

On Multidimensional Assignment Data Association for Simultaneous Robot Localization and Mapping

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Abstract—Data association or the correspondence problem is often considered as one of the key challenges in every state estimation algorithm in robotics. This paper introduces an efficient multi-dimensional assignment based data association algorithm for simultaneous localization and map building (SLAM) problem in mobile robot navigation. Data association in SLAM problem is compared with the data association in a multi-sensor multi-target tracking context and formulated as a 0-1 integer programming(IP) problem. A suboptimal dual frame assignment based data association scheme is thus formulated using a linear programming relaxation of the IP problem. Simulations were conducted to verify the superior nature of the new data association scheme over the conventional nearest neighbor data association algorithm in the presence of high clutter densities. Experimental results are also presented to verify the enhanced performance of the algorithm.

Keywords—tracking; localization; data association; robot navigation;

I. INTRODUCTION

The SLAM problem has often been recognized as one of the key challenges in building autonomous mobile vehicles capable of operating in complex unstructured environments. The goal of an autonomous vehicle performing SLAM is to start from an unknown location in an unknown environment, build a map (consisting of environment features) of its environment incrementally by using the uncertain information extracted from its sensors, whilst simultaneously using that map to localize itself with respect to a reference coordinate frame and navigate in real-time. A vehicle capable of performing SLAM using naturally occurring environmental features in its locality and sustaining for hours or possibly days in completely unknown environments will indeed be invaluable in several key areas of robotics such as autonomous vehicle operation in unstructured terrain, driver assistance systems, mining, surveying, cargo handling, autonomous under water explorations, aviation applications, autonomous planetary exploration and military applications. The first solution to the SLAM problem was due to Smith, Self and Cheeseman [1]. They emphasized the importance of map vehicle correlations in SLAM and introduced the Extended Kalman Filter (EKF) based stochastic framework, which estimated the vehicle pose and the map feature (landmark) locations in an augmented state vector. Although EKF based SLAM within the stochastic mapping framework gained a wide popularity among SLAM research community over the time, it

has several shortcomings. [2], [3] Notable shortcomings are its susceptibility to data association errors and inconsistent treatment of nonlinearities.

Data association or the correspondence problem is one of the most difficult problems encountered in SLAM even in stationary environments and much more challenging in dynamic environments. Almost every algorithm available today for state estimation has to deal with the correspondence problem in the form of maximum likelihood assignment or correlation search in establishing the correspondence between the elements of observations and the available tracks. Large uncertainties in vehicle pose, variable feature densities, dynamic objects in the environment, false alarms and clutter complicate data association in SLAM problem in many respects. An efficient data association scheme must also aid feature or track initialization, maintenance, termination and map management. Furthermore, in traversing large loops or cycles, robots face what is known as the “Cycle Detection Problem” or “Loop Closing Problem” which means identifying the return to a previously mapped region. This problem introduces a significant overhead to the data association algorithm in the form of very large search space, especially in mapping large areas. Therefore development of efficient and robust data association algorithms is a very important area in robot localization and mapping.

Feature based approach to SLAM, can be considered as a multi-sensor multi-target tracking problem [2]. It is highly sensitive to the fragility in data association (incorrect measurement to feature associations). Miss-associations can cause the map to be converged to an incorrect state and sometimes result inconsistency and divergence. An efficient data association scheme must also establish the difference amongst false alarms, new feature measurements and missed detections in addition to the basic function of associating currently available feature tracks with measurements. The most widely employed data association method in SLAM is the nearest neighbor data association algorithm [3]. It associates a track to the nearest observation in its validation region based on some distance measure. However, the nearest neighbor data association has several shortcomings. As such it fails even in low densities of false alarms and data association decisions once made cannot be reversed (Hard decisions). Techniques like joint probabilistic data association (JPDA) [4] are devised to provide a better solution in this respect. JPDA associates all the measurements falling inside the validation region of a track

to itself by a probabilistic weighting procedure and performs fairly well in moderate clutter. However, it can be computationally prohibitive in the manner of calculating weighting probabilities. On the other hand JPDA, in its standard form, does not explicitly provide a means of initiating tracks, which is vital for feature based map building applications.

Multiple Hypotheses Tracking (MHT), [5], is the most structured and optimal approach employing deferred logic available for multi-target tracking and data association. Deferred logic schemes allow, data association decisions to be deferred until a number of additional frames of measurements are received in successive scans and association decisions made in the past can be corrected as a consequence. MHT data association defers the association decisions in conflicting situations and forms all probable association hypotheses, which are then propagated through subsequent iterations in the belief that new information will most likely resolve the conflicts. Therefore MHT is capable of dealing with missed detections, false alarms and track initiation. However the hypotheses tree in MHT grows exponentially in time and therefore suffers from exponential memory and computational requirements.

In this paper we propose for the first time a multidimensional assignment based data association algorithm [14] for SLAM problem. The multidimensional assignment method proposed in this work, has been subjected to extensive experimental verifications and is believed to find widespread use in most future systems even going to the extent of replacing MHT. The use of this method is largely justified on the basis that nearest neighbor data association fails in most instances in SLAM when features are not sparsely distributed or in the presence of high, persistent clutter. Multidimensional assignment methods in multi target tracking and data association have comparable performance with MHT and lower computational complexity than MHT and JPDA.

This paper is organized in the following manner. Section II briefly reviews previous work on multi-dimensional assignment based data association approaches and formulates general two frame data association method. Section III extends the two frame data association for the EKF based stochastic SLAM framework. Section IV describes simulation results and a comparison between the nearest neighbor data association method and the multi-dimensional assignment based method for data association in SLAM. Section V provides conclusions drawn from this work and extensions for future work.

II. PROBLEM FORMULATION

A. Data Association as a Multiple Frame Multidimensional Assignment Problem

An alternative to the optimal MHT method is proposed in [6], in which multiple-frame data association in the context of multi-target tracking is formulated as a discrete optimization problem. This is further extended in [7] and [8] by expressing data association of multiple targets over multiple frames, in the form of a multidimensional assignment problem. The core attribute of these algorithms can be identified as the use of more than one previous frame of measurements in determining the best associations for the current frame.

In complexity theory, multidimensional (N dimensional) assignment problem for $N \geq 3$, is considered to be NP-hard. Therefore the various suboptimal methods [7] and [8] such as Lagrangian relaxation are used to obtain a sub optimal solution. Because of the computational complexity of multi-frame assignment based data association for large N, a two frame assignment scheme is proposed in this work for the data association in simultaneous localization and mapping of an autonomous vehicle. The entire algorithm is first formulated as a three dimensional assignment problem. Then it is reduced to a linear programming problem as detailed in [9] for a polynomial time suboptimal solution. Solution to the linear programming assignment problem thus formulated can be occasionally fractional and such fractional solutions, when encountered, are considered to be the association probabilities of a JPDA type update method.

B. General Two Frame Data Association

A rigorous formulation of a two frame data association scheme, which is a special case of multi-frame assignment data association, is presented in this section for feature based SLAM. For simplicity, an EKF based stochastic mapping approach is employed in this formulation to clearly demonstrate the data association algorithm, although the method can be easily extended to cater the Rao-Blackwellised particle filter based methods [10] and [11] as well. The algorithm utilizes two frames of measurements at times k and $k+1$ and tracks updated at time $k-1$ and assign measurements to tracks at time k based on the combined effect of measurement frames obtained at time k and $k+1$. The frames at time k and time $k+1$ are denoted as $frame_k$ and $frame_{k+1}$ respectively. Now the data association problem in the form of multidimensional assignment is to assign proper measurement combinations to existing tracks and new probable tracks in the two frames concerned in an efficient and optimal manner.

Let $\mathbf{Z}(k) = \{\mathbf{z}_i(k) \mid i = 0, 1, 2, \dots, n_k\}$ and $\mathbf{Z}(k+1) = \{\mathbf{z}_j(k+1) \mid j = 0, 1, 2, \dots, n_{k+1}\}$ be the sets of measurements obtained in $frame_k$ and $frame_{k+1}$ respectively. Here n_k and n_{k+1} are the number of measurements obtained in the frames k and $k+1$ respectively. The symbols $z_0(k)$ and $z_0(k+1)$ are dummy measurements used to accommodate the missed detections of targets in the respective measurement frames, so that targets not detected in the frames concerned could be assigned to them. Let the association variable $\eta(t, i, j)$ be defined in such a way that $\eta(t, i, j) = 1$ when measurement $\mathbf{z}_i(k)$ in $frame_k$ and measurement $\mathbf{z}_j(k+1)$ in $frame_{k+1}$ are associated with the target t , out of T existing targets and $\eta(t, i, j) = 0$ otherwise.

Let T^S denote, a partition of assigning T , existing targets to measurement pairs given by, $\Omega = \{(i, j) \mid \mathbf{z}_i(k) \in frame_k \text{ and } \mathbf{z}_j(k+1) \in frame_{k+1}, \forall i \& j\}$ Then the likelihood of T^S given by $\Lambda(T^S)$ can be calculated in the following manner.

$$\Lambda(T^S) = \Lambda_{true}(T^S)\Lambda_{false}(T^S) \quad (1)$$

Where $\Lambda_{true}(T^S)$ is the likelihood of measurements associating with true targets in T^S and $\Lambda_{false}(T^S)$ is the likelihood of measurements associating with false alarms in T^S . Given P_d as the detection probability of targets, $L(\mathbf{z}_i(k)|t)$, the likelihood of $\mathbf{z}_i(k)$ at $frame_k$ given the target t , $L(\mathbf{z}_j(k+1)|\mathbf{z}_i(k),t)$ the likelihood of $\mathbf{z}_j(k+1)$ at $frame_{k+1}$, given the target t updated by measurement $\mathbf{z}_i(k)$ at $frame_k$, the likelihood of the partition T^S can be determined as a nonlinear function φ , [14] in the following manner.

$$\Lambda(T^S) = \varphi(P_d, V_s, \varepsilon^k(i), \varepsilon^{k+1}(j), L(\mathbf{z}_i(k)|t), L(\mathbf{z}_j(k+1)|\mathbf{z}_i(k),t)) \quad (2)$$

Here $\varepsilon^k(i)$ indicates whether target t is detected at $frame_k$ ($\varepsilon^k(i) = 1$ if detected and $\varepsilon^k(i) = 0$ otherwise) and similarly $\varepsilon^{k+1}(j)$ indicates whether target t is detected at $frame_{k+1}$. Indices i and j denote the indices of measurements in $frame_k$ and $frame_{k+1}$ respectively. V_s is the surveillance region. Assuming that the false alarms are uniformly distributed in V_s as in [4], the normalized likelihood of T^S denoted by $\Lambda_N(T^S)$ is defined as follows.

$$\Lambda_N(T^S) = \Lambda(T^S) / (V_s^{-n_k} V_s^{-n_{k+1}}) \quad (3)$$

Now, the data association problem can be expressed as searching for T^S that minimizes minus of the log likelihood value of T^S denoted by $C(T^S)$. Let $c(t, i, j)$ denotes the negative value of the joint log likelihood of measurement $\mathbf{z}_i(k)$ in $frame_k$ and measurement $\mathbf{z}_j(k+1)$ in $frame_{k+1}$ be associated with existing target t . Hence, the two frame data association problem is the integer programming problem;

$$\text{Minimize } \sum_{i=0}^{n_k} \sum_{j=0}^{n_{k+1}} \sum_{t=1}^T \eta(t, i, j) c(t, i, j) \quad (4)$$

Subject to the constraints (5),(6),(7),(8),(9) and (10) imposed on the association variables.

a) Single source constraint for measurements:

Each measurement except dummy measurements can be assigned to only one target or not assigned to any target. However dummy measurements can be assigned to more than one target in a frame, as there can be several undetected targets in a particular measurement frame.

$$\sum_{t=1}^T \sum_{j=0}^{n_{k+1}} \eta(t, i, j) \leq 1 \text{ for } i = 1, 2, \dots, n_k \quad (5)$$

$$\sum_{t=1}^T \sum_{i=0}^{n_k} \eta(t, i, j) \leq 1 \text{ for } j = 1, 2, \dots, n_{k+1} \quad (6)$$

Constraint (5) therefore consists of n_k constraints and constraint (6) consists of n_{k+1} constraints.

b) Single return constraint for targets:

Each target can generate only one measurement in one measurement frame.

$$\sum_{i=0}^{n_k} \sum_{j=0}^{n_{k+1}} \eta(t, i, j) = 1 \text{ for } t = 1, 2, \dots, T \quad (7)$$

Constraint (7), therefore consists of T constraint equations.

c) Constraint on maximum number of dummy measurements

Maximum missed detections of targets in a measurement frame cannot exceed the number of existing targets.

$$\sum_{t=1}^T \sum_{j=1}^{n_{k+1}} \eta(t, 0, j) \leq T \text{ for } i = 1, 2, \dots, n_k \quad (8)$$

$$\sum_{t=1}^T \sum_{i=1}^{n_k} \eta(t, i, 0) \leq T \text{ for } j = 1, 2, \dots, n_{k+1} \quad (9)$$

(8) and (9) consists of n_k and n_{k+1} equations respectively.

d) Non-negativity Constraints: Since η is an association variable, η must be always nonnegative.

$$\eta(t, i, j) \geq 0 \text{ for } i = 0, 1, 2, \dots, n_k, \\ j = 0, 1, 2, \dots, n_{k+1} \text{ and } t = 1, 2, \dots, T \quad (10)$$

However this integer programming problem is a 3-dimensional assignment problem and therefore NP hard in complexity. Hence it is necessary to obtain a sub optimal solution by relaxing the integer constraints of $\eta(t, i, j)$ as shown in [6], [7], [8] and [9].

III. TWO FRAME DATA ASSOCIATION FOR FEATURE BASED SLAM

A. Basic SLAM Framework

The basic framework used in EKF SLAM algorithms represents both the vehicle and the landmark locations by absolute coordinates with reference to a coordinate frame. The major highlight of the formulation was its consistent probabilistic representation of robot's pose, landmark positions, uncertainties and their relationships using EKF. The methodology is still considered to be the primary framework of most feature-based stochastic SLAM algorithms [1], [3] and is also used in this work. A major attribute of this formulation is the map augmented state vector denoted by \mathbf{X} , a vector consisting of feature or landmark position vectors and the vehicle pose, \mathbf{x}_k at time k .

$$\mathbf{X} = [\mathbf{x}_k \quad \mathbf{M}_k]^T \quad (11)$$

Where $\mathbf{X} = [\mathbf{x}_k \quad x_1^k \quad y_1^k \quad x_2^k \quad y_2^k \quad \dots \quad x_n^k \quad y_n^k]^T$ and landmark position vectors $[x_i^k \quad y_i^k]^T, i = 1, 2, \dots, n$ denote the

absolute coordinates of the landmark locations with reference to a world coordinate frame. In general, the motion model of the vehicle is nonlinear and can be represented in the closed form as

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{v}(k) \quad (12)$$

Where \mathbf{u}_k is the control input at time k and $\mathbf{v}(k)$ is a zero mean temporally uncorrelated noise sequence with covariance matrix $\mathbf{Q}(k)$. Similarly when $\mathbf{w}(k)$ is also a zero mean temporally uncorrelated noise sequence with covariance matrix $\mathbf{R}(k)$, the observation model is represented by

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{x}_k, \mathbf{M}_k) + \mathbf{w}(k) \quad (13)$$

When the vehicle state covariance matrix is denoted by $\mathbf{P}(k | k)$, estimation process takes the following form.

$$\mathbf{X}(k | k-1) = [\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) \quad \mathbf{M}_{k-1}]^T \quad (14)$$

$$\mathbf{P}(k | k-1) = \mathbf{F}\mathbf{P}(k-1 | k-1)\mathbf{F}^T + \mathbf{Q}(k) \quad (15)$$

Where \mathbf{F} is the Jacobian ($\partial \mathbf{f} / \partial \mathbf{x}_k$) of the process model evaluated at time $k-1$. When true observations are available at time k , the state vector is updated after resolving correct observation to landmark association using an appropriate data association algorithm. Once this problem is solved, update of the map augmented state vector is carried out using the standard EKF equations. In the two frame assignment based data association, the integer programming problem (4) is NP hard in complexity. Thus the data association problem for SLAM is simplified by relaxing the integer constraint of η . Then the integer programming problem (4) would become a linear programming problem. Since the solution to the linear program can be occasionally sub optimal, a JPDA type update is used in rare cases having fractional solutions.

B. Modifications for Data Association in SLAM

Two steps must be completed prior to the integer programming problem formation and its linear program relaxation. 1) Reduce the number of variables associated with the linear program by data preprocessing and gating of measurements because only a small fraction of the association variables actually make sense in the data association solution. 2) Calculate the cost coefficients $c(t, i, j)$ of the linear programming problem. In the context of SLAM it is also important to consider possible tentative and confirmed features as outlined in [3] in performing data preprocessing. Gating is done in every frame to identify those landmarks that are falling outside the validation regions of the confirmed features and also allowing for the missed detections. Other observations are added to a tentative feature list. A validation matrix Ψ is constructed for each frame k and $k+1$. Fig. 1 shows a time instance in a simulation where several measurements are obtained and validated with the available features. The number 1 in particular cell of Fig. 1 shows that particular measurement is in the validation region of the corresponding target and 0 indicates otherwise. Consequently the variable $\eta(t, i, j)$ is retained in the set of variables only if both $\mathbf{z}_i(k)$ and $\mathbf{z}_j(k+1)$ fall inside the validation region of feature t which can be derived from the two validation matrices.

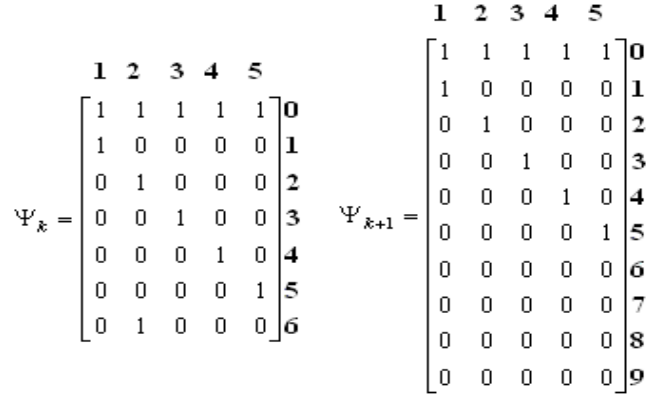


Figure 1. Validation matrices for two adjacent frames. The columns in validation matrices consist of available features and rows correspond to the measurements in respective frames.

Calculation of $c(t, i, j)$ is performed for both the confirmed feature list and the tentative feature lists by incorporating vehicle location uncertainty in the measurement model and assuming Gaussian densities using second order statistics for the likelihoods.

Since the number of features that must be processed in outdoor SLAM is fairly low (below 20), the scheme can be implemented faster. However, complexity of popular linear program solving methods such as simplex method is said to have a worst case exponentially growing complexity with the problem size. In the simulation of this data association algorithm, an efficient and much faster (having a polynomial complexity) interior point algorithm [13], known as Mehrotra's predictor corrector method [12] is used. One advantage of this data association algorithm is that there is a clear way of distinguishing track initiation. When converting constraint inequalities given by (5) into equations in linear programming solution algorithms, slack variables S_i are used. Therefore, n_k slack variables are required to convert the constraint inequality (5) into equation form. If $S_i = 1$ for any i , then that observation in frame k , $\mathbf{z}_i(k)$, can't be associated with any existing track and therefore most probably originated from a new track. Such tracks can be added into tentative tracks for further confirmation as in nearest neighbor data association for SLAM.

IV. SIMULATIONS AND EXPERIMENTS

A. Algorithm Performance in SLAM

Data association algorithm was tested in a simulated environment consisting of several point features. A scenario in which an autonomous vehicle performing EKF, feature based SLAM in this environment with the help of range bearing sensor (such as SICK LMS 291 measurement system) and encoders is considered. The simulation parameters used in the study included the clutter model [4] with clutter density of 0.002 returns per m^2 .

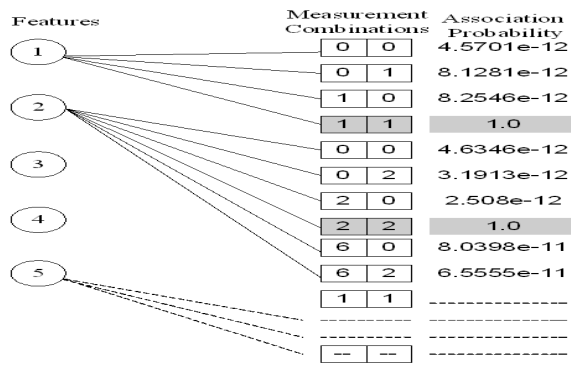


Figure 2. Feature to measurement association hypotheses. The figure shows possible association hypothesis in one instant. Measurement combinations shown by squares indicate the measurement indices in two consecutive frames of measurements.

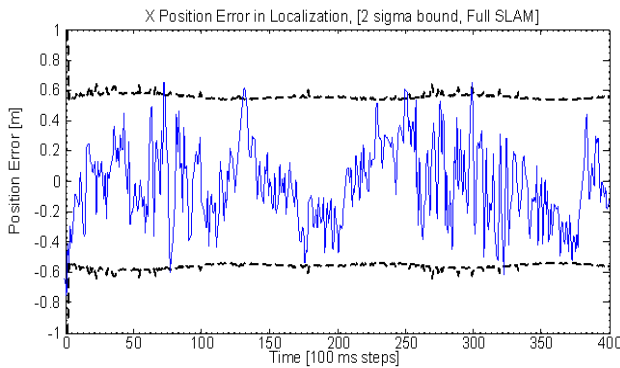


Figure 3. Lateral position error in SLAM with two frame data association. Estimated 2 sigma bounds of the localization error are shown by the dashed lines.

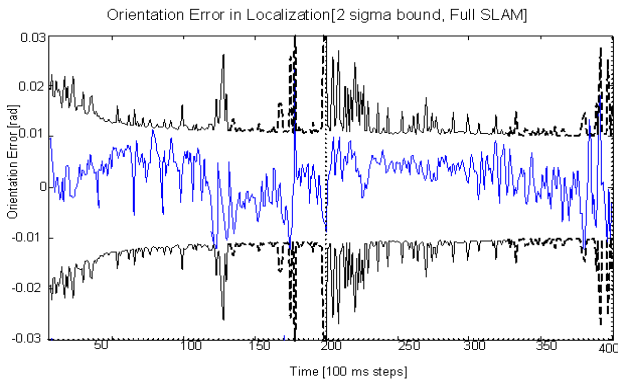


Figure 4. Orientation error in SLAM with two frame data association. Estimated 2 sigma bounds of the localization error are shown by the dashed lines.

Simulation results show that SLAM algorithm performance is quite satisfactory even under high clutter levels. A typical association scenario is elaborated in Fig. 2. The particular case consists of 22 association hypotheses, number of clutter returns in frame k and $k+1$ are 1 and 4 and number of true measurements in both cases is 5. Fig. 2 clearly shows when the correct measurement to target association is obtained, the

solution is equal to 1.0 and for other combinations of assignments it is negligibly small. Performance of the SLAM algorithm with this data association method is shown by Fig. 3 and Fig. 4. The errors are well bounded by the two sigma limits as illustrated.

B. Algorithm Performance in SLAM

The new data association algorithm was compared with the standard nearest neighbor data association scheme by performing several Monte-Carlo runs. Table 1 shows the comparison and it clearly justifies the superior performance of two frame assignment based data association method for SLAM in high clutter levels.

C. Complexity

The complexity of the linear program can be well reduced by the preprocessing steps described in Section III. The linear program solution to the assignment problem is obtained by a primal-dual infeasible-interior point approach known as Mehrotra's predictor corrector method [12]. The result established in [9] on the complexity of data association algorithm in multi target tracking is still valid here and therefore SLAM data association and preprocessing functions in two frame method has the worst case complexity that grows with the cube of the number of features considered. In this work several studies were conducted to compare the amount of computational resources required in the two frame method and the standard nearest neighbor method. A CPU time requirement for preprocessing and data association functions are determined for several Monte-Carlo runs by using a Pentium 4, 2.4 GHz, 512 MB RAM PC as illustrated in Table 2.

D. Experimental Results

The experimental verification of the above data association algorithm in SLAM is carried out by implementing the algorithm with data obtained by the Generic Outdoor Mobile Explorer (GenOME), a car like mobile robot (Fig 7).

TABLE 1. COMPARISON OF DATA ASSOCIATION METHODS

Data Association Algorithm	% Track Loss			
	False Alarm Density 0.01 at 99% Gate		False Alarm Density 0.5 at 99% Gate	
	Feature 1	Feature 2	Feature 1	Feature 2
Nearest Neighbor	2.7	2.8	36.5	36.2
Two frame assignment	1.0	1.1	9.4	9.2

TABLE 2. COMPARISON OF AVERAGE COMPUTATIONAL LOAD

Data Association Algorithm	CPU Time (s)	
	Number of Association hypotheses	
	50	100
Nearest Neighbor	0.018	0.042
Two frame assignment	0.098	0.214

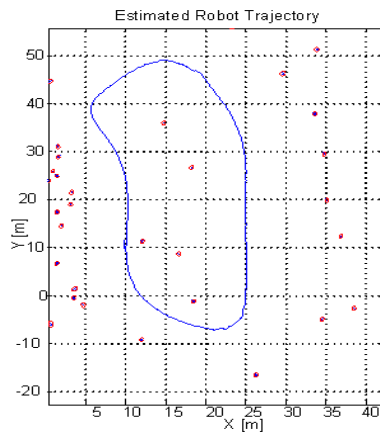


Figure 5. Estimated vehicle path (thick line) and feature locations(circles) (SLAM experiment in a campus car park).

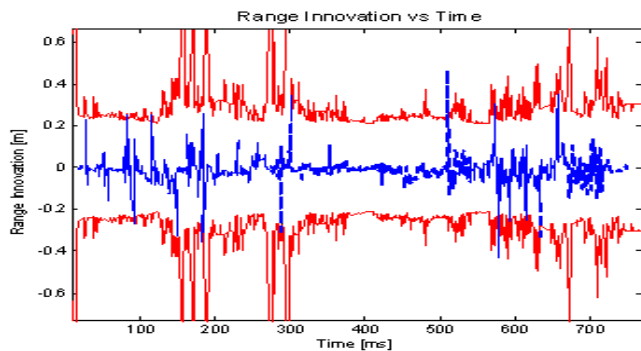


Figure 6. Filter performance :Range innovation(dashed line) and its two sigma bounds(thick line).



Figure 7. Mobile robot used in SLAM experiments

The vehicle is equipped with SICK LMS 291 laser measurement system, GPS, gyroscopes and wheel encoders. Range, bearing readings obtained from SICK LMS 291 and encoders are used to estimate the vehicle path and feature map(consisting of trees and lamp posts) at the same time. The estimated path and the feature locations of the SLAM experiment [3] performed in a campus car park is shown in Fig.5. Comparison with true vehicle path and estimated vehicle path and the well bounded innovations process shown in Fig 6 verify that the performance of SLAM is satisfactory with the new data association algorithm.

V. CONCLUSIONS

This paper introduces a new data association algorithm utilizing multiple frame multidimensional assignment for SLAM problem. The work establishes that the performance of data association in SLAM under two frame assignment is superior to standard hard logic nearest neighbor method in high clutter. Formulation of this data association algorithm for SLAM is also similar in single frame. However the performance is proved to be fairly low to justify the increase in computational overhead. However, the data association algorithm would be a suboptimal alternative to MHT if formulated in multiple frames. The encouraging results suggest that it would be desirable to examine the performance of this data association scheme when formulated in multiple frames.

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