

# An Analysis of the Bias Correction Problem in Simultaneous Localization and Mapping

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**Abstract**— Unmodeled systematic and nonsystematic errors in robot kinematics and measurement processes often cause adverse effects in several autonomous navigation tasks. In particular, accumulated sensor biases can render simultaneous localization and mapping (SLAM) algorithms of autonomous vehicles to perform very poorly especially in large unexplored terrains including cycles, as a result of the estimator divergence and inconsistency. One way to deal with this problem is the accurate modeling and precise calibration of sensors. However this may add up to longer setup and calibration times. Even after accurate calibration and modeling, sensor calibration may often be subject to drifts, rendering the efforts ineffective. Therefore, the correct and effective way to deal with this problem is explicit estimation of these parameters with other states. In this work we address the estimation theoretic sensor bias correction problem in SLAM using a simple unified framework and establish theoretically, the behavior and properties of the solution with special consideration to diminishing uncertainty, rates of convergence and observability.

Index Terms – localization, mapping, convergence

## I. INTRODUCTION

The simultaneous localization and mapping (SLAM) is still considered as one of the key challenges in achieving truly autonomous navigation capability for mobile platforms. The main goal of the SLAM problem is to investigate whether an autonomous vehicle starting from an unknown location in an unknown environment could build a map incrementally with the help of its sensors whilst simultaneously using that map to localize itself with reference to a global reference frame and navigate in real time. Application of SLAM for robot localization is quite important in several instances where global positioning systems (GPS) fail as a result of multipath effects of local terrain, periodic signal blockage by foliage, places having a restricted view of the sky, hilly terrain, partial satellite occlusion and active RF jamming. Demand for deploying SLAM is very high in number of outdoor applications, where autonomous exploration is to be carried out in completely unstructured and unexplored terrains having varying environmental conditions. Military reconnaissance, and surveillance, under water exploration, planetary exploration, mining, cargo handling and surveying are heading the list among several others. Self and Cheeseman [1], were the first to emphasize the importance of maintaining map vehicle correlations in SLAM and first to introduce the widely

used Extended Kalman Filter (EKF) based stochastic mapping framework. This formulation represents the robot pose, landmark or feature locations and their uncertainties in a consistent probabilistic framework. This was followed by the work of Mourtarlier and Chatila [3], Dissanayake et al. [2], work on visual navigation by Ayache and Faugeras [4] and more recent probabilistic and particle filter based methods [5], [6] and [7].

This work incorporates the feature based estimation theoretic EKF/stochastic mapping framework [3] for the purpose of formulating and studying the sensor and control bias estimation and compensation problem in detail. Nevertheless the framework has several shortcomings. It suffers from the inconsistent treatment of nonlinearities and is susceptible to data association errors at large. Further, the propagation of sensor uncertainties due to systematic and nonsystematic errors accentuate the problems and often results in inconsistent and inaccurate maps. As more and more features are added to the map, the existence of map vehicle correlations result in the spread of bias errors throughout the map causing divergence and instability in EKF. [2] Nonlinear transformations in the EKF also cause the Gaussian assumptions in the measurement and process models to be invalid, biased or underestimated. Persistent and slowly varying biases that exist due to modeling errors, sensor biases and imperfect calibrations also contribute to map divergence. In small scale SLAM implementations, it has been demonstrated that by adding more stabilizing process, non-linearity in state and measurement models can be offset. [1], [3] However, this will not always ensure consistent results in the presence of large sensor and control input biases, inevitably present in practical situations such as traversing large loops and revisiting (loop closing). [8] Thus, bias and modeling offsets [9], [10] and their cumulative effects cause significant problems in large-scale outdoor SLAM implementations. Furthermore, the bias errors impact tracking accuracy and data association significantly.

There are several bias estimation methods such as [11], [12] and [13] in the literature. However in this work, we have used the augmented state vector method [11] because of its simplicity and its similarity to the structure of SLAM's map augmented state vector. Since the number of bias terms is very small compared to the total number of state variables in SLAM, separate bias estimation methods such as [12] and [13]

are not used here as there is no significant increase of SLAM complexity by augmenting the SLAM state vector by comparatively few bias terms. The work presented here, analyses the properties of estimation theoretic bias estimation problem in SLAM using a class of linear process and observation models and verifies the established properties in a more general SLAM framework having all nonlinear process and observation models.

## II. SENSOR AND INPUT BIAS ESTIMATION

### A. EKF Based SLAM

The basic feature based EKF/SLAM framework [1] uses a map augmented state vector  $\mathbf{X}$ , consisting of concatenated 2D landmark position vectors,  $\mathbf{m}_k$  (known as the map) and the vehicle pose,  $\mathbf{x}_k$  at time  $k$  as follows.

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}_k^T & \mathbf{m}_k^T \end{bmatrix}^T \quad (1)$$

Here  $\mathbf{m}_k = [x_1^k \ y_1^k \ x_2^k \ y_2^k \dots \ x_n^k \ y_n^k]^T$ , the vehicle pose  $\mathbf{x}_k = [x_k \ y_k \ \theta_k]^T$  with  $x_k$ ,  $y_k$  and  $\theta_k$  denoting position coordinates and heading of the vehicle (middle of the rear axel) and  $[x_i^k \ y_i^k]^T$ ,  $i=1,2,\dots,n$  denoting the absolute feature position vectors with respect to a global coordinate frame. In general, the motion model of the vehicle is nonlinear and can be represented in closed form as;

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{v}(k) \quad (2)$$

Where,  $\mathbf{u}_{k-1} = [u_{k-1} \ \gamma_{k-1}]^T$  is the control input vector at time  $k-1$  comprising of, the speed input,  $u_{k-1}$  and the steering angle input,  $\gamma_{k-1}$  and  $\mathbf{v}(k) \sim N(0, \mathbf{Q}_v(k))$ . Assuming static landmarks, the map process model is:

$$\mathbf{m}_k = \mathbf{m}_{k-1} \quad (3)$$

Assuming a typical range bearing sensor such as SICK LMS 290 laser measurement system (LMS), the observation model is represented by

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{x}_k, \mathbf{m}_k) + \mathbf{w}(k) \quad (4)$$

Where,  $\mathbf{w}(k) \sim N(0, \mathbf{R}(k))$ . When the composite state vector covariance is  $\mathbf{P}(k|k)$  and the observation prediction is  $\hat{\mathbf{z}}(k|k-1)$  then the EKF predictor equations are as follows:

$$\mathbf{X}(k|k-1) = \begin{bmatrix} (\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}))^T & \mathbf{m}_{k-1}^T \end{bmatrix}^T \quad (5)$$

$$\mathbf{P}(k|k-1) = \mathbf{F}\mathbf{P}(k-1|k-1)\mathbf{F}^T + \mathbf{Q}(k) \quad (6)$$

$$\hat{\mathbf{z}}(k|k-1) = \mathbf{h}(\mathbf{x}_k, \mathbf{m}_k) \quad (7)$$

Where  $\mathbf{F}$  is the Jacobian ( $\partial\mathbf{f}/\partial\mathbf{x}_k$ ) of the process model and  $\mathbf{Q}(k)$  is the composite process noise covariance matrix. When true observations,  $\mathbf{z}(k)$  are available at time  $k$ , and after correct observation to feature associations are resolved using

an appropriate data association algorithm, the EKF update is done as usual using the following equations.

$$\mathbf{e}(k) = \mathbf{z}(k) - \hat{\mathbf{z}}(k|k-1) \quad (8)$$

$$\mathbf{X}(k|k) = \mathbf{X}(k|k-1) + \mathbf{K}(k)\mathbf{e}(k) \quad (9)$$

$$\mathbf{P}(k|k) = \mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k) \quad (10)$$

Where,  $\mathbf{e}(k)$  is the observation innovation,  $\mathbf{S}(k)$  is its covariance matrix and  $\mathbf{K}(k) = \mathbf{P}(k|k-1)\mathbf{H}^T\mathbf{S}^{-1}$  is the Kalman gain and  $\mathbf{H} = \partial\mathbf{h}/\partial\mathbf{X}(k|k-1)$  is the measurement Jacobian. Landmark track initiation, maintenance and deletion is carried out as in [2].

### B. Bias Estimation

Work by Clark [14] and Williams [11] are examples of earlier attempts to estimate vehicle parameters and input biases in the context of SLAM. In our work, a simplified rigorous framework is formulated for estimating control input and sensor biases present in SLAM algorithms in a consistent manner.

For simplicity, the derivation is given for a single sensor with range and bearing biases, together with control input biases, without loosing the generality of including many sensors. Let the lumped biases in the input vehicle velocity, steering angle and their covariance values be  $u_b(k)$ ,  $\gamma_b(k)$ ,  $\sigma_{ub}^2$  and  $\sigma_{\gamma b}^2$  respectively. Should these bias parameters be time varying it is straightforward to model their time varying characteristics in the given formulation. Suppose, the single sensor's (possibly a laser scanner) biases in the range, bearing and their covariance values be  $r_b(k)$ ,  $\alpha_b(k)$ ,  $\sigma_{rb}^2$  and  $\sigma_{\alpha b}^2$  respectively, a vector of biases,  $\mathbf{x}_b(k)$  is formed by concatenating all the biases as follows.

$$\mathbf{x}_b(k) = [u_b(k) \ \gamma_b(k) \ r_b(k) \ \alpha_b(k)]^T \quad (11)$$

The new composite state vector,  $\mathbf{X}$  including the bias vector is formed in the following manner.

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}_k^T & \mathbf{x}_b^T(k) & \mathbf{m}_k^T \end{bmatrix}^T \quad (12)$$

The process model of the map, (3) remains unchanged, while the new vehicle model, (13) and the bias model (14) take the following forms.

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, u_b(k-1), \gamma_b(k-1)) + \mathbf{v}(k) \quad (13)$$

$$\mathbf{x}_b(k) = \mathbf{x}_b(k-1) + \mathbf{v}_b(k) \quad (14)$$

Here,  $\mathbf{v}_b(k) \sim N(\mathbf{0}, \text{diag}(\sigma_{ub}^2, \sigma_{\gamma b}^2, \sigma_{rb}^2, \sigma_{\alpha b}^2))$ . The observation model is also modified to incorporate sensor biases as follows.

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{x}_k, \mathbf{m}_k, r_b(k), \alpha_b(k)) + \mathbf{w}(k) + \mathbf{v}_s(k) \quad (15)$$

Here,  $\mathbf{v}_s(k) \sim N(\mathbf{0}, \text{diag}(\sigma_{rb}^2, \sigma_{\alpha b}^2))$ . Now the estimation of vehicle pose, map and the bias parameters proceed in the same manner as described for without biases given by equations (1) to (10).

### III. PROPERTIES OF THE BIAS ESTIMATION PROBLEM IN SLAM

In this section we address three properties of the bias estimation in SLAM. They are the observability, convergence and stability. These issues are vital for a viable and robust estimation algorithm. A class of linear process and observation models [2] is used in the analysis.

#### A. Diminishing Uncertainty

In this work, we use linear and synchronous models as in [2] and [15] and assume constant bias vectors to investigate the uncertainty variation characteristics of sensor bias estimation in SLAM with following modifications. Linear vehicle model with input bias can be expressed as;

$$\mathbf{x}_k = \mathbf{F}_v \mathbf{x}_{k-1} + \mathbf{B}_v (\mathbf{u}_v(k) + \mathbf{u}_b(k)) + \mathbf{v}(k) \quad (16)$$

where,  $\mathbf{v}(k) \sim N(\mathbf{0}, \mathbf{Q}_v(k))$ ,  $\mathbf{B}_v$  is the transition matrix of the control vector and the vectors  $\mathbf{u}_v(k)$  and  $\mathbf{u}_b(k)$  denote the vehicle control input and the input bias vector. The process model of the  $i^{\text{th}}$  landmark vector  $\mathbf{L}_i(k) = [x_i^k \ y_i^k]^T$  is given by,

$$\mathbf{L}_i(k) = \mathbf{L}_i(k-1), \quad i=1,2,\dots,n \quad (17)$$

where  $n$  is the number of landmarks. When  $\mathbf{s}_b(k)$  is the bias vector of the sensor used to obtain landmark observations and assuming negligible noise injections into the bias vector, the evolution of bias vector is,

$$\mathbf{x}_b(k) = [\mathbf{u}_b^T(k) \ \mathbf{s}_b^T(k)]^T = \mathbf{x}_b(k-1) \quad (18)$$

Therefore the modified composite state vector is given by,

$$\mathbf{X}(k) = [\mathbf{x}_k^T \ \mathbf{u}_b^T(k) \ \mathbf{s}_b^T(k) \ \mathbf{L}_1^T(k) \ \dots \ \mathbf{L}_n^T(k)]^T \quad (19)$$

Linear observation model, when observing the  $i^{\text{th}}$  landmark:

$$\mathbf{z}(k) = \mathbf{H}_{\mathbf{L}_i}(k) \mathbf{L}_i(k) - \mathbf{H}_v(k) \mathbf{x}_k + \mathbf{s}_b(k) + \mathbf{w}(k) \quad (20)$$

Where  $\mathbf{H}_{\mathbf{L}_i}(k)$  and  $\mathbf{H}_v(k)$  are the parts of the observation model relating to the  $i^{\text{th}}$  landmark being observed and, with respect to the vehicle pose vector, and  $\mathbf{w}(k) \sim N(\mathbf{0}, \mathbf{R}(k))$ . Let  $\mathbf{H}(k)$  be the total Jacobian corresponding to the augmented state vector  $\mathbf{X}(k)$ , then the measurement prediction is given by;

$$\hat{\mathbf{z}}(k | k-1) = \mathbf{H}(k) \mathbf{X}(k | k-1) \quad (21)$$

Then the consequent prediction and update equations take the form of the equations (5) to (10). Now the composite covariance matrix,  $\mathbf{P}(k | k)$ , is denoted by (22), where subscripts m, v, u and s denote the entire map, the vehicle, the input bias vector and the sensor bias vector respectively. In (6) and (22), the process model  $\mathbf{F}$  and matrix partitions  $\mathbf{F}_b$  of  $\mathbf{F}$ ,  $\mathbf{P}_{bb}(k | k)$  of  $\mathbf{P}(k | k)$  and  $\mathbf{P}_{bm}(k | k)$  of  $\mathbf{P}(k | k)$  can be defined as (23), (24), (25) and (26).

$$\mathbf{P}(k | k) = \begin{bmatrix} \mathbf{P}_{vv}(k | k) & \mathbf{P}_{vu}(k | k) & \mathbf{P}_{vs}(k | k) & \mathbf{P}_{vm}(k | k) \\ \mathbf{P}_{vu}^T(k | k) & \mathbf{P}_{uu}(k | k) & \mathbf{P}_{us}(k | k) & \mathbf{P}_{um}(k | k) \\ \mathbf{P}_{vs}^T(k | k) & \mathbf{P}_{us}^T(k | k) & \mathbf{P}_{ss}(k | k) & \mathbf{P}_{sm}(k | k) \\ \mathbf{P}_{vm}^T(k | k) & \mathbf{P}_{um}^T(k | k) & \mathbf{P}_{sm}^T(k | k) & \mathbf{P}_{mm}(k | k) \end{bmatrix} \quad (22)$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_b & \mathbf{0}_{bm} \\ \mathbf{0}_{bm}^T & \mathbf{I}_{mm} \end{bmatrix} \quad (23)$$

$$\mathbf{F}_b = \begin{bmatrix} \mathbf{F}_v & \mathbf{B}_v & \mathbf{0}_{vs} \\ \mathbf{0}_{vu}^T & \mathbf{I}_{uu} & \mathbf{0}_{us} \\ \mathbf{0}_{vs}^T & \mathbf{0}_{us}^T & \mathbf{I}_{ss} \end{bmatrix} \quad (24)$$

$$\mathbf{P}_{bb}(k | k) = \begin{bmatrix} \mathbf{P}_{vv}(k | k) & \mathbf{P}_{vu}(k | k) & \mathbf{P}_{vs}(k | k) \\ \mathbf{P}_{vu}^T(k | k) & \mathbf{P}_{uu}(k | k) & \mathbf{P}_{us}(k | k) \\ \mathbf{P}_{vs}^T(k | k) & \mathbf{P}_{us}^T(k | k) & \mathbf{P}_{ss}(k | k) \end{bmatrix} \quad (25)$$

$$\mathbf{P}_{bm}(k | k) = [\mathbf{P}_{vm}^T(k | k) \ \mathbf{P}_{um}^T(k | k) \ \mathbf{P}_{sm}^T(k | k)]^T \quad (26)$$

Where  $\mathbf{0}_{bm}$  is a null matrix having number of rows equal to that of  $\mathbf{F}_b$  and number of columns equal to the dimension of the map vector. The matrices,  $\mathbf{I}_{uu}$ ,  $\mathbf{I}_{ss}$  and  $\mathbf{I}_{mm}$  are identity matrices of dimension  $(\dim(\mathbf{u}_b))^2$ ,  $(\dim(\mathbf{s}_b))^2$  and  $(\dim(\mathbf{m}))^2$  respectively.  $\mathbf{0}_{vs}$ ,  $\mathbf{0}_{vu}$  and  $\mathbf{0}_{us}$  are null matrices having appropriate dimensions. From (6), prediction of the composite state vector covariance can be determined as;

$$\begin{aligned} & \mathbf{P}(k | k-1) \\ &= \begin{bmatrix} \mathbf{F}_b \mathbf{P}_{bb}(k-1 | k-1) \mathbf{F}_b^T + \mathbf{Q}(k-1) & \mathbf{F}_b \mathbf{P}_{bm}(k-1 | k-1) \\ \mathbf{P}_{bm}^T(k-1 | k-1) \mathbf{F}_b^T & \mathbf{P}_{mm}(k-1 | k-1) \end{bmatrix} \end{aligned} \quad (27)$$

where,  $\mathbf{Q}(k-1)$  represents the covariance of the noise injections from the vehicle motion and bias terms. Expand the first sub matrix partition of (27) as follows;

$$\mathbf{F}_b \mathbf{P}_{bb}(k-1 | k-1) \mathbf{F}_b^T + \mathbf{Q}(k-1) = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_4 & \mathbf{P}_5 \\ \mathbf{P}_4^T & \mathbf{P}_2 & \mathbf{P}_6 \\ \mathbf{P}_5^T & \mathbf{P}_6^T & \mathbf{P}_3 \end{bmatrix} \quad (28)$$

where,

$$\mathbf{P}_1 = \mathbf{F}_v \mathbf{P}_{vv}(k-1 | k-1) \mathbf{F}_v^T + \mathbf{F}_v \mathbf{P}_{vu}(k-1 | k-1) \mathbf{B}_v^T + \mathbf{B}_v \mathbf{P}_{vu}(k-1 | k-1) + \mathbf{B}_v \mathbf{P}_{vu}(k-1 | k-1) \mathbf{B}_v^T + \mathbf{F}_v \mathbf{P}_{vs}(k-1 | k-1) \mathbf{F}_v^T + \mathbf{Q}_v(k) \quad (29)$$

$$\mathbf{P}_2 = \mathbf{P}_{uu}(k-1 | k-1) \quad (30)$$

$$\mathbf{P}_3 = \mathbf{P}_{ss}(k-1 | k-1) \quad (31)$$

$$\mathbf{P}_4 = \mathbf{F}_v \mathbf{P}_{vu}(k-1 | k-1) + \mathbf{B}_v \mathbf{P}_{uu}(k-1 | k-1) \quad (32)$$

$$\mathbf{P}_5 = \mathbf{F}_v \mathbf{P}_{vs}(k-1 | k-1) + \mathbf{B}_v \mathbf{P}_{us}(k-1 | k-1) \quad (33)$$

$$\mathbf{P}_6 = \mathbf{P}_{us}(k-1 | k-1) \quad (34)$$

As  $\mathbf{P}(0|0)$ ,  $\mathbf{Q}(k)$  and  $\mathbf{R}(k)$  are positive semi definite (PSD) matrices, by the properties of PSD matrices [16],  $\mathbf{S}(k)$  and  $\mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k)$  are also PSD matrices. Hence from (10);

$$\det(\mathbf{P}(k|k)) = \det(\mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k)) \quad (35)$$

$$\det(\mathbf{P}(k|k)) \leq \det(\mathbf{P}(k|k-1)) \quad (36)$$

Since any principal sub matrix of a PSD matrix is also a PSD;

$$\det(\mathbf{P}_{uu}(k|k)) \leq \det(\mathbf{P}_{uu}(k|k-1)) \quad (37)$$

$$\det(\mathbf{P}_{ss}(k|k)) \leq \det(\mathbf{P}_{ss}(k|k-1)) \quad (38)$$

But from (28) to (33)  $\mathbf{P}_{uu}(k|k-1) = \mathbf{P}_{uu}(k-1|k-1)$  and  $\mathbf{P}_{ss}(k|k-1) = \mathbf{P}_{ss}(k-1|k-1)$ . Thus from (37) and (38);

$$\det(\mathbf{P}_{uu}(k|k)) \leq \det(\mathbf{P}_{uu}(k-1|k-1)) \quad (39)$$

$$\det(\mathbf{P}_{ss}(k|k)) \leq \det(\mathbf{P}_{ss}(k-1|k-1)) \quad (40)$$

Since the determinant of a matrix is proportional to its volume, determinant of covariance matrices indicate the volume or size of their uncertainty ellipsoids and therefore it can be concluded that the uncertainty of bias parameters cannot increase in an update of the SLAM algorithm.

### B. Rates of Convergence

The rate of convergence of uncertainties associated with the bias estimation in SLAM is studied by using the one dimensional robot (monobot) SLAM algorithm. [17], [18] Here the one landmark continuous SLAM is considered for the analysis as follows.

$$\begin{bmatrix} \dot{x}(t) \\ \dot{u}_b(t) \\ \dot{s}_b(t) \\ \dot{L}_1(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ u_b(t) \\ s_b(t) \\ L_1(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} [u(t) + v(t)] \quad (41)$$

$x(t)$ ,  $u_b(t)$ ,  $s_b(t)$ ,  $L_1(t)$  and  $u(t)$  denote the position of the monobot, bias of the speed sensor, bias of the range sensor, landmark position, and speed input in the continuous space at time  $t$  and  $v(t) \sim N(0, q^2)$ . The state vector of 1D SLAM is given by  $\mathbf{x}(t) = [x(t) \ u_b(t) \ s_b(t) \ L_1(t)]^T$  and the observation model is given by

$$\mathbf{z}(t) = \mathbf{H}\mathbf{x}(t) + w(t) \quad (42)$$

where  $w(t) \sim N(0, r^2)$ ,  $\mathbf{R} = r^2$  and  $\mathbf{H} = [-1 \ 0 \ 1 \ 1]$  Let

$$\mathbf{G} = [1 \ 0 \ 0 \ 0]^T, \quad \mathbf{F} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{P}(0) = \text{diag}(0, \sigma_{u_b}^2, \sigma_{s_b}^2, r^2),$$

where  $\mathbf{P}(0)$  is the initial covariance matrix of the model (41) and  $\mathbf{I}$  is an identity matrix of size 4, the covariance matrix,  $\mathbf{P}(t)$  of the state vector in (41) is governed by the Riccati equation;

$$\dot{\mathbf{P}}(t) = \mathbf{F}\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}^T + \mathbf{G}\mathbf{Q}\mathbf{G}^T - \mathbf{P}(t)\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{P}(t) \quad (43)$$

The solution to this Riccati equation [19] is of the form  $\mathbf{P}(t) = \mathbf{M}(t)\mathbf{N}^{-1}(t)$ , where  $\mathbf{M}(t)$  and  $\mathbf{N}(t)$  are given by the following equations,

$$\begin{bmatrix} \dot{\mathbf{M}}(t) \\ \dot{\mathbf{N}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{F} & \mathbf{G}\mathbf{Q}\mathbf{G}^T \\ \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} & -\mathbf{F}^T \end{bmatrix} \begin{bmatrix} \mathbf{M}(t) \\ \mathbf{N}(t) \end{bmatrix} \quad (44)$$

$$\begin{bmatrix} \mathbf{M}(0) \\ \mathbf{N}(0) \end{bmatrix} = \begin{bmatrix} \mathbf{P}(0) \\ \mathbf{I} \end{bmatrix} \quad (45)$$

Then the characteristic equation  $\mathbf{C}(t)$  of the system and the  $m^{\text{th}}$  row  $n^{\text{th}}$  column element  $\mathbf{P}(m, n)$  of  $\mathbf{P}(t)$  for all  $m$  and  $n$  can be obtained by solving (44) and (45) as follows.

$$\begin{aligned} \mathbf{C}(t) = & r^2 q^2 (t\sigma_{bu}^2 + q^2)((\tau+1) + (\tau-1)e^{-2\tau t}) \\ & - 2\sigma_{bu}^2 r q (\sigma_{bs}^2 + r^2)(1 - e^{-\tau t})^2 + \\ & (1 - e^{-2\tau t})(tq^2\sigma_{bu}^2\sigma_{bs}^2 + q^4\sigma_{bs}^2 - \sigma_{bu}^2 r^2 q^2) \end{aligned} \quad (46)$$

$$\mathbf{P}(1, 1) = (1/\mathbf{C}(t)) q^3 r (t\sigma_{bu}^2 + q^2) (2(\sigma_{bs}^2 + r^2)(1 - e^{-\tau t})^2 + r q (1 - e^{-2\tau t})) \quad (47)$$

$$\mathbf{P}(1, 2) = (1/\mathbf{C}(t)) \sigma_{bu}^2 q^3 r (2(\sigma_{bs}^2 + r^2)(1 - e^{-\tau t})^2 + r q (1 - e^{-2\tau t})) \quad (48)$$

$$\mathbf{P}(1, 3) = (1/\mathbf{C}(t)) \sigma_{bs}^2 q^3 r (t\sigma_{bu}^2 + q^2)(1 - e^{-\tau t})^2 \quad (49)$$

$$\mathbf{P}(1, 4) = (1/\mathbf{C}(t)) q^3 r^3 (t\sigma_{bu}^2 + q^2)(1 - e^{-\tau t})^2 \quad (50)$$

$$\mathbf{P}(2, 2) = (1/\mathbf{C}(t)) \sigma_{bu}^2 r^2 q^4 ((\sigma_{bs}^2 + r^2)(1 - e^{-\tau t})^2 + r q (1 - e^{-2\tau t})) \quad (51)$$

$$\mathbf{P}(2, 3) = (1/\mathbf{C}(t)) q^3 r \sigma_{bu}^2 \sigma_{bs}^2 (1 - e^{-\tau t})^2 \quad (52)$$

$$\mathbf{P}2, 4) = (1/\mathbf{C}(t)) q^3 r^3 \sigma_{bu}^2 (1 - e^{-\tau t})^2 \quad (53)$$

$$\begin{aligned} \mathbf{P}(3, 3) = & (1/\mathbf{C}(t)) \sigma_{bs}^2 \{r^2 q^2 (t\sigma_{bu}^2 + q^2 - \sigma_{bu}^2)(1 - e^{-2\tau t}) - \\ & 2\sigma_{bu}^2 r^3 q (1 - e^{-\tau t})^2 + r q^3 (t\sigma_{bu}^2 + q^2)(1 + e^{-2\tau t})\} \end{aligned} \quad (54)$$

$$\begin{aligned} \mathbf{P}(3, 4) = & (1/\mathbf{C}(t)) \sigma_{bs}^2 r q \{2\sigma_{bu}^2 r^2 (1 - e^{-\tau t})^2 \\ & - r q (t\sigma_{bu}^2 + q^2)(1 - e^{-2\tau t})\} \end{aligned} \quad (55)$$

$$\begin{aligned} \mathbf{P}(4, 4) = & (1/\mathbf{C}(t)) r^2 \{q^2 (t\sigma_{bu}^2 \sigma_{bs}^2 + q^2 \sigma_{bs}^2 - r^2 \sigma_{bu}^2)(1 - e^{-2\tau t}) - \\ & 2\sigma_{bu}^2 \sigma_{bs}^2 r q (1 - e^{-\tau t})^2 + r q^3 (t\sigma_{bu}^2 + q^2)(1 + e^{-2\tau t})\} \end{aligned} \quad (56)$$

Here,  $\tau = q/r$  and  $1/\tau$  is the time constant of the model. All other terms of  $\mathbf{P}(t)$  can be derived from (46) to (56) using the symmetry of  $\mathbf{P}(t)$ . The above equations show that the covariance terms of  $\mathbf{x}(t)$  converge exponentially initially at a time constant of  $1/\tau$  and then asymptotically to the steady state covariance according to the expressions, (46) to (56). The above relatively slow asymptotic convergence of the estimated uncertainty terms in the SLAM bias estimator is one of the reasons for the poor performance of the estimation algorithm. This convergence rate is not affected by the biases or their uncertainties and depends only on the information available to the filter as in standard EKF SLAM. [17]. The steady state covariance of the  $\mathbf{x}(t)$ ,  $\mathbf{P}(\infty)$  can be obtained

from (46) to (56) by applying the limit as  $t \rightarrow \infty$ .

$$\mathbf{P}^{(\infty)} = \begin{pmatrix} \frac{1}{\sigma^*} \\ \sigma^* \end{pmatrix} \begin{bmatrix} qr(2(\sigma_{bs}^2 + r^2) + rq) & 0 & \sigma_{bs}^2 qr & qr^3 \\ 0 & 0 & 0 & 0 \\ \sigma_{bs}^2 qr & 0 & \sigma_{bs}^2 (r^2 + rq) & -\sigma_{bs}^2 r^2 \\ qr^3 & 0 & -\sigma_{bs}^2 r^2 & r^2(\sigma_{bs}^2 + rq) \end{bmatrix} \quad (57)$$

Where  $\sigma^* = r^2(\tau + 1) + \sigma_{bs}^2$ . Thus we can conclude (1). The rate of convergence of state uncertainties does not depend on the bias parameters or their initial uncertainty. But this depends only on the amount of process noise and the amount of information available to the estimator, as in standard 1D SLAM. (2) In the limit, the bias associated with the vehicle speed is perfectly known. (3) In the limit, the state uncertainties are affected only by the initial sensor bias uncertainty. They do not depend on the initial vehicle speed input bias uncertainty.

### C. Observability of the Estimator

In full SLAM, because of the growing correlations among the constituent states and as a result of partial observability [18], the full reconstruction of the map state vector is not possible with typical measurement models. The SLAM with bias estimation also suffers from the above and also shows marginal stability as a result of the filter's steady state dependence on initial bias uncertainty. We also analyze the observability of 1D SLAM for the verification. The angle  $\psi$  between the controllable and observable subspaces is an indication of the closeness of observable states from fully revising the process state. This angle is also a measure of partial observability of the problem. For this analysis we use the discrete versions of the  $n$  landmark SLAM process and observation models given in (41) and (42). Let  $\mathbf{O}$  and  $\mathbf{C}$  denote the observability and controllability matrices of the above system. Let  $\text{Im}(\mathbf{X})$  denote a basis of the image of the space specified by  $\mathbf{X}$ .

$$\text{Im}(\mathbf{C}) = \begin{bmatrix} 1 \\ \mathbf{0}_{(n+3) \times 1} \end{bmatrix} \quad (58)$$

$$\text{Im}(\mathbf{O}^T) = \begin{bmatrix} \mathbf{1}_{1 \times n} & 0 \\ \mathbf{0}_{1 \times n} & 1 \\ -\mathbf{1}_{1 \times n} & 0 \\ -\mathbf{1}_{n \times n} & \mathbf{0}_{n \times 1} \end{bmatrix} \quad (59)$$

Thus using the vector transformations and projections we can derive the angle between these subspaces,  $\psi$  as;

$$\psi = \cos^{-1} \left( \sqrt{\frac{n}{2n+1}} \right) \quad (60)$$

Since  $\psi$  is a monotonically decreasing function of  $n$ , as the number of landmarks increases, the observable subspace gets closer to the controllable part of the state space. However  $\psi$

cannot be reduced below  $\pi/4$ . Thus it is important to propose methods of providing full observability to the bias estimation problem in SLAM.

In the 2D nonlinear SLAM, can be verified by symbolic manipulation of models (11) to (15) and the observability Grammian, that the SLAM with sensor bias estimation becomes fully observable when observing more than one 2D known landmark and observing the landmarks which are in the state vector at the same time. Thus it is important to note that in all the simulations carried out in section IV, the full observability of the problem is preserved by always observing two known 2D landmarks.

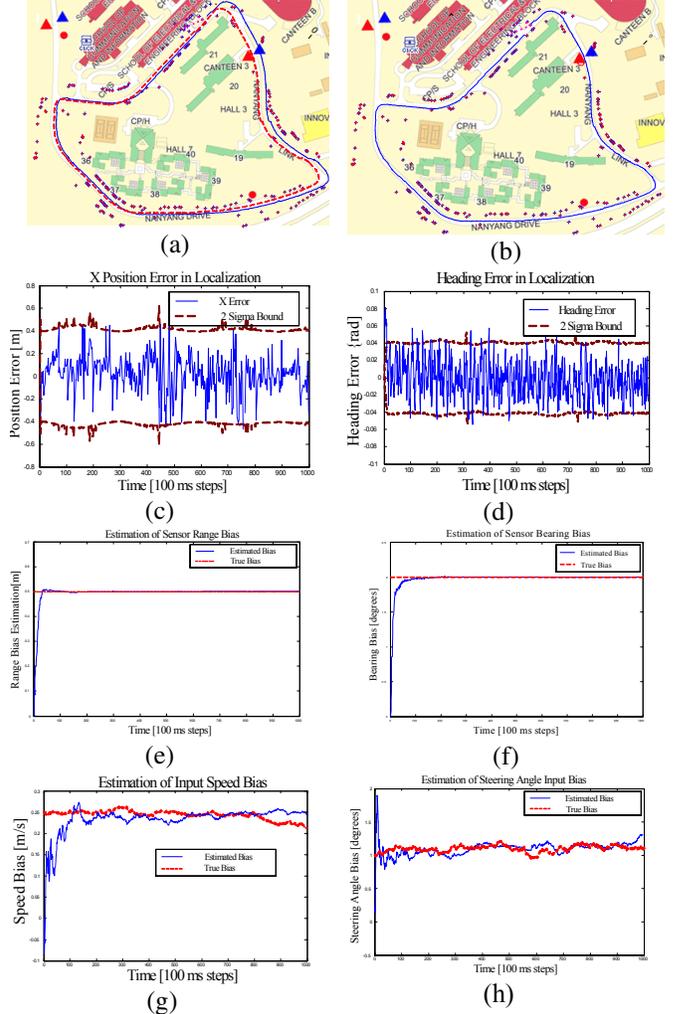


Figure 1 – Figure 1 (a) shows the map divergence in the case of large scale SLAM in loop closing experiments. Dashed line marks the true path robot should travel and the thick line is the estimated robot trajectory without bias compensation. Figure 1 (b) represents the same experiment in loop closing with on line bias compensation. Figures 1 (c) and 1(d) show the 2D SLAM, localization error in x and heading. Figures 1(e)-(h) show sensor bias estimates. Dashed lines in Figures 1(c) and (d) show 2 sigma bounds. Thick lines Figures 1(e)-(h) show the estimated biases and dashed lines show the true biases.



Fig 2 – The mobile robot platform

#### IV. SIMULATIONS AND EXPERIMENTS

Fig 1 shows SLAM simulation and experimental results indicating SLAM with bias estimation. Results of Fig 1 (a) and Fig 1 (b) indicates the outdoor SLAM experiments done in a campus neighborhood (over 1.1km stretch) are obtained from experiments conducted at a campus neighborhood with the help of GENOME (Generic Outdoor Mobile Explorer), a car like mobile robot. (Fig. 2.-(a)) equipped with laser measurement system (SICK LMS 290) and encoders. The results show the improvements in bias compensation in SLAM when using biased sensors. Fig 1 (c), (d), (e), (f), (g) and (h) show simulations done in a similar setting with a robot navigating in a circular path of 20m radius. Simulations are done for 2D SLAM with biased sensors (LMS range bias 0.5m, angle bias 2 degrees, speed input bias 0.25m/s and steering angle bias 1 degree and the random walks in the above biases are taken as  $0, 0, 0.2 \text{ m}/\sqrt{hr}, 2^0/\sqrt{hr}$ )

#### V. CONCLUSIONS AND FUTURE WORK

A detailed theoretical study of the estimation theoretic sensor bias correction problem in SLAM is carried out and the properties of the solution established for the first time. It is established that the sensor bias estimation problem in SLAM in fact has a solution, which converges with diminishing uncertainty over the time. Conditions for convergence and observability problem are also discussed. Conditions that affect the estimation problem are also verified with simulations and experimental results. The nonlinearities in the process and observation models, asynchronous nature of observations and the violation of flat ground assumption, result in the non-smooth variation of bias error characteristics. In general, detailed study of the bias correction problem highlights the importance of bias correction for successful implementation of large scale SLAM involving large loops using low cost, poor quality sensors. The above preliminary studies are a prelude to the study of the much complex problem of SLAM using low cost inertial sensors. This study, which is of paramount importance in various large scale outdoor robotics applications including servicing, agricultural and rescue robots, is pursued as future work based on the same theoretical principals developed.

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