measurement likelihoods,  $\theta_d$  and  $\theta_{fa}$ , remain unknown quantities. Note that both the likelihoods and the map state, m, are constrained to exist within the bounded limits [0, 1]. The following section therefore presents a particle filter solution to the proposed problem.

#### V. PARTICLE FILTER IMPLEMENTATION

This section outlines the formulation for the filtering of an occupancy random variable using the signal intensity information from a measurement. The objective of this filter is to propagate the posterior density of the occupancy variable,  $p(m_t, \Theta_t | y^t)$ . Assuming the prior,  $p(m_{t-1}, \Theta_{t-1} | y^{t-1})$  to be represented by a set of particles  $\{x_{t-1}^i, w_{t-1}^i\}$ ,

$$p(m_{t-1}, \Theta_{t-1} | y^{t-1}) = \sum_{i=1}^{N} w_{t-1}^{(i)} \delta_{x_{t-1}^{(i)}}(m_{t-1}, \Theta_{t-1}).$$
(15)

As we have no *a priori* knowledge, we assume the measurement likelihoods as uniformly distributed,

$$\theta_d = U(0,1)$$
$$\theta_{fa} = U(0,1)$$

and the prior on the map as p(m) = 0.5.

## A. Measurement Model

This section outlines the derivation of both measurement likelihoods,  $p(z_t = D|m)$  and  $p(z_t = D|\bar{m})$ , which are commonly referred to as the 'probability of detection',  $\theta_d$ and 'probability of false alarm',  $\theta_{fa}$ .  $\theta_d$  is a function of the *estimated* mean target SNR,  $\overline{\Re}$ , an assumed target SNR fluctuation model,  $p(\Re|\overline{\Re})$ , as well as the detection threshold, T.  $P_{fa}$  is generally a constant design parameter. As discussed previously in section IV-A, the measurement data S(M)is assumed to consist of a vector of K consecutive (in time or range) independent signal intensity measurements,  $s_k$ , for each bearing angle, shown previously in figure 2. A priori signal distribution assumptions are made on both  $p(s_k|\Omega_m, m_k)$  and  $p(s_k|\Omega_{\bar{m}}, \bar{m}_k) \quad \forall k \in \{1 \dots K\}$ , where the distribution moments  $\Omega_m$  and  $\Omega_{\bar{m}}$  are generally assumed unknown and must be estimated using the signal intensity information.

1) The False Alarm Likelihood Measurement: Given the distribution  $p(s_k | \mathbf{\Omega}_{\bar{m}}, \bar{m}_k) \quad \forall k \in \{1 \dots K\}$  with known  $\Omega_{\bar{m}}$ , an optimal detection threshold  $T_k$ , calculated through a function of  $\Omega_{\bar{m}}$  can be derived to obtain a constant  $\theta_{fa}$ . In the case of unknown  $\Omega_{\bar{m}}$ , with certain distribution assumptions (Exponential, Rayleigh, Weibull, K-distributed), an adaptive  $T_k$  which is independent of the unknown parameters  $\Omega_{\bar{m}}$  can be obtained and a constant  $P_{fa}$  maintained. Therefore, constant false alarm rate (CFAR) detectors maintain the predefined false alarm likelihood,  $\theta_{fa}$ , if the K consecutive intensity measurements are IID samples from  $p(x_k | \Omega_{\bar{m}}, \bar{m})$   $k \in \{1 \dots K\}$ , and more importantly, given that the distribution assumption on  $p(s | \Omega_{\bar{m}}, \bar{m})$ , is valid. The density of equation **??** will thus become deterministic.

2) The Detection Likelihood Measurement: To make an estimate of  $\theta_d$ , we must first estimate the targets mean SNR,  $\Re$ . Taking the measured intensity in bin k,  $\{s_k | m\}$ , as the signal + noise measurement (assuming the existence of a target), we must therefore estimate the local noise intensity at that bin to generate an SNR estimate. Using leading and lagging windows, N intensity measurements are used to generate this estimate. From the signal detection literature [16], there are numerous adaptive methods of generating local estimates of the unknown noise distribution parameters. Whilst most signal processing literature considers the probability of detection for targets of known SNR, this work requires  $\theta_d$  to be estimated from the measured intensity information using an assumed distribution  $p(s|\mathbf{\Omega}_m, m)$ , but with an *unknown* mean SNR. Thus, given an estimate of the local noise intensity in bin k,  $\{s_k | \bar{m}\}$ , generated by the detector,  $\bar{\Re}_k$  can be estimated as,

$$\bar{\Re}_k = \frac{\{s_k|m\} - \{s_k|\bar{m}\}}{\{s_k|\bar{m}\}}.$$
(16)

(17)

An ordered-statistics approach [17], which is adopted in this work, has been shown to be most robust in situations of high clutter and multi-target situations, as is commonly encountered in a field robotics environment. Under certain distribution assumptions, using an ordered statistic noise estimate, closed form solutions exist for  $\theta_d$ ,

 $\theta_d = \left(1 + \frac{T_k \{s_k | \bar{m}\}}{1 + \bar{\mathfrak{R}}_k}\right)^{-N}$ 

with,

$$p(s|\mathbf{\Omega}_{\bar{m}} = \mu, \bar{m}) = \frac{1}{\mu} \exp(-s/\mu)$$

$$p(s|\mathbf{\Omega}_{m} = \{\mu, \Re\}, m) = \frac{1}{\mu} \exp\left((-s/\mu) + \Re\right) I_{0}\left(2\sqrt{\frac{\Re s}{\mu}}\right)$$

$$p(\Re|\bar{\Re}) = \frac{2\Re}{\bar{\Re}} \exp(-\Re^{2}/\bar{\Re}).$$

Thus, approximating each estimate as a Gaussian random variable, we can generate a Gaussian distributed estimate of

 $\theta_d$ . Given an estimate,  $\bar{S}_k$  of the mean SNR in range bin k, we can estimate the measurement likelihood, which is then sampled during the occupancy measurement update as seen in equation **??**.

#### **VI. EXPERIMENTS**

A log odds model is dependant on the order of the measurements -  $DDD\bar{D}\bar{D}$ 

In this section the proposed filter is analyzed using real data collected from a 77GHz FMCW radar in an outdoor environment. The first example propagates an existence variable in each range bin of the radar (K = 800) at a single bearing angle. A sample spectrum from this data was shown previously in figure 2. It is assumed that there are three reflections present in the data, two from targets and one from multipath effects. As shown in figure 4, with no a priori information available, the variables are initialized with a uniform distribution. In this example  $N_r$  and  $N_l$  (as seen in equations ?? and ??) are chosen to be 20. An ordered statistics approach is used to generate a local estimate of the noise level in each range bin, a leading and lagging window width of 20 bins, with 2 guard cells at each side of the cell-under-test. Using a swerling I fluctuation assumption [15], and a  $P_{fa}$ of  $10^{-6}$ , the mean and variance of the detection probability can be estimated according to equation 17 for every range bin, prior to the target presence hypothesis decision. Once the hypothesis,  $H_0: \overline{m}$  or  $H_1: m$ , is made at each range bin, the measurement likelihoods,  $p(z_t = D|m)$  and  $p(z_t = D|\bar{m})$ can be defined. Samples from these densities are shown in figure 5. Note in this most conservative measurement likelihood, every range bin is treated independently. That is, for each time step t, each peak in the data (under both null and alternate hypotheses), is treated as a potential target, and no model of the sensor beam propagation properties is assumed. Mixing these samples with the prior samples and resampling according to equations ?? - ??, we get the posterior density samples in each range bin as seen in figure 6, with an associated variance seen in figure 7. Large uncertainty exists in regions of missed detections as there is in adequate information to distinguish between empty space or a missed detection. Figures 8 and 9 shows the same corresponding plots after 5 iterations. It can be seen that the variance remains practically zero for most cells however due to high variance SNR estimates, particulary at 5m and 150m, there remains some uncertainty in the existence estimate. From this plot it can be seen that the filter is most certain of target presence at approximately 135m, as the existence estimate is close to unity and its associated variance is close to zero.

The second experiment was conducted in an outdoor carpark within the university campus. A picture showing an overview of the carpark is seen in fig 11. Due to the difficulty in getting an accurate "ground truth" for outdoor occupancy grid mapping algorithms, a comparison map was constructed from two back-to-back LMS SICK laser sensors, as shown in figure 12. A picture of the vehicle 'Johnny 5' is shown in figure 10. With built-up surroundings, the GPS



Fig. 4. The uniform a priori occupancy samples at each range bin with the mean (dashed) plotted



Fig. 5. The measurement samples based on the estimated probability of detection and the output hypothesis of the detector. Samples represent particle approximations of  $p(z_t = D|m)$  or  $p(z_t = D|\bar{m})$ , in the case of  $H_1$  and  $H_0$  hypotheses respectively. Larger spreads of particles indicate increased uncertainty in that cell. The dashed plot represents the mean value of  $P_d$  in each range bin, independent of the detection hypothesis, thus accommodating for potential missed detections.



Fig. 6. The posterior existence samples after a single filter iteration, with the dashed line representing the mean occupancy in each bin.



Fig. 7. Variance of the posterior occupancy samples, in each range bin k, after a single filter iteration.



Fig. 8. The posterior occupancy samples, after five filter iterations, with the dashed line again representing the mean existence in each bin.



Fig. 9. Variance of the posterior occupancy samples, after five filter iterations. The filter is most certain of landmark existence at 135m as there is low variance on a high occupancy estimate.



Fig. 10. Test platform 'Johnny 5'.

data acquired was not accurate to determine ground truth location. As the laser scans frequently obtained returns from the ground, automatic matching techniques failed. Thus the consecutive laser scans were manually matched to determine the true location of the vehicle at each update.

The environment contains numerous objects of varying dimensions with fluctuating probabilities of detection. In this environment, missed detections of small targets such as lamp posts and trees are common. Although a Log-Odds update, such as that seen in [18], presents a computationally simplistic approach, its range ( $[0,\infty]$ ) makes it susceptible to high amplitude false alarms. Furthermore, there is no context of "uncertainty" for such a filter, thus all detections are equally trusted and scaled according to the same measurement model. Figure ?? shows the resulting maps based on the standard logodds implementation, with all occupancy estimates treated as being statistically equivalent. Figures 13 and 14 show the expected value of the existence variables on the same map along with their associated variances. Again low variance estimates can be extracted in regions of unbiased SNR estimates, which occur mostly in regions of low clutter. When a feature is present within some clutter (perhaps a bush) an increased uncertainty is present in the existence estimate. Finally figure 15 shows a cross section through the map, plotting the existence estimate along with its 2 sigma bounds.

## A. Actual Probability of False Alarm

Can use an evidential approach, measure pdf deviation from the assumed..or add a noise to the estimate. Has a large affect on the calculated occupancy posterior.

## VII. CONCLUSION

This paper presented an alternate method of tracking a features "quality" by considering the sensor signal processing. It showed that the measurement likelihood typically used in occupancy grid algorithms should in fact be modeled as a density function as opposed to a deterministic function, which is normally the case. By examining the measurement

Entrance Passage

Fig. 11. Overview of testing ground for comparison with the radar generated maps for figures **??** and 13 (Cars were absent at time of scan).



Fig. 12. The corresponding laser map for the carpark environment shown in figure 11.

model and using signal detection theory, it is shown that the existence random variable can be calculated in closed form without the need of heuristic models. However, the measurement likelihood used in the existence calculation is in itself an estimated entity, and is thus modeled by a density function rather than a discrete variable. The standard discrete Bayes estimation framework therefore no longer applies to the occupancy grid problem and a particle filter approach is proposed. Using discrete Bayes solutions for each particle with random samples from the measurement likelihood, a particle set representing the posterior existence can be generated. Weights for these posterior particles are also obtained by propagating prior and measurement weights through the



Fig. 13. A plot of the mean existence value on the map. The model considered missed detections of objects with low radar cross section unlike that of the log odds model.



Fig. 14. A plot of the variance of the occupancy estimates. This allows us to statistically quantify the occupancy estimates on the map which is not considered in previous methods.

discrete Bayes filter. The resulting set of posterior particles is then resampled and the particle count is reduced to the original number. By virtue of the static state assumption, these particles then represent the prior distribution for the following iteration.

This concept was demonstrated for a MMWR sensor which is typically used in an outdoor environment. The sensor gives access to unprocessed range data, allowing for custom feature detectors to be developed. The framework then allows for the accurate assignment of map existence probabilities, irrespective of the hypothesis chosen by the detector.

However, significant work needs to be carried out to



Fig. 15. A cross section through the map showing the estimate occupancy with 2 sigma bounds. For illustration purposes the bounds are not cropped at the variable bounded limits of 0 and 1.

accurately estimate  $P_{fa}$  in regions of the map which deviate from the homogenous exponential assumption. Since for most practical scenarios  $P_d \gg P_{fa}$ , any Bayesian based mapping method will assign almost unity existence to any detection. However, when the assumption on the noise distribution is violated and  $P_{fa}$  deviates from its theoretical value, an evidential method should be applied as there would be large uncertainty as to the true false alarm probability. This would further improve the mapping accuracy. Further work also would integrate three dimensional uncertainties (in Euclidean location as well as existence) into data association decisions and SLAM algorithms.

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