Finite-Set Statistics and SLAM

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Purpose

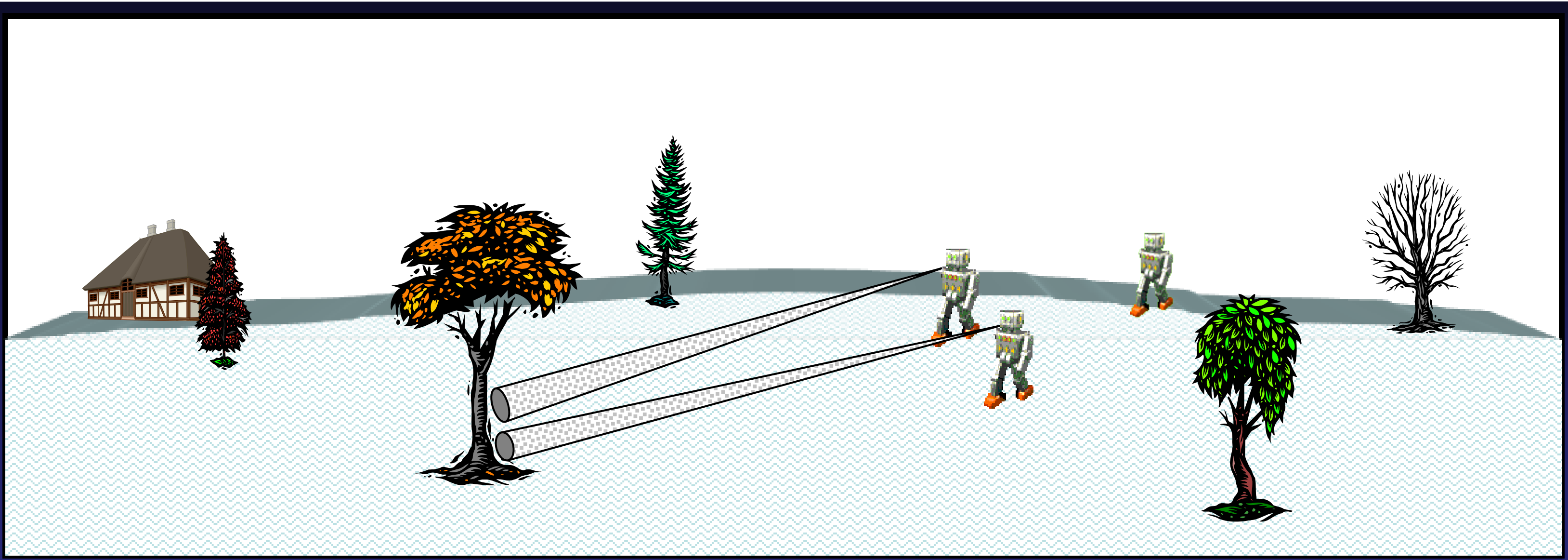
Describe the elements of a new, practical, Bayes-optimal, and theoretically unified foundation for multisensor-multitarget problems: "Finite-Set Statistics" (FISST).

Finite-set statistics is the basis for a fundamentally new, Bayes-optimal, and theoretically unified approach to SLAM and related robotics problems that is the focus of this workshop:

- Mullane, Vo, Adams, Vo: "A random-finite-set approach to Bayesian SLAM, *IEEE T-Robotics*, (27)2: 268-282, 2011.
- Mullane, Vo, Adams, Vo: *Random Finite Sets in Robotic Map Building and SLAM*, Springer, 2011.

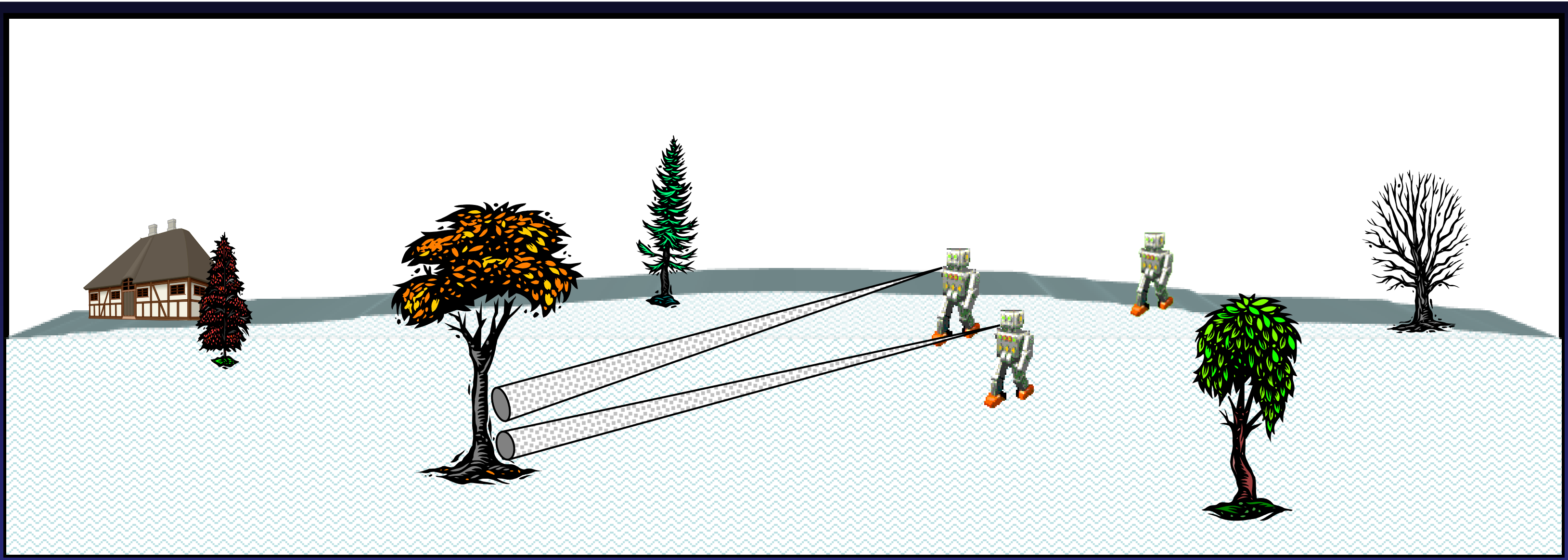
My purpose here is contextual: *to provide an overview of FISST and to explain its pertinence for SLAM and similar applications*

Simultaneous Localization and Mapping (SLAM)



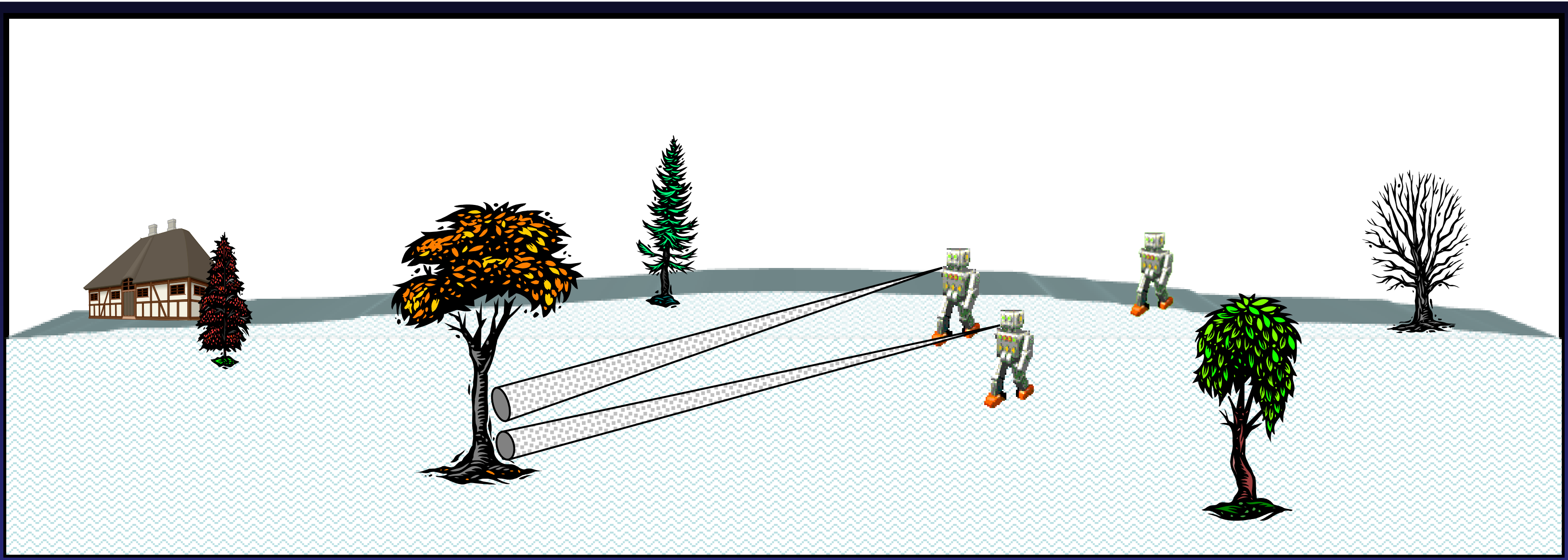
- **Multiple moving robots explore an unfamiliar environment without access to GPS or *a priori* map (terrain, architectural) information**
- **Without human intervention and by employing only their oboard sensors, the robots must detect and localize uknown stationary landmarks (“features”)**
- **From these landmarks they must construct, *on-the-fly*, a local map of the environment**
- **Then they must situate themselves within this map—along with any unknown, moving, and possibly noncooperative targets**

Important Points to Consider



- **The landmarks will be unknown, and of unknown, varying number**
- **The robots will be unknown and of unknown, varying number**
- **The sensor measurements—whether generated by robots, targets, landmarks, or clutter—will be varying and of varying number**
- **There is generally no *a priori* way to order the robots, the landmarks, the targets, or the measurements**

The Theoretical Challenge



- Vector representations of SLAM scenarios are problematic
- How can we measure the degree of deviation between between the actual map and a SLAM algorithm's estimate of it (which will differ not only in estimates of individual landmarks, but in their number)?
- How can we claim that the algorithm's estimate is "optimal" in a Bayesian sense?

The Approach: Finite-Set Statistics

- Formulate SLAM problems in terms of random finite set (RFS) theory
- Generalize “Statistics 101” concepts to multitarget realm: multitarget probability laws, multitarget integro-differential calculus
- From formal statistical models of sensors & targets, create RFS multisensor-multitarget measurement models
- From formal statistical models of target motions (including appearance & disappearance) create RFS multitarget motion models
- From the RFS motion & measurement models, construct “true” multitarget Markov densities and likelihood functions
- From the Markov density & likelihood function, construct an optimal solution: a multisensor-multitarget Bayes recursive filter
- Construct principled approximations of the optimal filter—e.g., PHD filter, CPHD filter, multi-Bernoulli filter, etc.

Topics

- Overview
- Single-sensor, single-target Bayes filter
- RFS multi-object calculus
- RFS modeling of multisensor-multitarget systems
- Multisensor-multitarget recursive Bayes filter
- Approximate multitarget Bayes filters
- Conclusions

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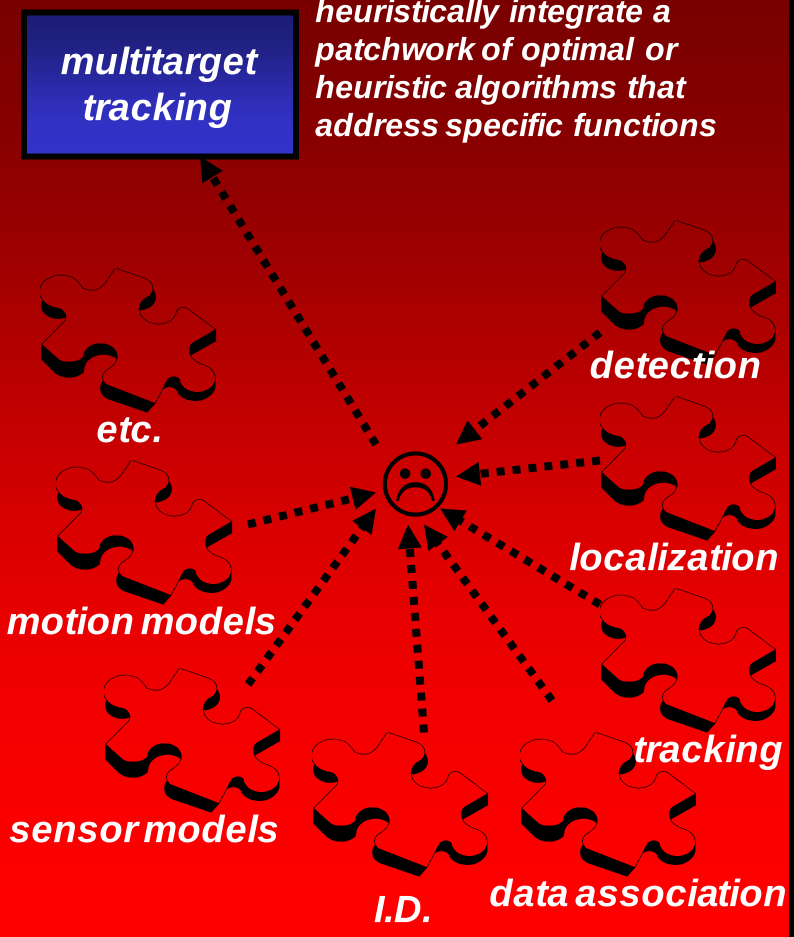
Top-Down versus Bottom-Up Multitarget Data Fusion

usual "bottom-up" approach to multitarget information fusion



multitarget tracking

heuristically integrate a patchwork of optimal or heuristic algorithms that address specific functions

☹

detection

etc.

localization

motion models

tracking

sensor models

I.D.

data association

"top-down," system-level approach

multiple sensors, multiple targets = single system

↓

specify statistical multitarget measurement & motion models

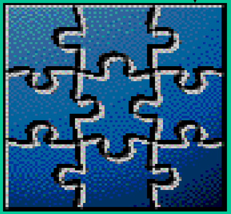
↓

Bayesian-probabilistic formulation of problem

↓ *requires random set theory*

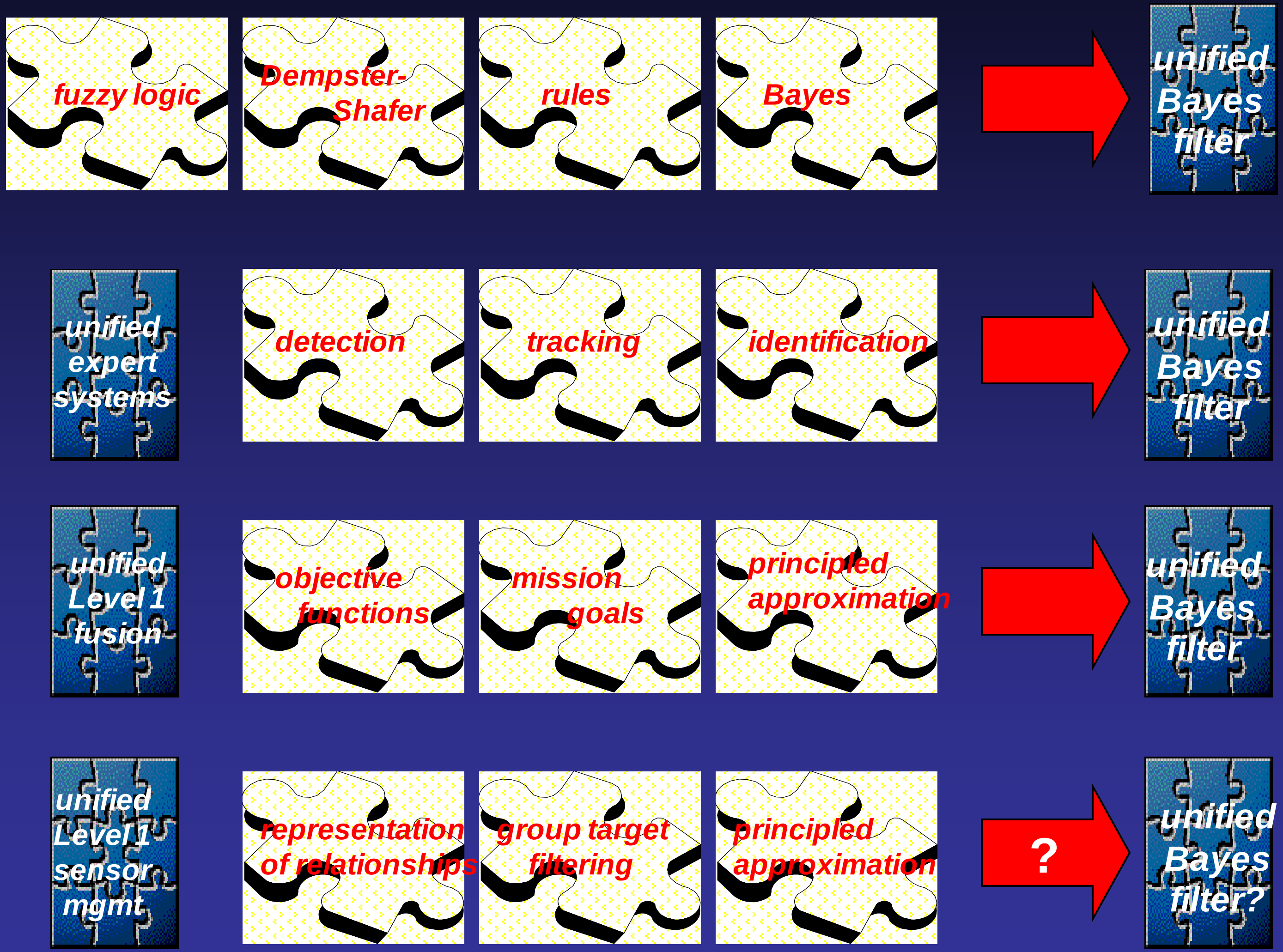
multisensor-multitarget recursive Bayes filter

↓ *models & functions subsumed in optimally integrated algorithm*



need a calculus for computational tractability!

The FISST Research Program



Advance 1:
unification of expert systems theory

fuzzy logic
Dempster-Shafer
rules
Bayes
unified Bayes filter

Advance 2:
unification of Level 1 fusion

unified expert systems
detection
tracking
identification
unified Bayes filter

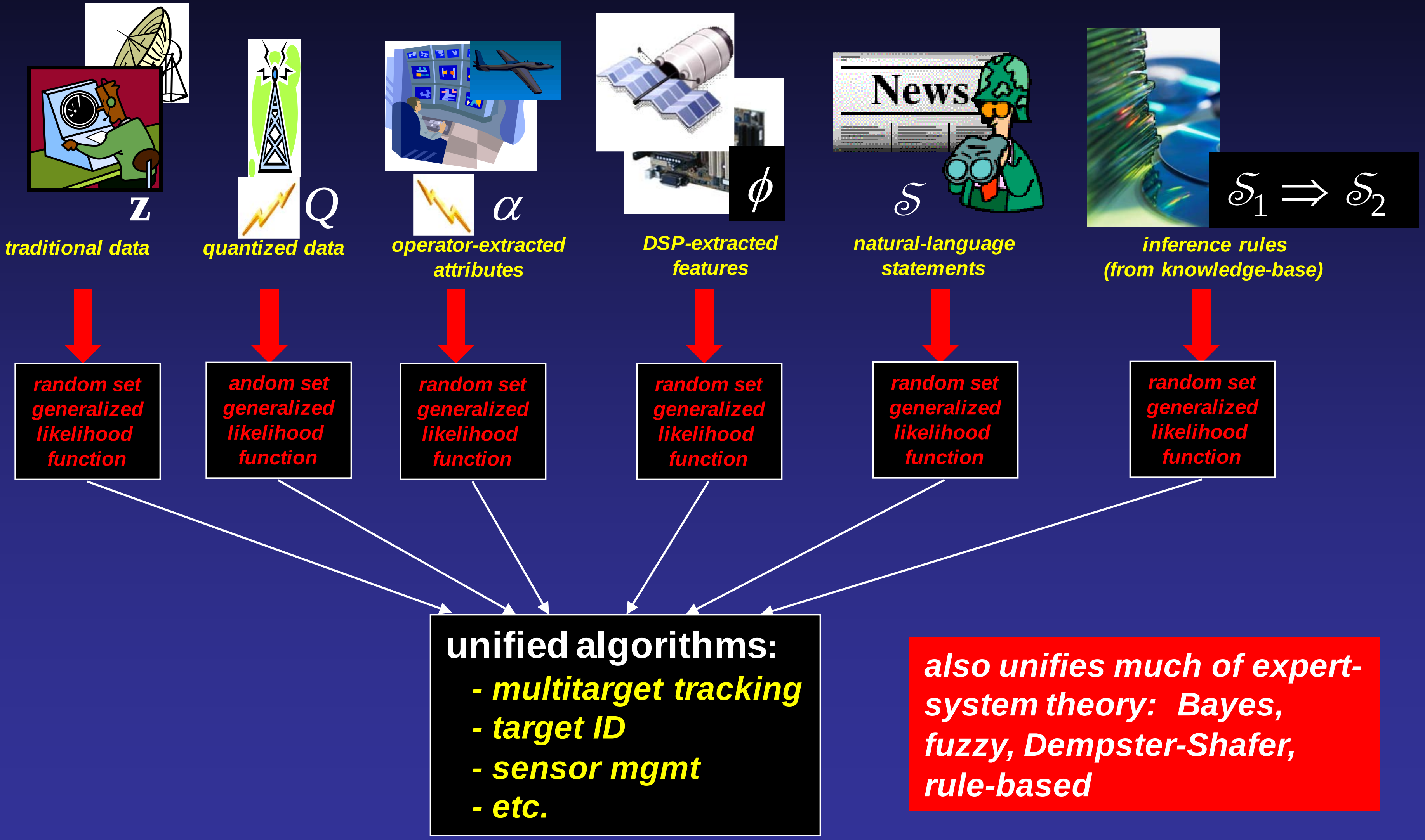
Advance 3:
unification of Level 1 sensor mgmt

unified Level 1 fusion
objective functions
mission goals
principled approximation
unified Bayes filter

Advance 4?
beginnings of a foundation for Levels 2/3?

unified Level 1 sensor mgmt
representation of relationships
group target filtering
principled approximation
?
unified Bayes filter?

Unified Information Fusion



$\mathbf{z}$
traditional data

Q
quantized data

α
operator-extracted attributes

ϕ
DSP-extracted features

$\mathcal{S}$
natural-language statements

$\mathcal{S}_1 \Rightarrow \mathcal{S}_2$
inference rules (from knowledge-base)

random set generalized likelihood function

andom set generalized likelihood function

random set generalized likelihood function

random set generalized likelihood function

random set generalized likelihood function

random set generalized likelihood function

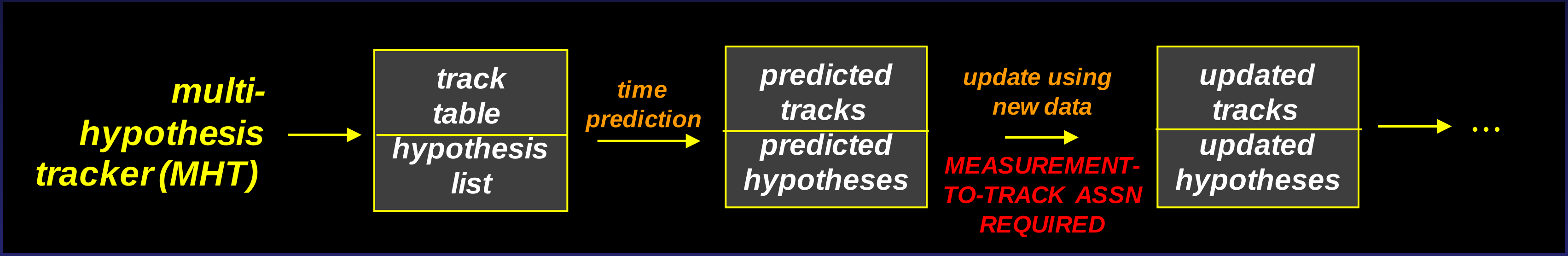
unified algorithms:

- *multitarget tracking*
- *target ID*
- *sensor mgmt*
- *etc.*

also unifies much of expert-system theory: Bayes, fuzzy, Dempster-Shafer, rule-based

Approximate Multitarget Filters: MHT, PHD, CPHD

given time-sequence of measurement-sets: $Z^{(k)} : Z_1, \ldots, Z_k$



multi-hypothesis tracker (MHT) → track table / hypothesis list —*time prediction*→ predicted tracks / predicted hypotheses —*update using new data*→ updated tracks / updated hypotheses → …

MEASUREMENT-TO-TRACK ASSN REQUIRED

PHD filter

(introduced 2000)

filter on probability hypothesis densities (PHDs)

$$\ldots \to D_{k|k}(\mathbf{x}|Z^{(k)}) \to D_{k+1|k}(\mathbf{x}|Z^{(k)}) \to D_{k+1|k+1}(\mathbf{x}|Z^{(k+1)}) \to \ldots$$

no measurement-to-track association required

CPHD filter

(introduced 2006)

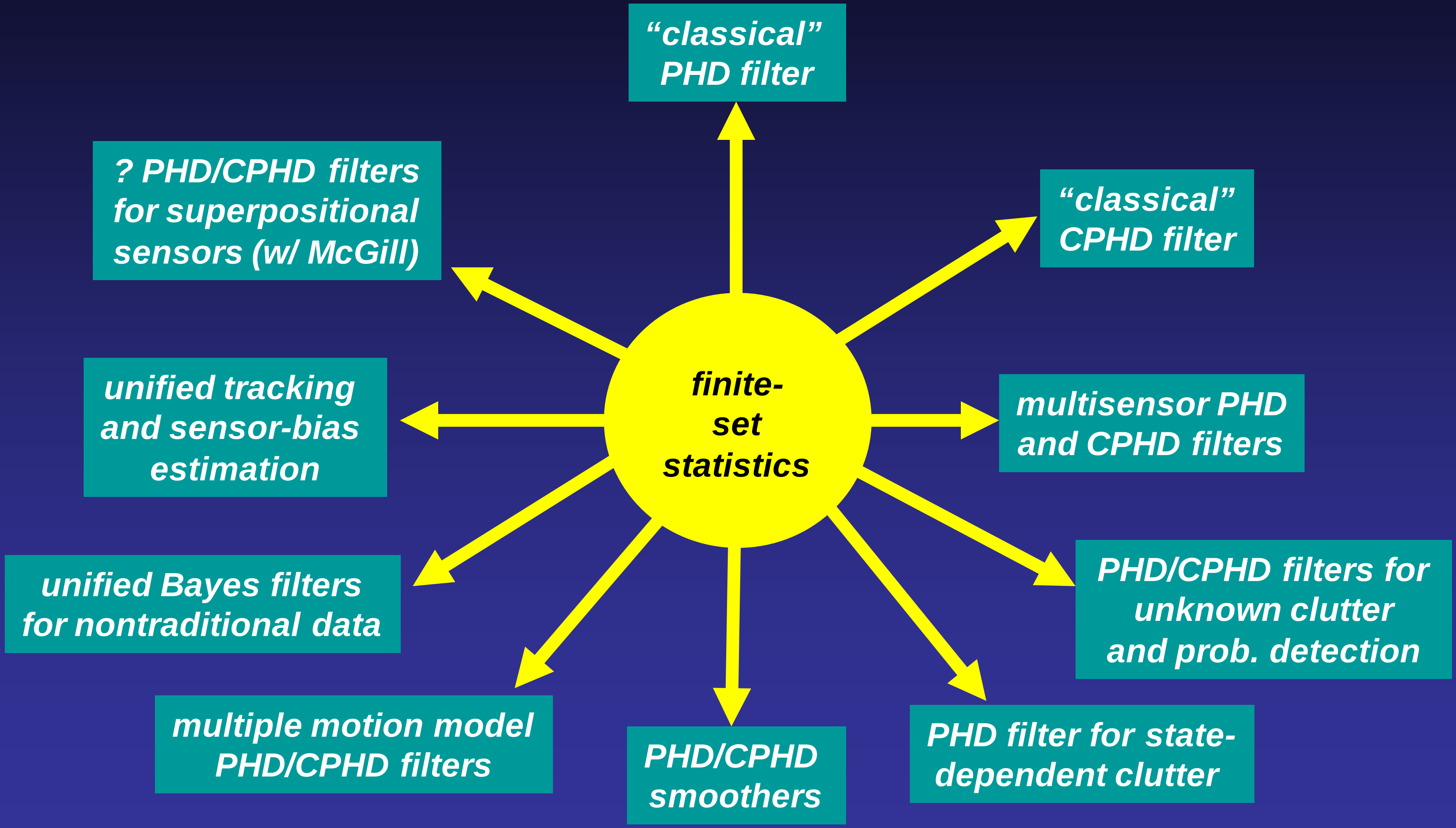
filter on PHDs

$$\begin{tikzcd}
\ldots \arrow[r] & D_{k|k}(\mathbf{x}|Z^{(k)}) \arrow[r] & D_{k+1|k}(\mathbf{x}|Z^{(k)}) \arrow[r] \arrow[dr] & D_{k+1|k+1}(\mathbf{x}|Z^{(k+1)}) \arrow[r] & \ldots \\
\ldots \arrow[r] & p_{k|k}(n|Z^{(k)}) \arrow[r] & p_{k+1|k}(n|Z^{(k)}) \arrow[r] \arrow[ur] & p_{k+1|k+1}(n|Z^{(k+1)}) \arrow[r] & \ldots
\end{tikzcd}$$

filter on target-number distributions

no measurement-to-track association required

Algorithms Derived Using Finite-Set Statistics



finite-set statistics

"classical" PHD filter

"classical" CPHD filter

multisensor PHD and CPHD filters

PHD/CPHD filters for unknown clutter and prob. detection

PHD filter for state-dependent clutter

PHD/CPHD smoothers

multiple motion model PHD/CPHD filters

unified Bayes filters for nontraditional data

unified tracking and sensor-bias estimation

? PHD/CPHD filters for superpositional sensors (w/ McGill)

Selected Applications

PHD and CPHD filters

ground moving target indicator (GMTI) radar multitarget tracking
(real-time “Bold Avenger” NATO exercise, 2007, FGAN/FKIE and EADS)

direction-of-arrival (DOA) tracking of dynamic signal sources

underwater active-acoustic tracking
(22 km, 6 hr, real-time pipeline tracking with a UUV, SeeByte and British Petroleum)

simultaneous localization and mapping (SLAM)

tracking multiple moving targets in terrain

group-target tracking

tracking in video images

multitarget tracking in distributed networks

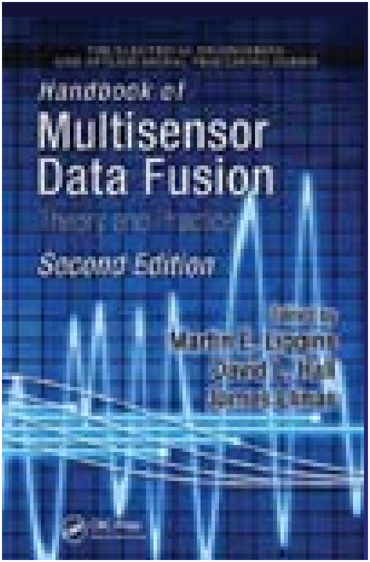
passive coherent localization (PCL) tracking

multisensor-multitarget sensor management

Primary References

Overview: Chapter 16

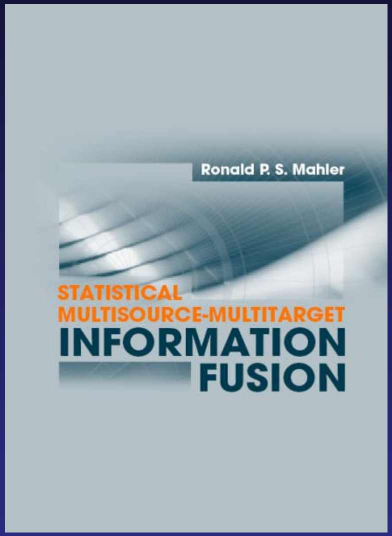
CRC Press 2008



Level 1 Fusion Textbook:

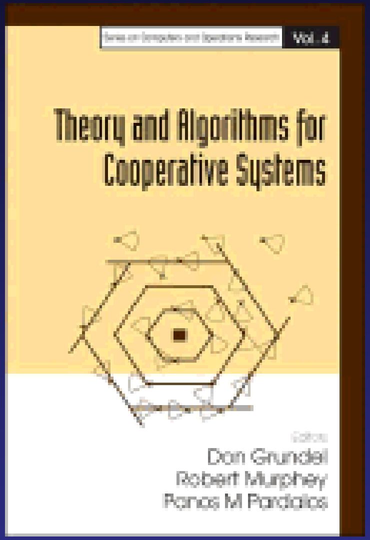
Artech House 2007

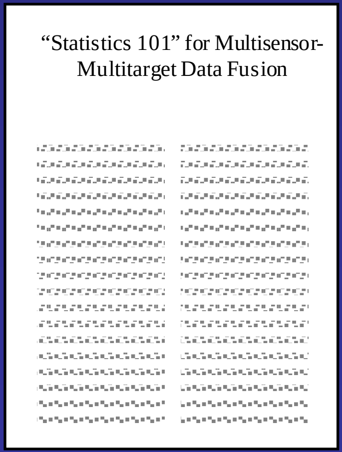
2007 Mignogna Data Fusion Award



Sensor Mgmt: Chapter 12

World Scientific 2004



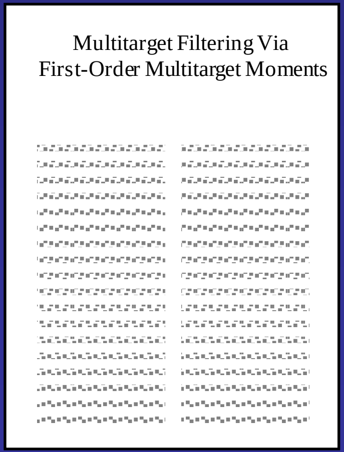


"Statistics 101" for Multisensor-Multitarget Data Fusion

invited tutorial:

IEEE AES Mag. 2004

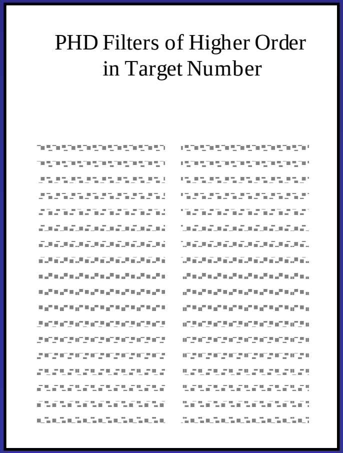
2005 IEEE AESS Mimno Award



Multitarget Filtering Via First-Order Multitarget Moments

PHD Filter Theory:

IEEE Trans. AES 2003



PHD Filters of Higher Order in Target Number

CPHD Filter Theory:

IEEE Trans. AES 2007

2007 IEEE AESS Carlton Award

The Random Set Filtering Website

RFS Filtering Website

- **United Kingdom mirror** **Prof. Daniel Clark, D.E.Clark@hw.ac.uk**
  - http://randomsets.eps.hw.ac.uk/index.html

- **Australian mirror** **Prof. Ba-Ngu Vo, ba-ngu.vo@uwa.edu.au**
  - http://randomsets.ee.unimelb.edu.au/index.html

RFS-SLAM Website

- **Prof. Martin Adams, martin@ing.uchile.cl**
  - http://www.cec.uchile.cl/~martin/Martin_research_18_8_11.html

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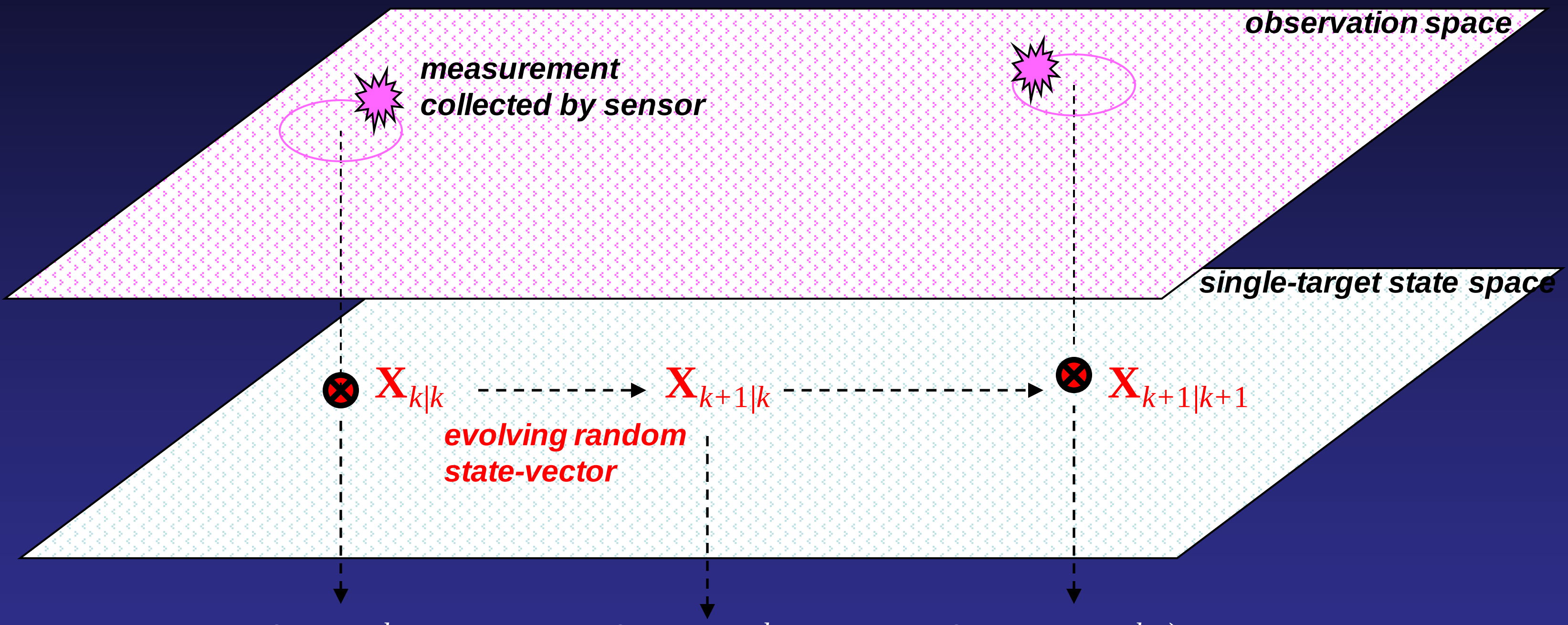
Foundation: The Single-Target Bayes Filter



observation space

measurement collected by sensor

single-target state space

$\mathbf{X}_{k|k}$ $\mathbf{X}_{k+1|k}$ $\mathbf{X}_{k+1|k+1}$

evolving random state-vector

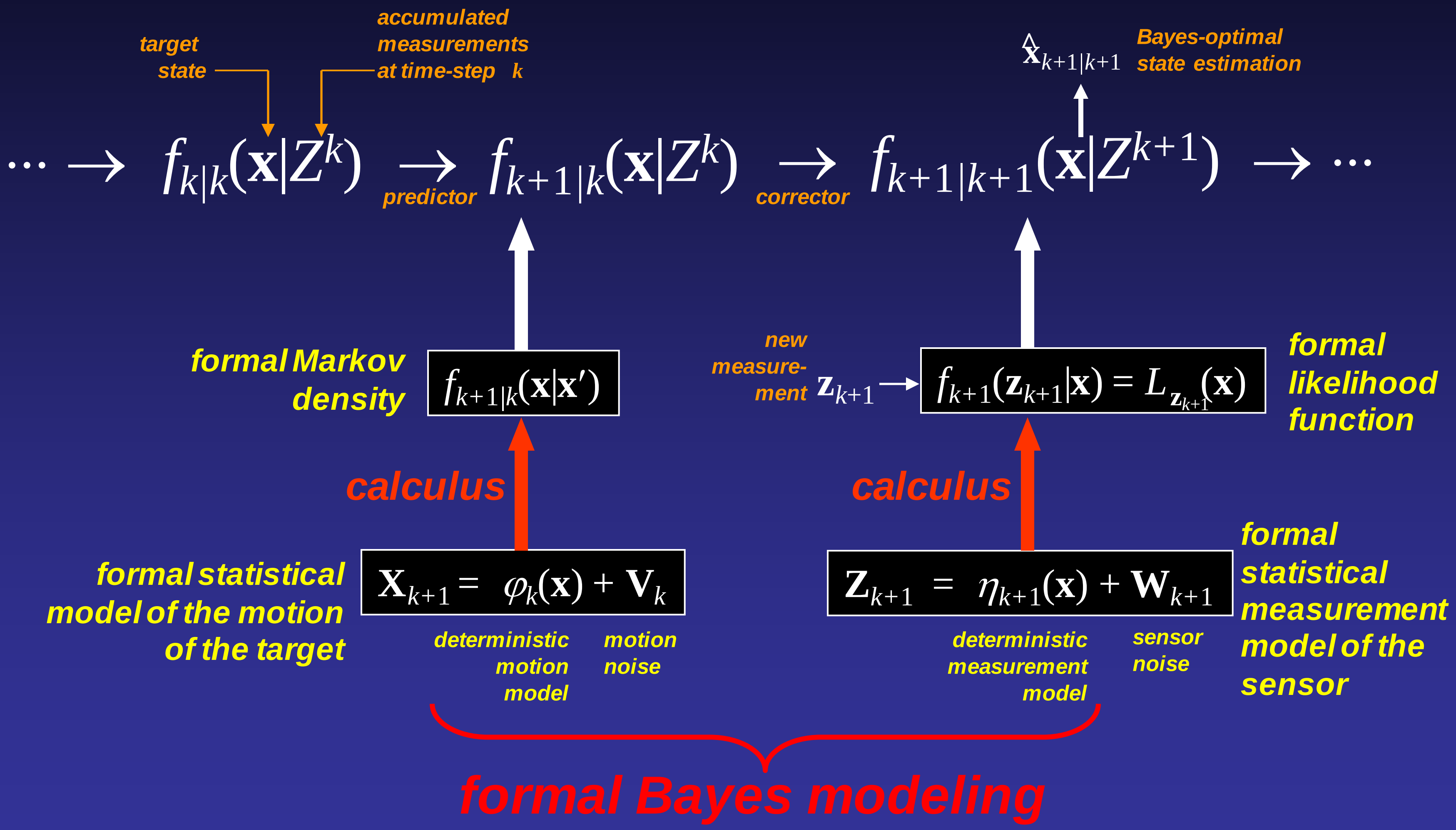
$$\cdots \longrightarrow f_{k|k}(\mathbf{x}|Z^k) \longrightarrow f_{k+1|k}(\mathbf{x}|Z^k) \longrightarrow f_{k+1|k+1}(\mathbf{x}|Z^{k+}) \longrightarrow \cdots$$

Bayes probability distribution on target state $\mathbf{x}$

time-update (predictor)

measurement-update (corrector)

Foundation: The Single-Target Bayes Filter, 2



target state

accumulated measurements at time-step k

$\hat{\mathbf{x}}_{k+1|k+1}$ Bayes-optimal state estimation

$$\cdots \rightarrow f_{k|k}(\mathbf{x}|Z^k) \xrightarrow[\text{predictor}]{} f_{k+1|k}(\mathbf{x}|Z^k) \xrightarrow[\text{corrector}]{} f_{k+1|k+1}(\mathbf{x}|Z^{k+1}) \rightarrow \cdots$$

formal Markov density $f_{k+1|k}(\mathbf{x}|\mathbf{x}')$

new measurement $\mathbf{z}_{k+1} \rightarrow f_{k+1}(\mathbf{z}_{k+1}|\mathbf{x}) = L_{\mathbf{z}_{k+1}}(\mathbf{x})$ formal likelihood function

calculus

calculus

formal statistical model of the motion of the target

$$\mathbf{X}_{k+1} = \varphi_k(\mathbf{x}) + \mathbf{V}_k$$

deterministic motion model

motion noise

$$\mathbf{Z}_{k+1} = \eta_{k+1}(\mathbf{x}) + \mathbf{W}_{k+1}$$

formal statistical measurement model of the sensor

deterministic measurement model

sensor noise

formal Bayes modeling

Special Case: The Kalman Filter

$$\begin{tikzcd}
{} \arrow[r] & N_{P_{k|k}}(\mathbf{x}-x_{k|k}) \arrow[r] & N_{P_{k+1|k}}(\mathbf{x}-\mathbf{x}_{k+1|k}) \arrow[r] & N_{P_{k+1|k+1}}(\mathbf{x}-\mathbf{x}_{k+1|k+1}) \arrow[r] & {} \\
 & & \boxed{N_{Q_k}(\mathbf{x}-F_k\mathbf{x}')} \arrow[u] & \boxed{N_{R_{k+1}}(\mathbf{z}_{k+1}-H_{k+1}\mathbf{x})} \arrow[u] & \\
 & & \boxed{\mathbf{x}_{k+1} = F_k\mathbf{x} + \mathbf{V}_k} \arrow[u] & \boxed{\mathbf{z}_{k+1} = H_{k+1}\mathbf{x} + \mathbf{W}_{k+1}} \arrow[u] &
\end{tikzcd}$$

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Mathematical Core of Single-Target Bayes Filter

formal Bayes modeling

(prevent model-mismatch due to a heuristics-generated fictitious sensor!)

$$p_{\mathbf{X}}(S)$$

probability measures / probability-mass functions of random vector $\mathbf{X}$

$$\frac{\delta p_{\mathbf{X}}}{\delta \mathbf{x}}(S)$$

integrals & derivatives

$$\int f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}$$

single-target filtering

$$f_{\mathbf{X}}(\mathbf{x})$$

probability density functions of random vector $\mathbf{X}$

ordinary calculus permits derivation of concrete algorithm formulas

Mathematical Core of Multitarget Bayes Filter

formal Bayes modeling

(prevent model-mismatch due to a heuristics-generated fictitious sensor!)

$\beta_\Psi(S)$ ***belief-mass functions** of random finite set* Ψ

$$\frac{\delta\beta_\Psi}{\delta Y}(S)$$

set integrals & derivatives

$$\int f_\Psi(Y)\delta Y$$

multitarget filtering

$f_\Psi(Y)$ ***multi-object probability density** functions of random finite set* Ψ

$$\frac{\delta G_\Psi}{\delta Y}[h]$$

set integrals & functional derivatives

$$\int f_\Psi(Y)\delta Y$$

principled approximation

$G_\Psi[h]$ ***probability generating functionals** (p.g.fl.'s) of random finite set* Ψ

Multi-Object Calculus, 1

set integrals

$$\int f_\Psi(Y)\delta Y = \sum_{n=0}^{\infty} \frac{1}{n!} \int f_\Psi(\{\mathbf{y}_1,\ldots,\mathbf{y}_n\}) d\mathbf{y}_1 \cdots d\mathbf{y}_n$$

functional power

$$h^Y = \prod\nolimits_{\mathbf{y}\in Y} h(\mathbf{y})$$

probability generating functional (p.g.fl.)

$$G_\Psi[h] = \int h^Y \cdot f_\Psi(Y)\delta Y$$

set integral

Dirac delta function

functional derivatives

$$\frac{\delta G_\Psi}{\delta \mathbf{y}}[h] = \lim_{\varepsilon\to 0} \frac{G_\Psi[h+\varepsilon\cdot\delta_{\mathbf{y}}] - G_\Psi[h]}{\varepsilon}$$

$$\frac{\delta G_\Psi}{\delta Y}[h] = \frac{\delta^n G_\Psi}{\delta\mathbf{y}_1\cdots\delta\mathbf{y}_n}[h]$$

Multi-Object Calculus, 2

multitarget distribution

$$f_\Psi(Y) = \boxed{\frac{\delta G_\Psi}{\delta Y}[0]}$$

functional derivative

probability hypothesis density (PHD)

$$D_\Psi(\mathbf{y}) = \frac{\delta G_\Psi}{\delta \mathbf{y}}[1] = \int f_\Psi(\{\mathbf{y}\} \cup Y)\delta Y$$

multitarget calculus permits derivation of concrete algorithm formulas

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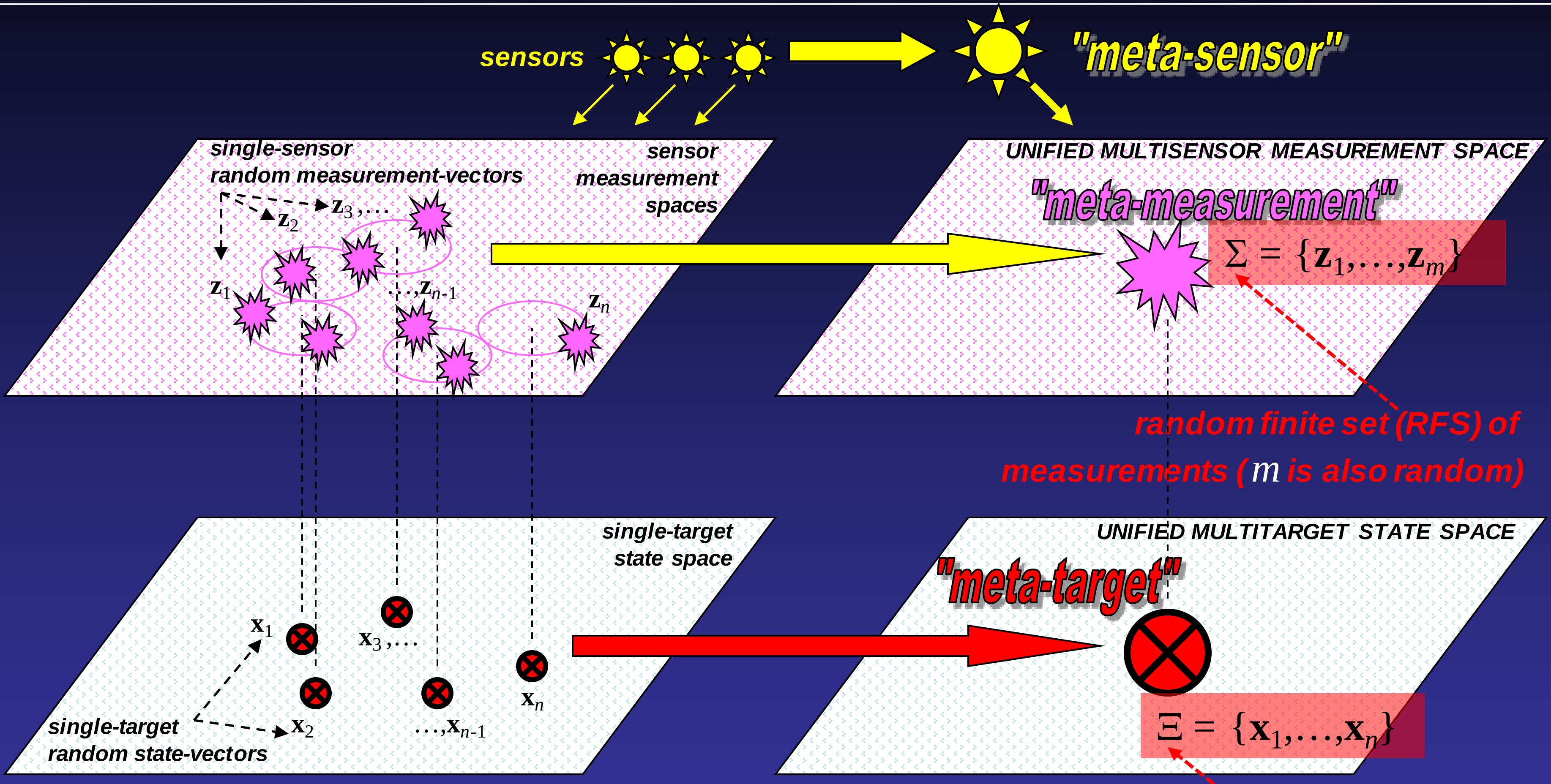
Multisensor-Multitarget Statistics



sensors

"meta-sensor"

single-sensor random measurement-vectors

sensor measurement spaces

UNIFIED MULTISENSOR MEASUREMENT SPACE

"meta-measurement"

$\Sigma = \{z_1, \ldots, z_m\}$

random finite set (RFS) of measurements (m is also random)

single-target state space

single-target random state-vectors

UNIFIED MULTITARGET STATE SPACE

"meta-target"

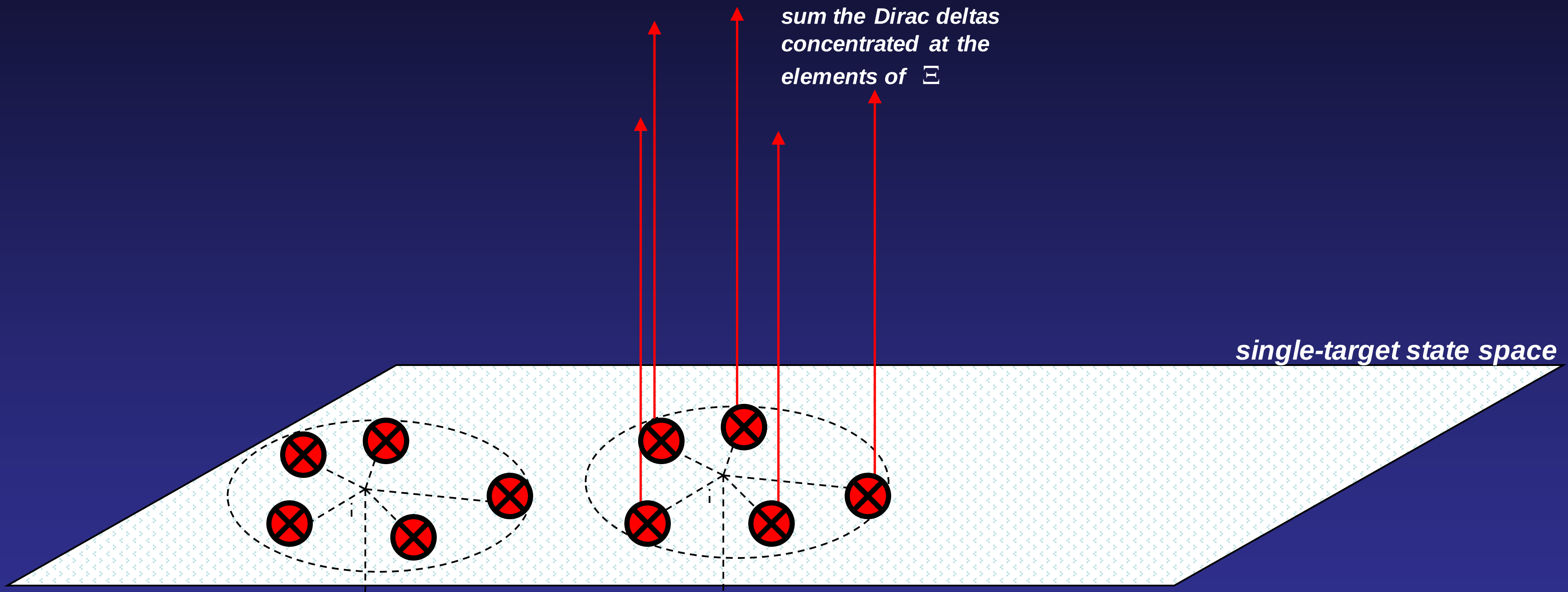
$\Xi = \{x_1, \ldots, x_n\}$

random finite set (RFS) of targets (n is also random)

multisensor, multitarget transformed to single-sensor, single-target

Statistical Representation of a Multitarget System

equivalent notations for a (multidimensional) simple point process



sum the Dirac deltas concentrated at the elements of Ξ

single-target state space

Ξ

random state-set

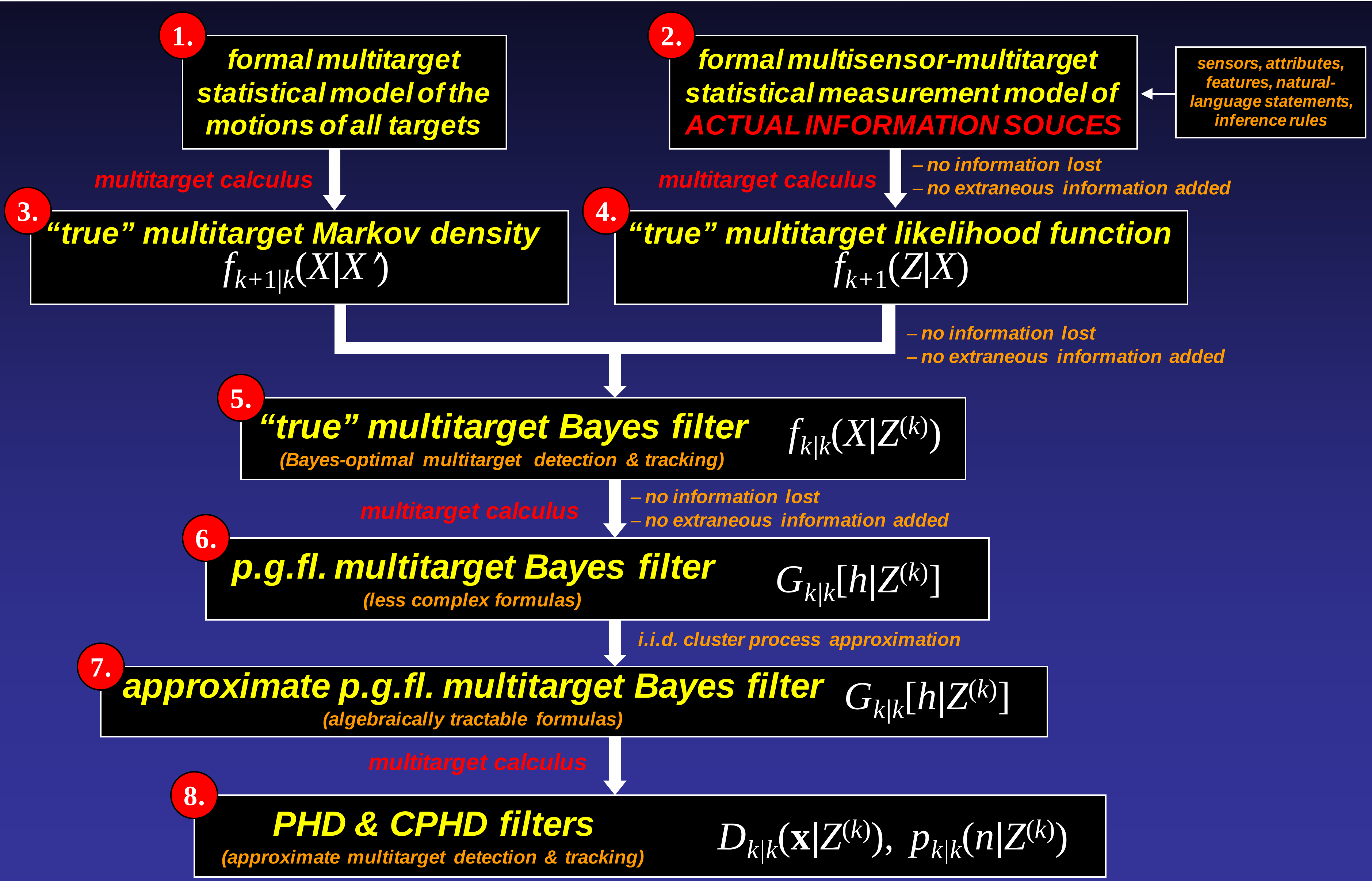
a particular formulation of a random point process (stochastic geometry formulation

$$\delta_\Xi(\mathbf{x}) = \sum_{\mathbf{y}\in\Xi} \delta_\mathbf{y}(\mathbf{x})$$

random density/ random measure

(point process theory formulation)

Systematic Multitarget Modeling & Approximation



1. *formal multitarget statistical model of the motions of all targets*

2. *formal multisensor-multitarget statistical measurement model of ACTUAL INFORMATION SOUCES*

sensors, attributes, features, natural-language statements, inference rules

multitarget calculus

multitarget calculus

– no information lost
– no extraneous information added

3. *"true" multitarget Markov density* $f_{k+1|k}(X|X')$

4. *"true" multitarget likelihood function* $f_{k+1}(Z|X)$

– no information lost
– no extraneous information added

5. *"true" multitarget Bayes filter* $f_{k|k}(X|Z^{(k)})$

(Bayes-optimal multitarget detection & tracking)

multitarget calculus

– no information lost
– no extraneous information added

6. *p.g.fl. multitarget Bayes filter* $G_{k|k}[h|Z^{(k)}]$

(less complex formulas)

i.i.d. cluster process approximation

7. *approximate p.g.fl. multitarget Bayes filter* $G_{k|k}[h|Z^{(k)}]$

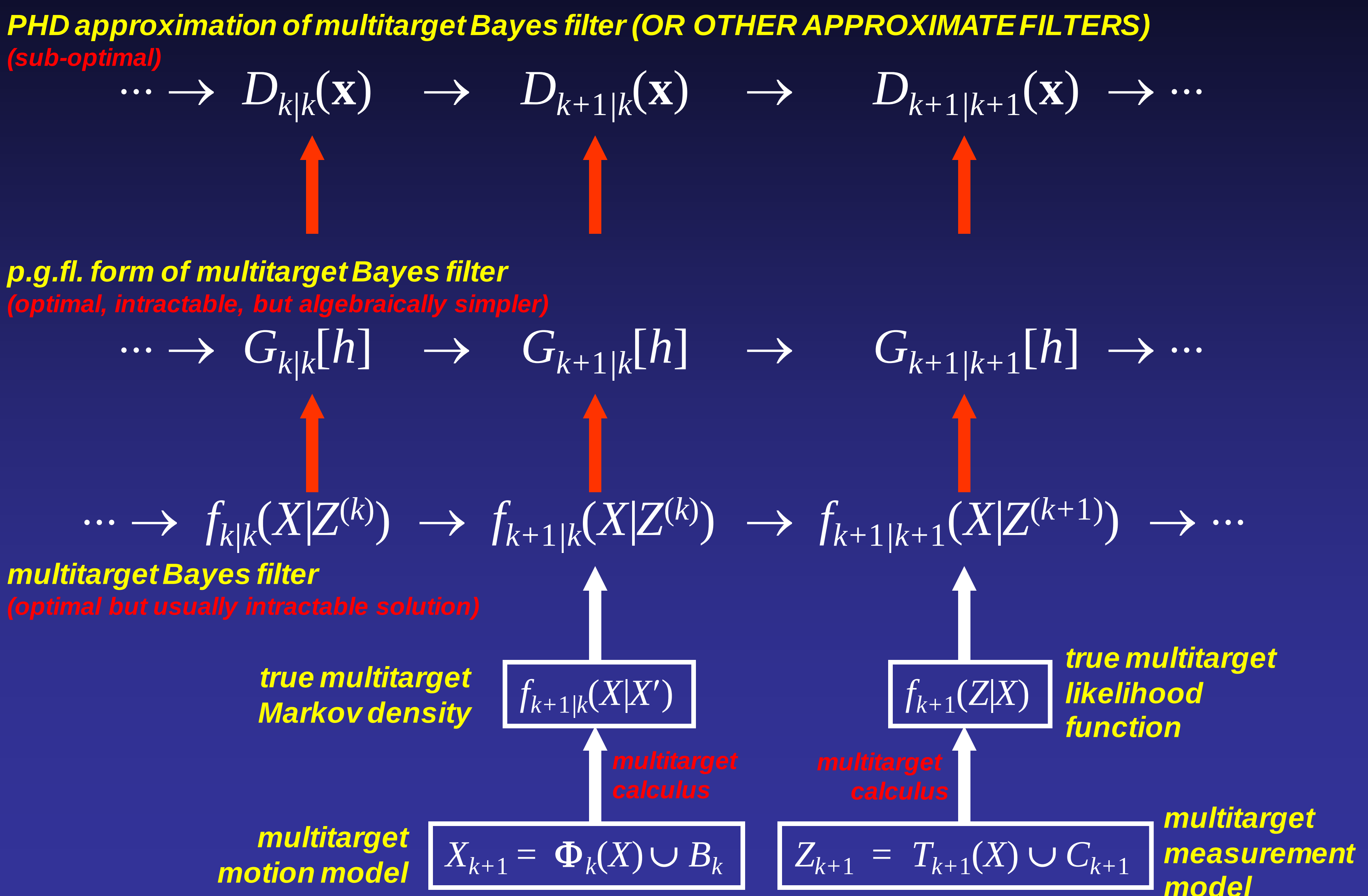
(algebraically tractable formulas)

multitarget calculus

8. *PHD & CPHD filters* $D_{k|k}(\mathbf{x}|Z^{(k)}),\ p_{k|k}(n|Z^{(k)})$

(approximate multitarget detection & tracking)

Systematic Multitarget Modeling & Approximation, 2



PHD approximation of multitarget Bayes filter (OR OTHER APPROXIMATE FILTERS)

(sub-optimal)

$$\cdots \to D_{k|k}(\mathbf{x}) \to D_{k+1|k}(\mathbf{x}) \to D_{k+1|k+1}(\mathbf{x}) \to \cdots$$

p.g.fl. form of multitarget Bayes filter

(optimal, intractable, but algebraically simpler)

$$\cdots \to G_{k|k}[h] \to G_{k+1|k}[h] \to G_{k+1|k+1}[h] \to \cdots$$

$$\cdots \to f_{k|k}(X|Z^{(k)}) \to f_{k+1|k}(X|Z^{(k)}) \to f_{k+1|k+1}(X|Z^{(k+1)}) \to \cdots$$

multitarget Bayes filter

(optimal but usually intractable solution)

true multitarget Markov density

$$\boxed{f_{k+1|k}(X|X')}$$

$$\boxed{f_{k+1}(Z|X)}$$

true multitarget likelihood function

multitarget calculus

multitarget calculus

multitarget motion model

$$\boxed{X_{k+1} = \Phi_k(X) \cup B_k}$$

$$\boxed{Z_{k+1} = T_{k+1}(X) \cup C_{k+1}}$$

multitarget measurement model

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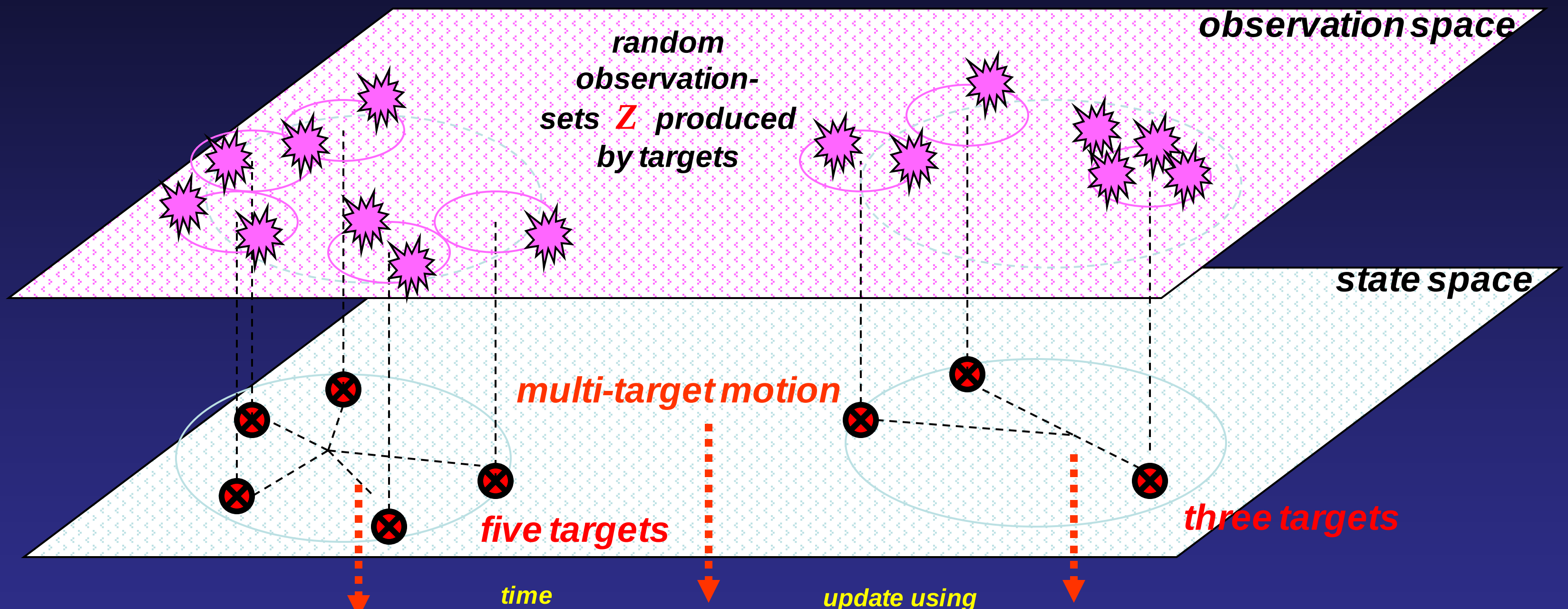
The Multitarget Bayes Filter



multitarget Bayes filter $\longrightarrow f_{k|k}(X|Z^{(k)}) \xrightarrow{\text{time prediction}} f_{k+1|k}(X|Z^{(k)}) \xrightarrow{\text{update using new data}} f_{k+1|k+1}(X|Z^{(k+1)}) \longrightarrow \cdots$

multitarget probability density function

$f_{k|k}(\emptyset|Z^{(k)})$ *(probability that there are no targets present)*

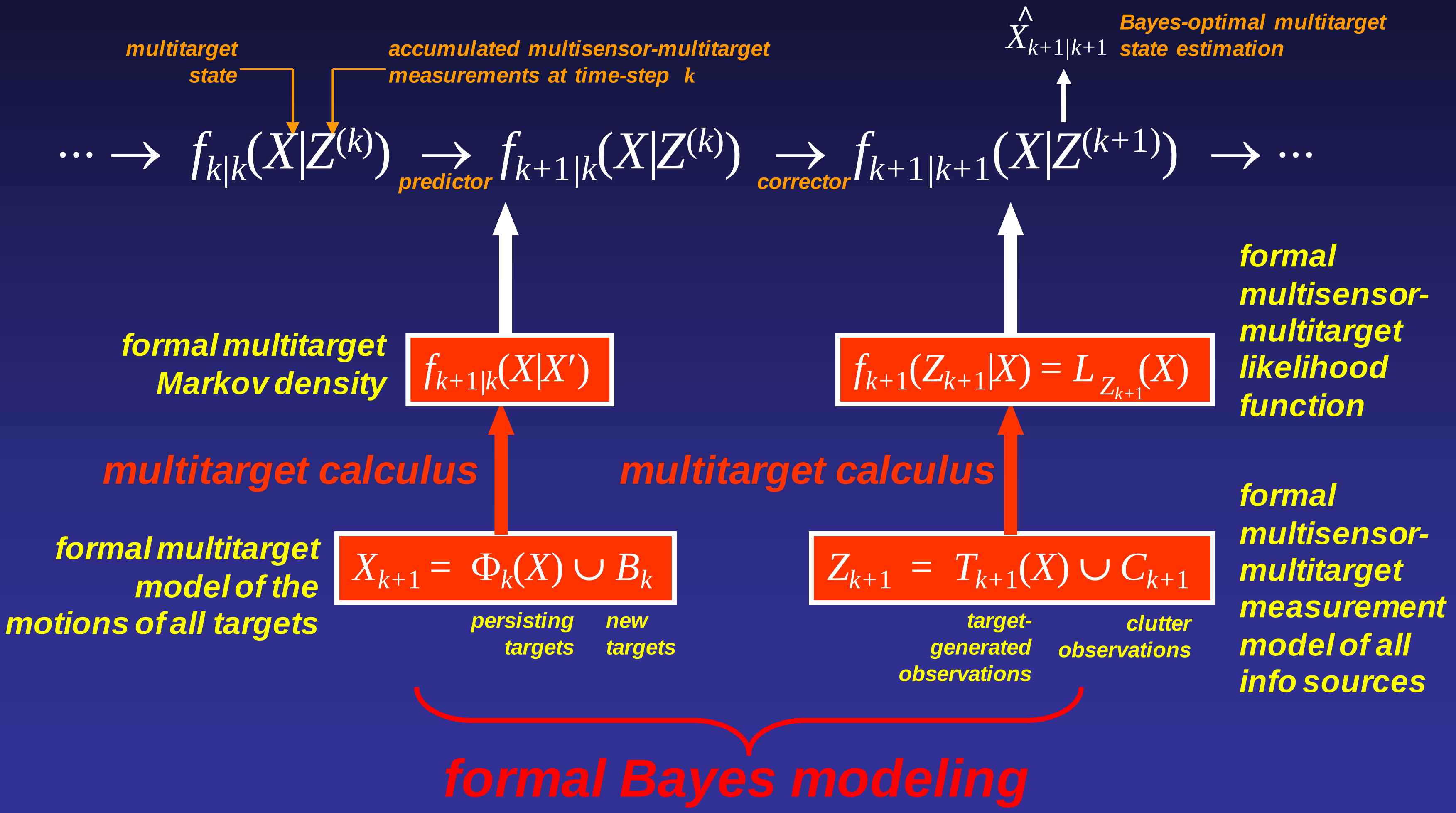
$f_{k|k}(\{\mathbf{x}_1\}$ *(probability of one target with state* $\mathbf{x}_1$*)*

$f_{k|k}(\{\mathbf{x}_1 ,\mathbf{x}_2\}|Z^{(k)})$ *(probability of two targets with states* $\mathbf{x}_1 ,\mathbf{x}_2$*)*

$\cdots$

$f_{k|k}(\{\mathbf{x}_1 ,\ldots, \mathbf{x}_n\}|Z^{(k)})$ *(probability of* n *targets with states* $\mathbf{x}_1 ,\ldots, \mathbf{x}_n$*)*

The Multitarget Bayes Filter, 2



multitarget state

accumulated multisensor-multitarget measurements at time-step k

$\hat{X}_{k+1|k+1}$ *Bayes-optimal multitarget state estimation*

$$\cdots \to f_{k|k}(X|Z^{(k)}) \xrightarrow[\text{predictor}]{} f_{k+1|k}(X|Z^{(k)}) \xrightarrow[\text{corrector}]{} f_{k+1|k+1}(X|Z^{(k+1)}) \to \cdots$$

formal multitarget Markov density

$$f_{k+1|k}(X|X')$$

$$f_{k+1}(Z_{k+1}|X) = L_{Z_{k+1}}(X)$$

formal multisensor-multitarget likelihood function

multitarget calculus

multitarget calculus

formal multitarget model of the motions of all targets

$$X_{k+1} = \Phi_k(X) \cup B_k$$

persisting targets

new targets

$$Z_{k+1} = T_{k+1}(X) \cup C_{k+1}$$

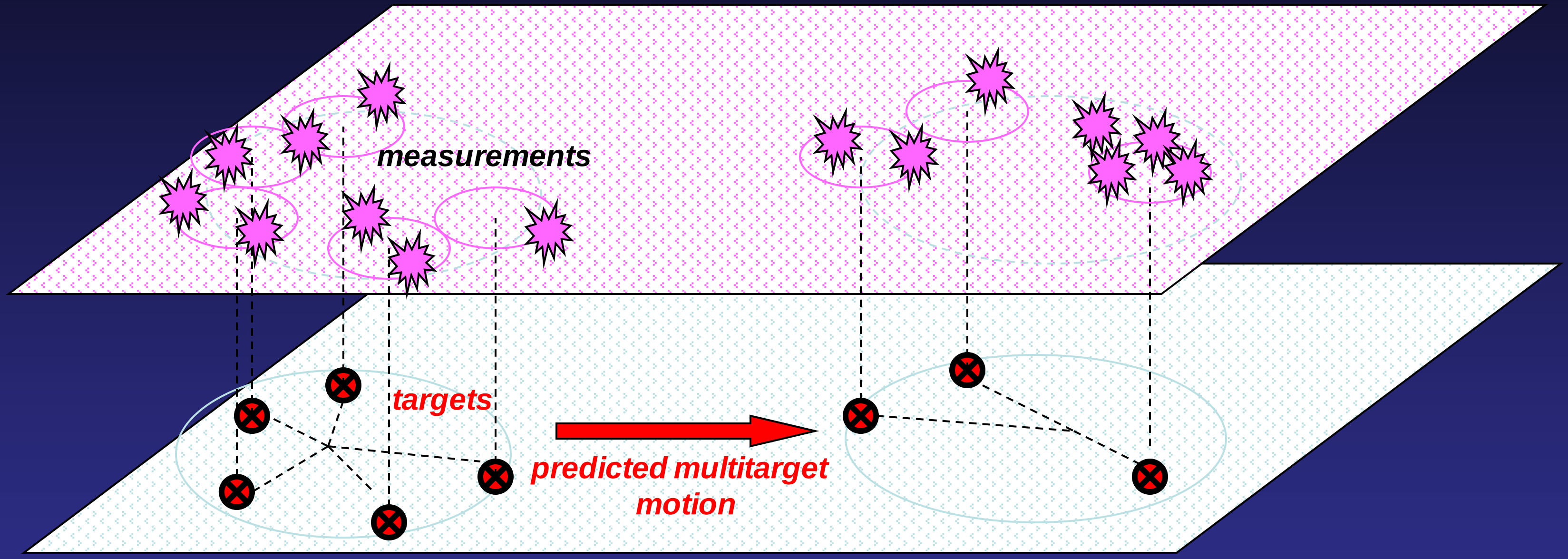
target-generated observations

clutter observations

formal multisensor-multitarget measurement model of all info sources

formal Bayes modeling

Conventional Multitarget Filtering (multi-hypothesis correlation trackers)



multitarget Bayes filter (optimal) → $f_{k|k}(X|Z^{(k}$ … $)$ … $(k+1))$ → …

time

update using

intractable in general!

approximation

approximation

approximation

multi-hypothesis correlator tracker →

track … hypothesis list

time

predicted … hypotheses

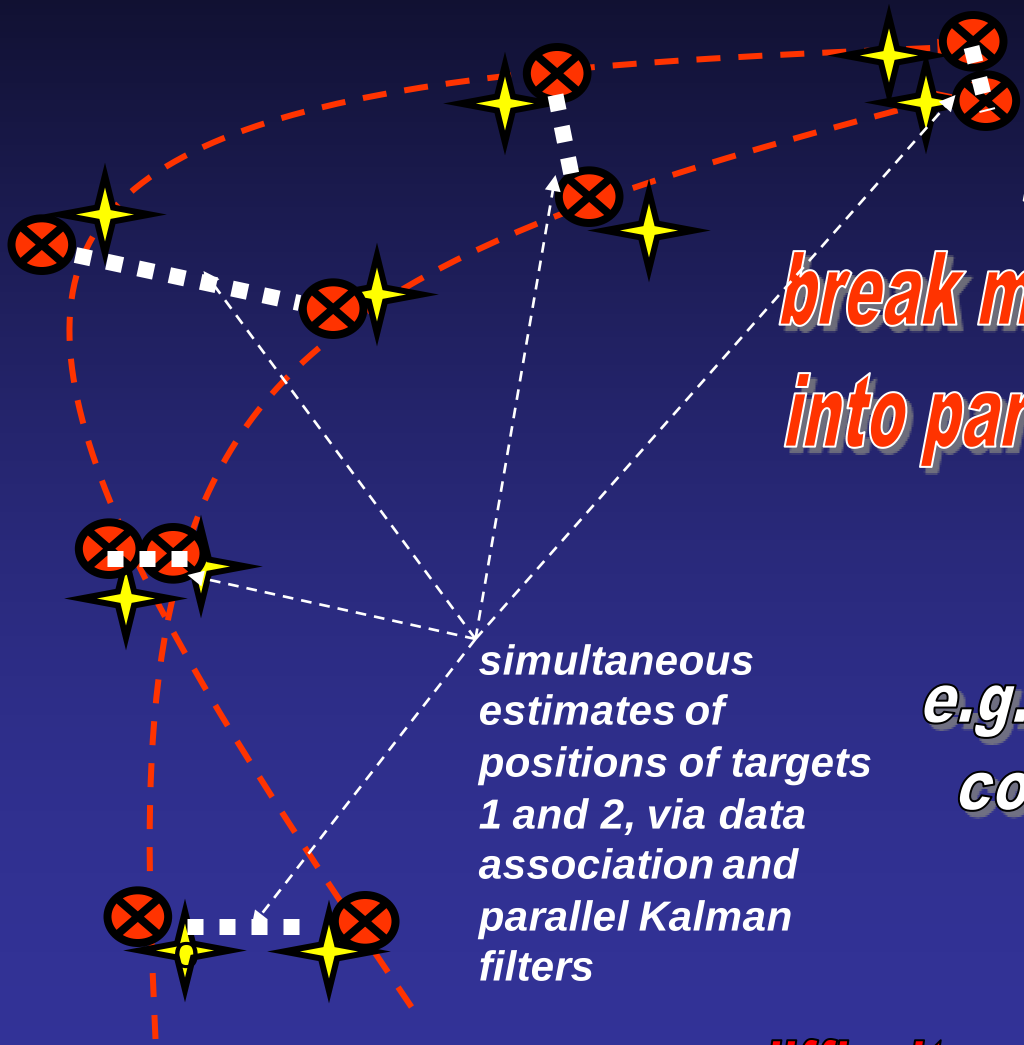
update us…

…ata

updated … hypotheses

"tracking barrier" challenges

Conventional Multitarget Tracking



simultaneous estimates of positions of targets 1 and 2, via data association and parallel Kalman filters

bottom-up: break multitarget problem into parallel single-target problems

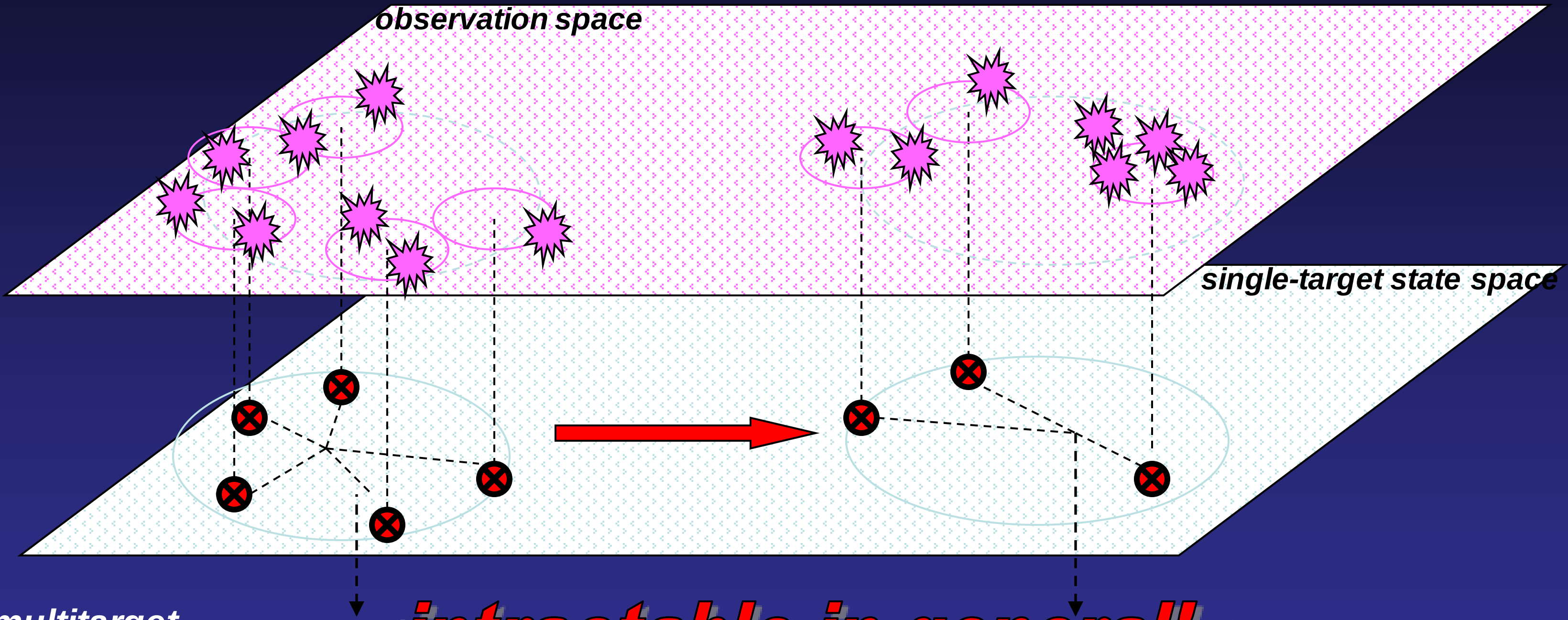
e.g., multi-hypothesis correlator trackers (MHTs)

difficulty: combinatorially complex

Topics

- Overview
- Single-sensor, single-target Bayes filter
- RFS multi-object calculus
- RFS modeling of multisensor-multitarget systems
- Multisensor-multitarget recursive Bayes filter
- **Approximate multitarget Bayes filters**
- Conclusions

Probability Hypothesis Density (PHD) Filter



multitarget Bayes filter (optimal) $\cdots \longrightarrow f_{k|k}(X|Z^{(k}$ … $)) \longrightarrow$ … $^{+1)}) \longrightarrow \cdots$

intractable in general!

compress to multi-target first moment — compress to multi-target first moment — compress to multi-target first moment

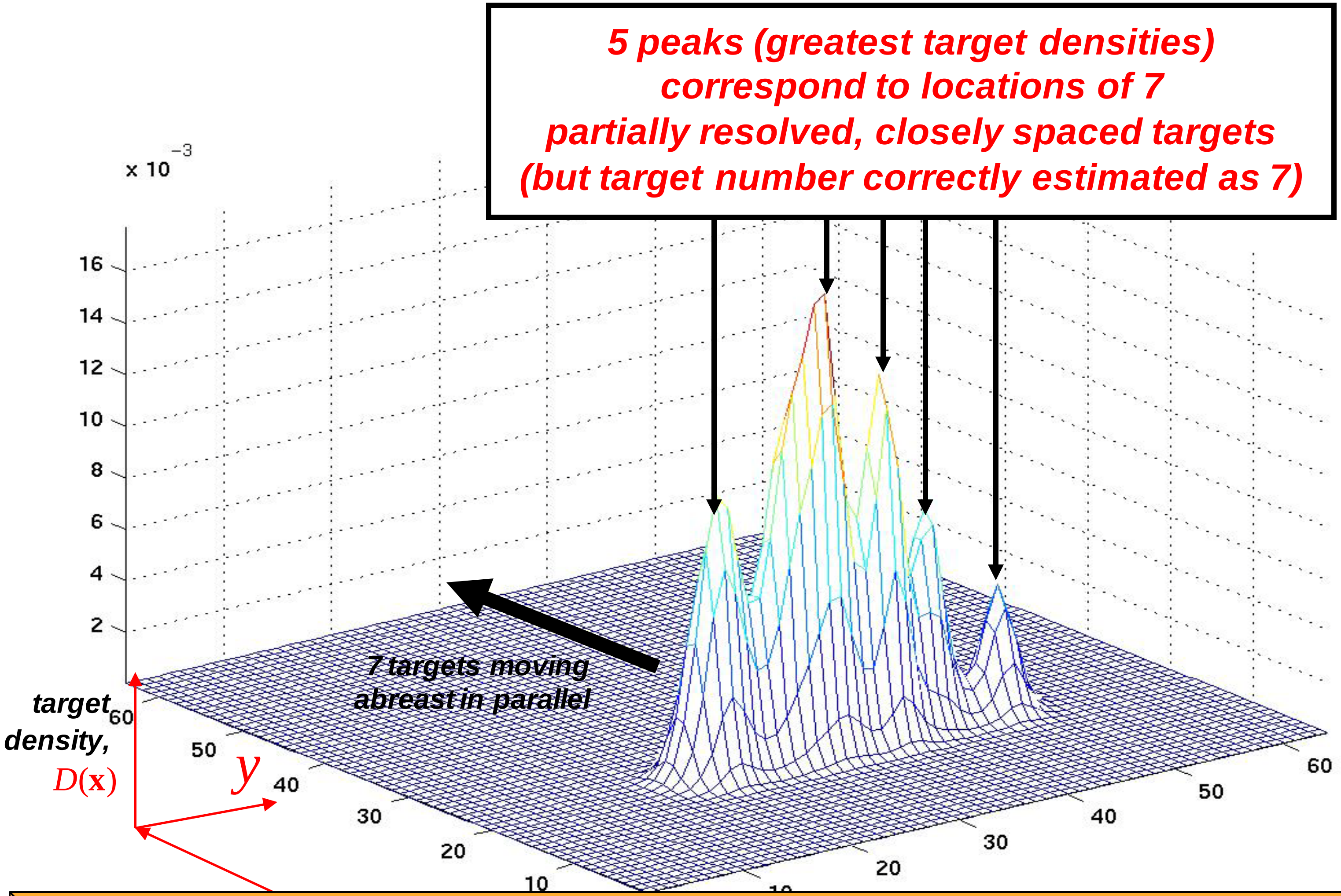
1st-moment (PHD) filter $\longrightarrow D_{k|k}(\mathbf{x}|Z^{(k)}) \longrightarrow D_{k+1|k}(\mathbf{x}|Z^{(k)}) \longrightarrow D_{k+1|k+1}(\mathbf{x}|Z^{(k+1)}) \longrightarrow \ldots$

time-update step (predictor)

data-update step (corrector)

computational complexity $O(mn)$, n ***= no. targets,*** m ***= no. measurements***

Example of a PHD



5 peaks (greatest target densities) correspond to locations of 7 partially resolved, closely spaced targets (but target number correctly estimated as 7)

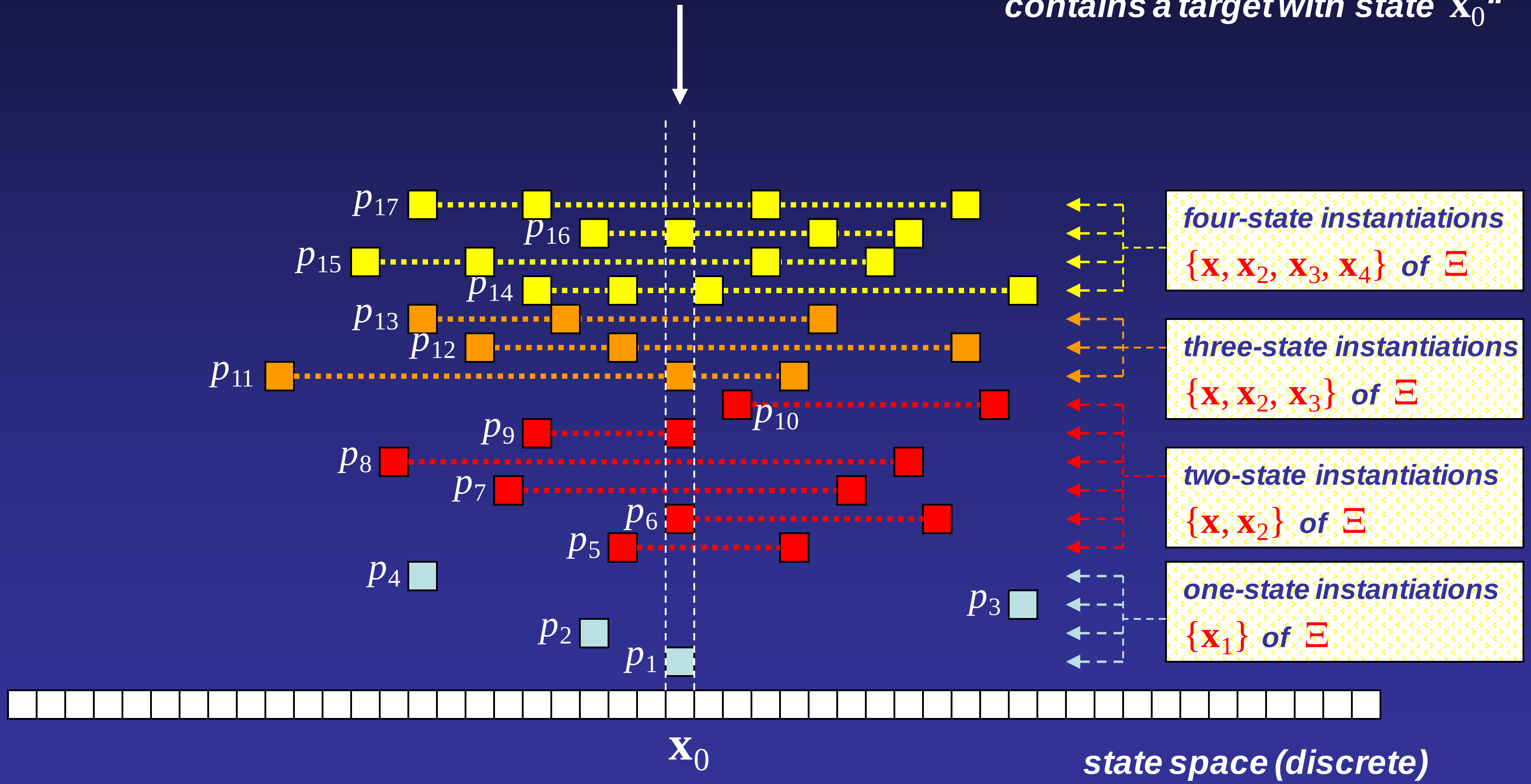
7 targets moving abreast in parallel

PHD represents targets first as a group, then as individual targets

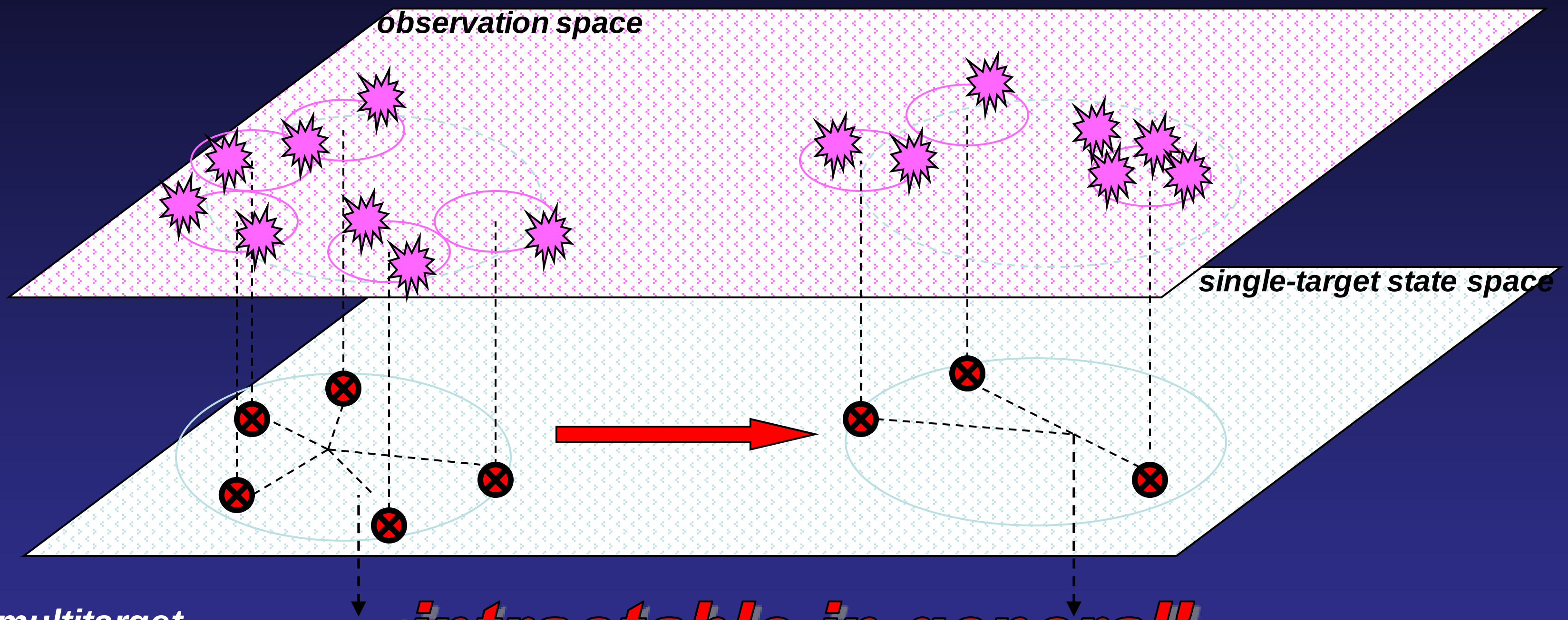
Probability Hypothesis Density: Picture

$$D_\Xi(\mathbf{x}_0) = \Pr(\mathbf{x}_0 \in \Xi) = p_1 + p_6 + p_9 + p_{11} + p_{16}$$

= *probability of the hypothesis: "the multitarget system contains a target with state* $\mathbf{x}_0$*"*



The Cardinalized PHD (CPHD) Filter



multitarget Bayes filter … → $f_{k|k}(X|Z^{(k}$ **intractable in general!** $^{+1)})$ → …

compress compress compress

CPHD filter

$$\begin{tikzcd}
\cdots \arrow[r] & D_{k|k}(\mathbf{x}|Z^{(k)}) \arrow[r] & D_{k+1|k}(\mathbf{x}|Z^{(k)}) \arrow[r] \arrow[dr] & D_{k+1|k+1}(\mathbf{x}|Z^{(k+1)}) \arrow[r] & \cdots \\
\cdots \arrow[r] & p_{k|k}(n|Z^{(k)}) \arrow[r] & p_{k+1|k}(n|Z^{(k)}) \arrow[r] \arrow[ur] & p_{k+1|k+1}(n|Z^{(k+1)}) \arrow[r] & \cdots
\end{tikzcd}$$

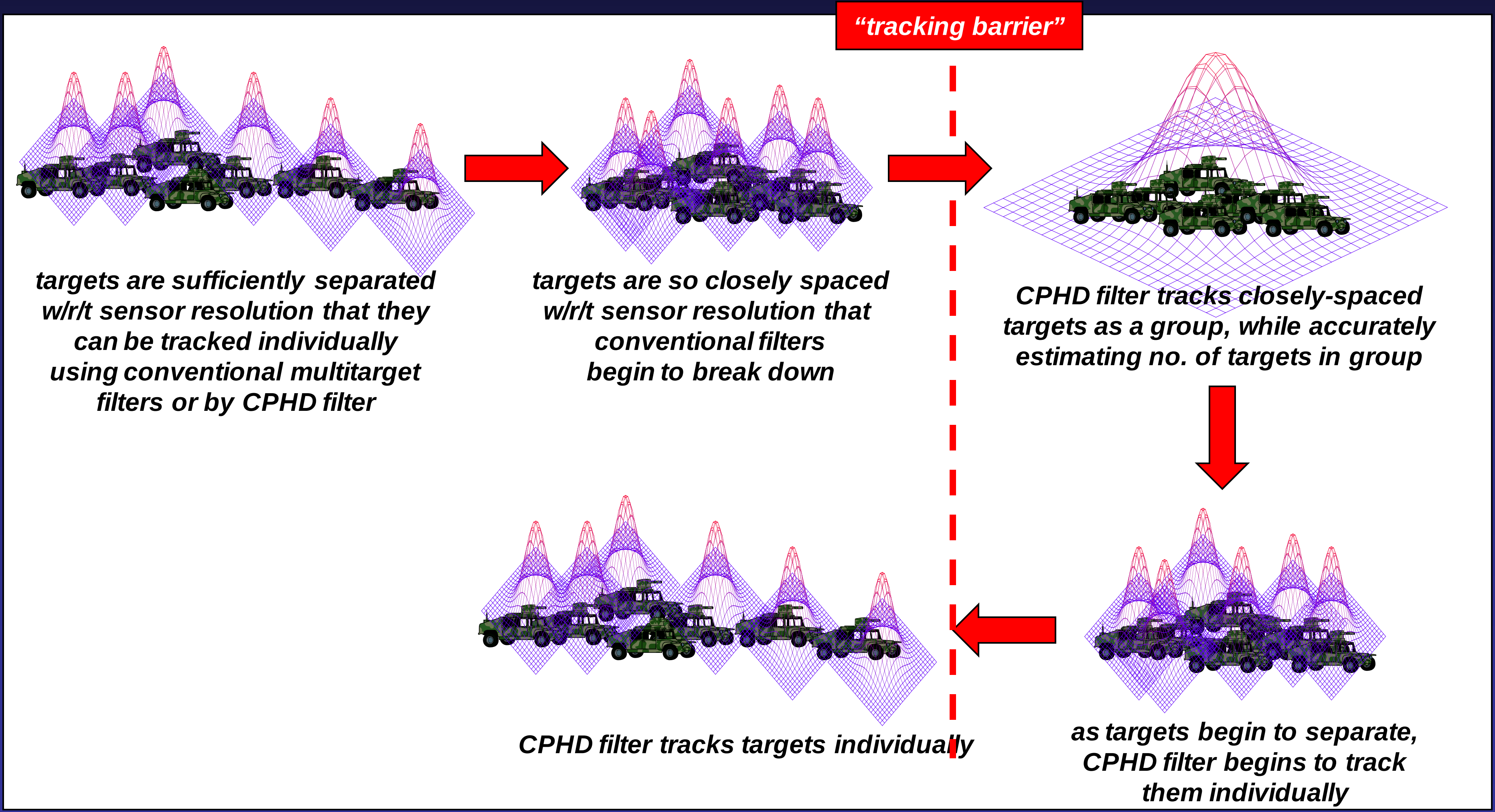
PHD

target-no. distribution

computational complexity $O(m^3n)$, n = **no. targets,** m = **no. measurements**

The PHD/CPHD Filters and Closely-Spaced Targets

PHD / CPHD filters permit detection and tracking of multiple targets when conventional approaches begin to perform poorly



targets are sufficiently separated w/r/t sensor resolution that they can be tracked individually using conventional multitarget filters or by CPHD filter

targets are so closely spaced w/r/t sensor resolution that conventional filters begin to break down

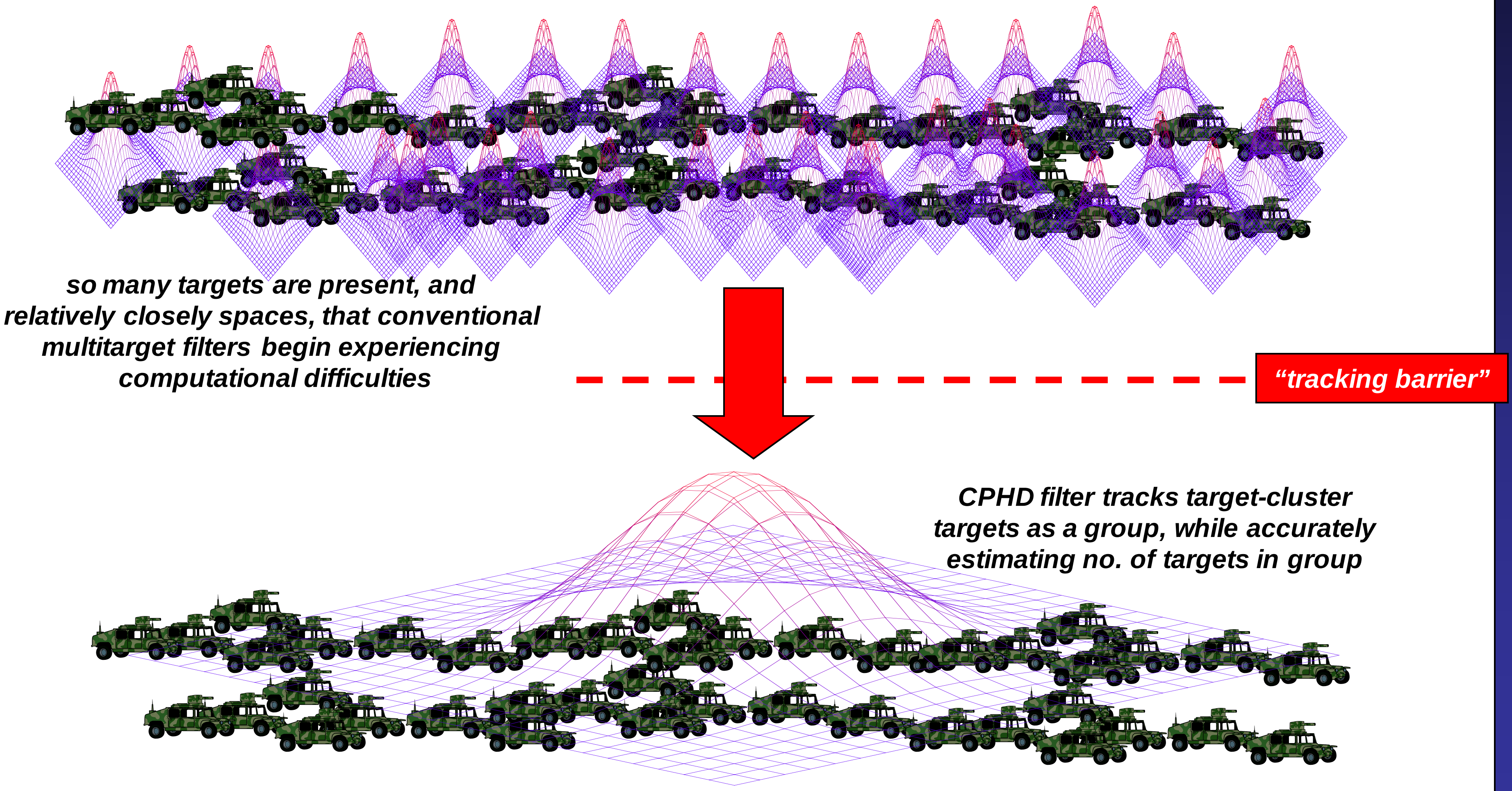
CPHD filter tracks closely-spaced targets as a group, while accurately estimating no. of targets in group

as targets begin to separate, CPHD filter begins to track them individually

CPHD filter tracks targets individually

The PHD/CPHD Filter and Large Target Clusters

PHD / CPHD filter permits tracking of dense target clusters when conventional approaches begin to perform poorly



so many targets are present, and relatively closely spaces, that conventional multitarget filters begin experiencing computational difficulties

"tracking barrier"

CPHD filter tracks target-cluster targets as a group, while accurately estimating no. of targets in group

Conclusions

- **Finite-set statistics is the basis for a new, Bayes-optimal, and theoretically unified approach to SLAM**

- **Permits a more principled way of approaching SLAM**

- **Promising new SLAM algorithms**

- **For more details on finite-set statistics**
  - **Handbook of Multisensor Data Fusion, *2nd Ed., Chapter 16***
  - **Statistical Multisource-Multitarget Information Fusion**
  - ***"Statistics '101' for multisensor, multitarget data fusion"***
  - ***papers listed in bibliography of the workshop paper***

Thank You!